SAMPLE CONTENT

PERFECT

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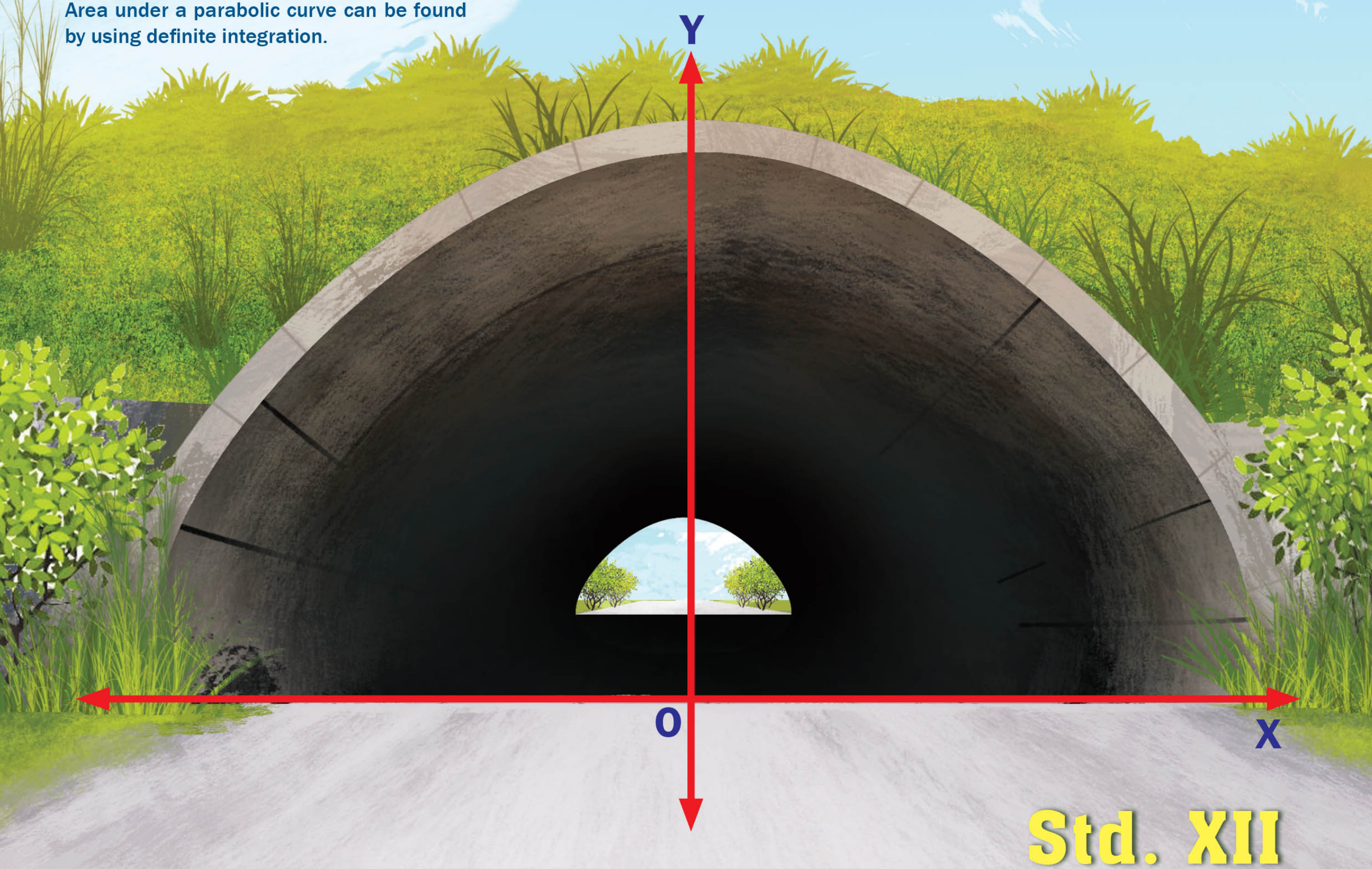
Part II

Based on Latest Paper Pattern and Textbook

Application of definite integration:
Area under a parabolic curve can be found by using definite integration.



Std. XII
(Sci. & Arts)

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PERFECT
MATHEMATICS & STATISTICS Part - II
Std. XII Sci. & Arts

Salient Features

- Written as per **Latest Board Paper Pattern**
- Exhaustive coverage of entire syllabus
- Covers answers to all exercises and miscellaneous exercises given in the textbook.
- All derivations and theorems covered
- Tentative marks allocation for all problems
- Relevant Previous Years' Board Questions: **March 1996 to July 2023**
- Each chapter contains:
  - 'Precise Theory' for every topic
  - 'MCQs' for practice
  - 'Topic Test' at the end of each chapter for self-assessment
  - 'Competitive Corner' to give the glimpse of prominent competitive examinations
- Includes Important Features for holistic learning:
  - ***Illustrative Example***
  - ***Smart Check***
  - ***Smart Recap***
- Q.R. codes provide:
  - Solutions of Topic Tests
  - Model Question Paper along with Solution

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PREFACE

Perfect Mathematics & Statistics Part – II, Std. XII Sci. & Arts is intended for every Maharashtra State Board aspirant of Std. XII, Science & Arts. The scope, sequence, and level of the book are consistent with the latest textbook released by Maharashtra State board.

At this crucial juncture in their lives, when the students are grappling with the pressures of cracking a career-defining board examination, we wanted to create a book that not only develops the necessary knowledge, tools, and skills required to excel in the examination, but also enables students to appreciate the beauty of the subject and piques their curiosity.

We believe that students respond favourably to meaningful content, if it is presented in a way that is easy to read and understand, rather than being mired down with facts and information. Consequently, we have always placed the highest priority on writing clear and lucid explanations of fundamental concepts. Moreover, special care has been taken to ensure that the topics are presented in a logical order.

The primary purpose of this book is to assist the students in preparing for the board examination. However, this is closely linked to other goals: to exemplify how important and how incredibly interesting mathematics is, and to help the student become an expert thinker and problem solver.

Practice, practice & more practice is the key to score high in mathematics!

To help the students, this book amalgamates problems that are rich in both variety and number which provides the student with ample practice, ensuring mastery of each concept.

The scope of the book extends beyond the State Board examination as it also offers a plethora of Multiple Choice Questions (MCQs) in order to familiarize the students with the pattern of competitive examinations.

In addition, the chapter-test have been carefully crafted to focus on concepts, thus providing the students with a quick opportunity for self-assessment and giving them an increased appreciation of chapter-preparedness.
'***Model Question Paper'*** based on latest paper pattern is provided along with solution which can be accessed through QR code to help students assess their preparedness for final board examination.

*Our **Perfect Mathematics & Statistics Part – II, Std. XII Sci. & Arts** adheres to our vision and achieves several goals: **building concepts**, **developing competence to solve problems, recapitulation, self-study, self-assessment** and **student engagement**—all while encouraging students toward cognitive thinking.*

The flow chart on the adjacent page will walk you through the key features of the book and elucidate how they have been carefully designed to maximize the student learning.

We hope the book benefits the learner as we have envisioned.

\- Publisher

Edition: Sixth

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.
Please write to us on: mail@targetpublications.org

KEY FEATURES

Illustrative Example

Illustrative Example provides a detailed approach towards solving a problem.

Smart Check

Smart Check is a technique to verify the answers. This is our attempt to cross-check the accuracy of the answer. Smart check is indicated by ✓ symbol

One Mark Questions

These questions require very short solutions with direct application of mathematical concepts.

Topic Test

Topic Test covers questions from the chapter for self-evaluation purpose. This is our attempt to provide the students with revision and help them assess their knowledge of chapter.

Competitive Corner

Competitive Corner presents questions from prominent [JEE (Main), MHT CET] competitive exams based entirely on the syllabus covered in the chapter. This is our attempt to introduce students to MCQs asked in competitive exams.

Smart Recap

Smart Recap given at the end of the book includes important and relevant concepts and formulae in the chapters. This is our attempt to offer students a handy tool to solve problems and ace the last minute revision.

QR Codes

QR code provides:

i. Solutions to the Topic Tests

ii. Model Question Paper with Solution

Board Questions

Includes relevant Board questions from March 1996 to July 2023

PAPER PATTERN

- There will be a single paper of 80 Marks in Mathematics and Statistics.
- Duration of the paper will be 3 hours.

Section A: **(20 Marks)**

This section will contain Multiple Choice Questions and Very Short Answer(VSA) type of questions.
There will be 8 MCQs, each carrying two marks
and 4 VSA type of questions, each carrying one mark.
Students will have to attempt all these questions.

Section B: **(16 Marks)**

This section will contain 12 Short Answer (SA-I) type of questions, each carrying 2 marks.
Students will have to attempt any 8 questions.

Section C: **(24 Marks)**

This section will contain 12 Short Answer (SA-II) type of questions, each carrying 3 marks.
Students will have to attempt any 8 questions.

Section D: **(20 Marks)**

This section will contain 8 Long Answer (LA) type of questions, each carrying 4 marks.
Students will have to attempt any 5 questions.

Distribution of Marks According to the Type of Questions

Type of questions		No of questions	Total Marks (with option)
MCQ	2 Marks each	Q.No. 1 (i to viii)	16 Marks
VSA	1 Mark each	Q.No. 2 (i to iv)	4 Marks
SA - I	2 Marks each	Q.No. 3 to 14	24 Marks
SA - II	3 Marks each	Q.No. 15 to 26	36 Marks
LA	4 Marks each	Q.No. 27 to 34	32 Marks

CONTENTS

Chapter No.	Chapter Name	Marks with option	Page No.
1	Differentiation	9	1
2	Applications of Derivatives	9	83
3	Indefinite Integration	10	128
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5	Application of Definite Integration	4	251
6	Differential Equations	8	273
7	Probability Distributions	5	326
8	Binomial Distribution	5	354
	Smart Recap		378
•	**Scan the given Q. R. Code in *Quill – The Padhai App* to view the Model Question Paper with Solution.**		

[Reference: Maharashtra State Board of Secondary and Higher Secondary Education, Pune - 04**]**

7 Probability Distributions

SYLLABUS

- Random variables.
- Types of random variables
- Probability distribution of random variable.
  - Discrete random variable
  - Probability mass function
  - Expected values and variance
  - Continuous random variable
  - Probability density function
  - Cumulative distribution function

LET'S RECALL

Basic Terminology

Random experiment (trial):
Any action in which all the possible results are known in advance, but none of them can be predicted is called a random experiment.

Sample space:
The set of all possible outcomes of a random experiment is called the sample space of that experiment and is denoted by S.
Every element of the sample space is called the sample point. The number of sample points in a sample space S are denoted by n(S).

Event:
An event is a subset of the sample space and is usually denoted by capital letters A, B, C, etc.

Equally likely outcomes:
The outcomes of a random experiment are called equally likely, if all of them have equal preference. In the experiment of tossing an unbiased coin, the outcomes 'Head' and 'Tail' are equally likely.

Complement of an event:
The set of all outcomes in a sample space excluding those of an event is called the complement of an event. It is denoted by A′, B′, C′, etc.

Using a Venn Diagram we represent the Sample Space (S), an Event (A), the Complement of an Event (A′) etc., as shown above.

Some commonly used events and their notations are given below:

Event	Notation
Not A	$\overline{A}$ or A′ or A^c
at least one of A, B occurs	$A \cup B$
both A and B occur	$A \cap B$
A occurs but not B	$A \cap B'$
B occurs but not A	$A' \cap B$
neither A nor B occur	$A' \cap B'$
at least one of A, B, C	$A \cup B \cup C$
Exactly one of A and B	$(A \cap \overline{B}) \cup (\overline{A} \cap B)$
All three of A, B, C	$A \cap B \cap C$
Exactly two of A, B and C	$(A \cap B \cap \overline{C}) \cup$ $(A \cap \overline{B} \cap C) \cup$ $(\overline{A} \cap B \cap C)$

Exhaustive events:
Two events A and B defined on a sample space S are said to be exhaustive if $A \cup B = S$.

Mutually Exclusive events:
Two events A and B defined on a sample space S are called mutually exclusive events if they can not occur simultaneously, i.e., $A \cap B = \phi$. In such a case A and B are disjoint sets.

Note:
If two events A and B defined on a sample space S are mutually exclusive and exhaustive, then they are called complementary events.
That is, if $A \cup B = S$ and $A \cap B = \phi$, then A and B are called complementary events.

Tossing a coin/coins:
If one coin is tossed n times or n coins are tossed together, then $n(S) = 2^n$.

Throwing a die/dice:
If one die is thrown n times or n dice are thrown together, then $n(S) = 6^n$

Playing cards:
A full pack of playing cards, also called the deck, consists of 52 cards.
It does not contain any joker.
Amongst these 52 cards, there are 4 different suits, each containing 13 cards.

Of these four suits, 2 are red and 2 are black.
The two red suits are the hearts (♥) and the diamonds (♦).
The two black suits are the spades (♠) and the clubs (♣).
In each suit, each of the thirteen cards is assigned a definite number, from 1 to 13, called its denomination.
In each suit, there are 3 picture (face) cards, viz. a jack, a queen and a king.
The card with denomination 1 is called an ace.

Probability:
If an event 'A' is defined over a sample space 'S' then the probability of event 'A' is given by $P(A) = \frac{n(A)}{n(S)}$, where n(A) is the number of elements in event A and n(S) is the number of elements in sample space S.

Fundamental rule of probability:
If one particular task can be done in 'm' ways and a second particular task can be done in 'n' ways, then both the task can be done together in '$m \times n$' number of ways.

Note:
If A be any event defined over a sample space S, then
i. $0 \le P(A) \le 1$
ii. $P(S) = 1$

Theorems:
i. If 'A' is an event defined over a sample space 'S' and A′ is the complement of the event 'A' then $P(A') = 1 - P(A)$.
ii. If 'A' and 'B' are any two events defined over a sample space 'S', then
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
iii. If events A and B are mutually exclusive, then $P(A \cap B) = 0$, thus $P(A \cup B) = P(A) + P(B)$

Conditional probability:
$$P(A/B) = \frac{P(A \cap B)}{P(B)} \text{ and } P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Multiplication theorem:
If A and B are two events defined on the sample space S, then the probability of occurrence of both the events is given by $P(A \cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$

Note:
If A and B are independent events (the occurrence of one event does not depend on the occurrence of other event), then
$P(A \cap B) = P(A) \cdot P(B)$

Bayes' theorem:
If $B_1, B_2, ..., B_n$ are n mutually exclusive and exhaustive events and if A is an arbitrary event consequent to these B_i's, then for each i,
$i = 1, 2, ..., n$
$$P(B_i/A) = \frac{P(B_i)P(A/B_i)}{\sum_{i=1}^{n} P(A \cap B_i)}$$

LET'S STUDY

Random Variable

Definition:
A random variable is a function from the sample space S to the set of real numbers.
Random variables are usually denoted by capital letter such as X,Y.
Symbolically, $X: S \to R$
We will often use the abbreviation 'r.v.' in place of random variable.
If the outcome of an experiment is expressed in a descriptive mode, we may modify the outcome suitably so that, it is expressed in the numerical form.
For example:
i. Instead of saying that H or T appears on tossing a coin, we would say '1' for head and '0' for tail. In such a case, 1 and 0 become random variables.
ii. In case of a die, the event reading on its uppermost face, being already a number, is automatically a random variable.
iii. Rainfall in cms, in a city is already a random variable, however, heavy rainfall or scanty rainfall is not a random variable.

Types of Random Variables

There are two types of random variables.
i. Discrete random variable
ii. Continuous random variable

Discrete Random Variable:
A random variable X is said to be discrete, if it takes either finite or countably infinite values. Thus, a discrete random variable takes only isolated values.
For example:
i. Number of children in a family, will take values say 0, 1, 2, 3, ..., etc.
Here, the random variable associated with this experiment takes values 0, 1, 2, 3 etc.
ii. Profit made by an investor in a day.
Here, the r.v. X associated with the above experiment may take positive (profit), negative (loss) or fractional values.
But the fractional value can be measured at best to the nearest paise.
If an investor has a profit of ₹ 140 and 50 paise, then X takes the value ₹ 140.5 but it can never take value like ₹ 140.573 etc. Hence, discrete r.v. takes isolated value.
In other words, its values jump from one possible value to the next but it never takes up the intermediate values.

Continuous Random Variable:
A random variable X is said to be continuous if it can take any value in a given interval. Thus, possible values of a continuous random variable are uncountably infinite.
For example:
i. Heights of students in a class.
Here, the r.v. X denotes the heights of students, which may take any value within the given range of heights.
ii. Failure time of electronic component.
The outcomes in this case are given by the time-to-time failure which may assume any non-negative real value, say, any value from 0 to ∞.

Values of continuous r.v. move continuously, from one possible value to another, without jump.

Remarks:
i. Values of a discrete r.v. are obtained by counting whereas those of continuous r.v. are obtained by measurement.
ii. Simple mathematical tools such as summation and difference are required to study the basic properties of discrete r.v., while continuous r.v. requires calculus methods such as integrals and derivatives.

Probability Mass Function (p.m.f.) of a Discrete Random Variable

The probability distribution of a discrete random variable is a list of probabilities associated with each of its possible values. It is also sometimes called the probability function or the probability mass function.
For example:
Suppose you flip two unbiased coins simultaneously. This simple experiment can have four possible outcomes: HH, HT, TH and TT.
$\therefore$ Sample space, S = {HH, HT, TH, TT}
Now, let the variable X represents the number of heads that result from this experiment.
Since coins are unbiased, each element in S is equally likely, i.e., has an equal probability $\frac{1}{4}$.
The variable X can take on the values 0, 1 or 2.
This can be shown as follows:

Outcome	Probability of outcome	Value of X
HH	$\frac{1}{4}$	2
HT	$\frac{1}{4}$	1
TH	$\frac{1}{4}$	1
TT	$\frac{1}{4}$	0

Note that sum of all probabilities is 1.

Definition:
If X is a discrete r.v. defined on a sample space S and the range of a random variable X assumes discrete values $x_1, x_2, x_3, \ldots, x_n$, then the function $p_i = P(X = x_i)$ is called the probability mass function of X if
i. $p_i \ge 0$, i = 1, 2, 3, …., n and
ii. $\sum_{i=1}^{n} p_i = 1$

The p.m.f. assigns a probability $[P(X = x_i)]$ for each of the possible value x_i of the variable.
$P(X = x_i)$ is read as probability that the r.v. X assumes the value x_i.

Probability Distribution:
A probability distribution is a table or an equation that links each outcome of a statistical experiment with its probability of occurrence.
Let X be a discrete r.v. defined on a sample space S. Suppose $\{x_1, x_2, \ldots, x_n\}$ is its range set and $p_i = P(X = x_i)$ is the p.m.f. of X.
The set of ordered pairs (x_i, p_i), i = 1, 2, …., n is called the probability distribution of X.
The probability distribution of X can be represented in a tabular form as follows:

$X = x$	x_1	x_2	…………..	x_n	Total
$P(X = x)$	p_1	p_2	…………..	p_n	1

In order to verify whether the given function is p.m.f. or not, we must check the following:
i. $P(X = x) \ge 0 \ \forall \ x$ and
ii. $\sum_{x} P(X=x) = 1$

The given function is not a p.m.f. if either (i) or (ii) or both are not satisfied.

Cumulative Distribution Function (c.d.f.) of a Discrete r.v.:
All random variables (discrete and continuous) have a cumulative distribution function. It is a function giving the probability that the random variable X is less than or equal to x, for every x.
For example:
Two coins are tossed simultaneously and the results are noted.
Let X be a r.v. defined as,
X = number of heads appearing on the coins.
Consider the following table that gives probability distribution of a r.v. X.

$X = x$	0	1	2
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Suppose, we want to find the probability that value of X is at most 1 or X takes values less than or equal to 1.
This means X takes values 0 or 1.
Symbolically, $P(X \le 1) = P(X = 0)$ or $P(X = 1)$
$= P(X = 0) + P(X = 1)$
$= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

The probabilities of the type $P(X \leq x)$ are called cumulative probabilities.

Like a probability distribution, a cumulative probability distribution can be represented by the following table:

No. of heads: $X = x$	0	1	2
Probability: $P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
Cumulative probability: $P(X \leq x)$	$\frac{1}{4}$	$\frac{3}{4}$	1

Definition:

Let X be a discrete r.v. suppose $\{x_1, x_2, \ldots, x_n\}$ is the range set of X and $p_1, p_2, \ldots, p_n$ are the respective probabilities of values of X. The cumulative distribution function of X at some fixed values x is denoted by $F(x)$ and is defined as $F(x) = P[X \leq x], x \in R$.

In particular, $F(x_i) = P[X \leq x_i] = p_1 + p_2 + \ldots + p_i$; $i = 1, 2, \ldots, n$.

Expected Value, Variance and Standard Deviation of a Discrete r.v.

For a discrete random variable, let us study its expected value, variance and standard deviation.

Expected Value:

The sum of the product of the values within the range of the discrete random variable and their respective probabilities of occurrence is called the expected value of a discrete random variable.

Definition:

If X is a discrete random variable assuming values $x_1, x_2, \ldots, x_n$, with respective probabilities of occurrence $p_1, p_2, \ldots, p_n$ such that $p_1 + p_2 + \ldots + p_n = 1$, then the Mathematical Expectation or Expected value denoted by E(X) or μ is given by

$$E(X) = x_1p_1 + x_2p_2 + \ldots + x_n p_n = \sum_{i=1}^{n} x_i p_i$$

Variance:

The variance of a discrete random variable X measures the spread or variability of the distribution.

If X is a discrete random variable assuming values $x_1, x_2, \ldots, x_n$, with respective probabilities of occurrence $p_1, p_2, \ldots, p_n$, and $\mu = E(X)$, then the variance of X, denoted by Var(X) or σ^2, is defined as

$$\sigma^2 = Var(X) = E[X - E(X)]^2 = E(X^2) - [E(X)]^2$$

where $E(X) = \sum_{i=1}^{n} x_i p_i$ and $E(X^2) = \sum_{i=1}^{n} x_i^2 p_i$

$$\therefore \quad \sigma^2 = E(X - \mu)^2 = \sum_{i=1}^{n} p_i (x_i - \mu)^2$$

Note:

i. $Var(X) \geq 0$, always.

Standard Deviation:

The standard deviation denoted by σ is the positive square root of the variance.

Thus, $\sigma = \sqrt{Var(X)} = \sqrt{E(X - \mu)^2}$

Remarks:

i. Standard deviation of X has the same unit as that of X, whereas in computation of variance unit gets squared.

ii. The variance or the standard deviation is a measure of the spread or dispersion of the values of a random variable about its expected value.

EXERCISE 7.1

1. Let X represent the difference between number of heads and number of tails obtained when a coin is tossed 6 times. What are possible values of X? **[2 Marks]**

***Solution*:**

X represent the difference between number of heads and number of tails.

Sample space of the experiment is

S = {(0 heads, 6 tails), (1 head, 5 tails), (2 heads, 4 tails), (3 heads, 3 tails), (4 heads, 2 tails), (5 heads, 1 tail), (6 heads, 0 tails)}

The value of X corresponding to these outcomes are as follow:

X(0 heads, 6 tails) = 0 – 6 = – 6

X(1 heads, 5 tails) = 1 – 5 = – 4

X(2 heads, 4 tails) = 2 – 4 = – 2

X(3 heads, 3 tails) = 3 – 3 = 0

X(4 heads, 2 tails) = 4 – 2 = 2

X(5 heads, 1 tails) = 5 – 1 = 4

X(6 heads, 0 tails) = 6 – 0 = 6

∴ Possible values of X are {–6, – 4, –2, 0, 2, 4, 6}

2. An urn contains 5 red and 2 black balls. Two balls are drawn at random. X denotes number of black balls drawn. What are possible values of X? **[1 Mark]**

***Solution*:**

5 red + 2 black = 7 balls

X denote the number of black balls drawn.

Sample space of the experiment is

S = {RR, BR, RB, BB}

The value of X corresponding to these out comes are as follow:

X (RR) = 0

X(BR) = X(RB) = 1

X(BB) = 2

∴ Possible values of X are {0, 1, 2}

3. State which of the following are not the probability mass function of a random variable. Give reasons for your answer. **[1 Mark Each]**

i.

X	0	1	2
P(X)	0·4	0·4	0·2

ii.

X	0	1	2	3	4
P(X)	0·1	0·5	0·2	–0·1	0·2

iii.

X	0	1	2
P(X)	0·1	0·6	0·3

iv.

Z	3	2	1	0	–1
P(Z)	0·3	0·2	0·4	0	0·05

v.

Y	–1	0	1
P(Y)	0·6	0·1	0·2

vi.

X	0	–1	–2
P(X)	0·3	0·4	0·3

***Solution*:**

i. Here, $p_i > 0, \forall i = 1, 2, 3$
Now consider,
$$\sum_{i=1}^{3} p_i = 0.4 + 0.4 + 0.2 = 1$$
$\therefore$ Given distribution is p.m.f.

ii. Here, $P(X = 3) = -0.1 < 0$
$\therefore$ Given distribution is not p.m.f.

iii. Here, $p_i > 0, \forall i = 1, 2, 3$
Now consider,
$$\sum_{i=1}^{3} p_i = 0.1 + 0.6 + 0.3 = 1$$
$\therefore$ Given distribution is p.m.f.

iv. Here, $p_i \geq 0, \forall i = 1, 2, \ldots, 5$
Now consider,
$$\sum_{i=1}^{5} p_i = 0.3 + 0.2 + 0.4 + 0 + 0.05 = 0.95 \neq 1$$
$\therefore$ Given distribution is not p.m.f.

v. Here, $p_i > 0, \forall i = 1, 2, 3$
Now consider,
$$\sum_{i=1}^{3} p_i = 0.6 + 0.1 + 0.2 = 0.9 \neq 1$$
Given distribution is not p.m.f.

vi. Here, $p_i > 0, \forall i = 1, 2, 3$
Now consider,
$$\sum_{i=1}^{3} p_i = 0.3 + 0.4 + 0.3 = 1$$
$\therefore$ Given distribution is p.m.f.

4. Find the probability distribution of
i. number of heads in two tosses of a coin. **[2 Marks]**
ii. number of tails in the simultaneous tosses of three coins. **[2 Marks]**
iii. number of heads in four tosses of a coin. **[2 Marks]**

***Solution*:**

i. Let X denote the number of heads.
Sample space of the experiment is
S = {HH, HT, TH, TT}
The values of X corresponding to these outcomes are as follows.
X(HH) = 2
X(HT) = X(TH) = 1
X(TT) = 0
$\therefore$ X is a discrete random variable that can take values 0, 1, 2.
The probability distribution of X is then obtained as follows:

X	0	1	2
$P(X = x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

ii. Let X denote the number of tails.
Sample space of the experiment is
S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}
The values of X corresponding to these outcomes are as follows.
X(HHH) = 0
X(HHT) = X(HTH) = X(THH) = 1
X(TTH) = X(THT) = X(HTT) = 2
X(TTT) = 3
$\therefore$ X is a discrete random variable that can take values 0, 1, 2, 3.
The probability distribution of X is then obtained as follows:

X	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

iii. Let X denote the number of heads.
Sample space of the experiment is
S = {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, THTH, HTHT, THHT, HTTT, THTT, TTHT, TTTH, TTTT}

Page no. **31** to **39** are purposely left blank.

To see complete chapter buy **Target Notes** or **Target E-Notes**

$\therefore$ c [log 3 – 0] = 1

$\therefore$ $c = \dfrac{1}{\log 3}$

b. $E(X) = \int_{-\infty}^{\infty} x f(x) = \int_{1}^{3} x f(x)\,dx$

$= \int_{1}^{3} x \dfrac{c}{x}\,dx$

$= c\int_{1}^{3} 1\,dx = \dfrac{1}{\log 3}[x]_1^3$

$= \dfrac{1}{\log 3}[3-1] = \dfrac{2}{\log 3}$

c. $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$= \int_{1}^{3} x^2 f(x)\,dx$

$= \int_{1}^{3} x^2 \cdot \dfrac{c}{x}\,dx$

$= c\int_{1}^{3} x\,dx = \dfrac{1}{2\log 3}\left[x^2\right]_1^3$

$= \dfrac{1}{2\log 3}[9-1] = \dfrac{8}{2\log 3} = \dfrac{4}{\log 3}$

$\therefore$ $Var(X) = E(X^2) - [E(X)]^2$

$= \dfrac{4}{\log 3} - \left(\dfrac{2}{\log 3}\right)^2$

$= \dfrac{4}{(\log 3)} - \dfrac{4}{(\log 3)^2}$

$= \dfrac{4\log 3 - 4}{(\log 3)^2} = \dfrac{4(\log 3 - 1)}{(\log 3)^2}$

MISCELLANEOUS EXERCISE – 7

I. Choose the correct option from the given alternatives: **[2 Marks Each]**

1. P.d.f. of a c.r.v. X is $f(x) = 6x(1-x)$, for $0 \le x \le 1$ and = 0, otherwise (elsewhere). If P (X < a) = P (X > a), then a =

(A) 1 (B) $\dfrac{1}{2}$
(C) $\dfrac{1}{3}$ (D) $\dfrac{1}{4}$

2. If the p.d.f of a c.r.v. X is $f(x) = 3(1-2x^2)$, for $0 < x < 1$ and = 0, otherwise (elsewhere), then the c.d.f of X is F(x) =

(A) $2x - 3x^2$ (B) $3x - 4x^3$
(C) $3x - 2x^3$ (D) $2x^3 - 3x$

3. If the p.d.f of a c.r.v. X is $f(x) = \dfrac{x^2}{18}$, for $-3 < x < 3$ and = 0, otherwise then P (| X | < 1) =

(A) $\dfrac{1}{27}$ (B) $\dfrac{1}{28}$
(C) $\dfrac{1}{29}$ (D) $\dfrac{1}{26}$

4. If a d.r.v. X takes values 0, 1, 2, 3, . . . with probability $P(X = x) = k(x+1)\cdot 5^{-x}$, where k is a constant, then P(X = 0) =

(A) $\dfrac{7}{25}$ (B) $\dfrac{16}{25}$ (C) $\dfrac{18}{25}$ (D) $\dfrac{19}{25}$

5. If p.m.f. of a d.r.v. X is $P(X = x) = \dfrac{{}^5C_x}{2^5}$, for $x = 0, 1, 2, 3, 4, 5$ and = 0, otherwise
If a = P (X ≤ 2) and b = P (X ≥ 3), then

(A) a < b (B) a > b
(C) a = b (D) a + b

6. If p.m.f. of a d.r.v. X is $P(X = x) = \dfrac{x}{n(n+1)}$, for $x = 1, 2, 3, \ldots$, n and = 0, otherwise then E (X) =

(A) $\dfrac{n}{1}+\dfrac{1}{2}$ (B) $\dfrac{n}{3}+\dfrac{1}{6}$
(C) $\dfrac{n}{2}+\dfrac{1}{5}$ (D) $\dfrac{n}{1}+\dfrac{1}{3}$

[Note: *The question has been modified.***]**

7. If p.m.f. of a d.r.v. X is $P(x) = \dfrac{c}{x^3}$, for $x = 1, 2, 3$ and = 0, otherwise (elsewhere) then E (X) =

(A) $\dfrac{343}{297}$ (B) $\dfrac{294}{251}$
(C) $\dfrac{297}{294}$ (D) $\dfrac{294}{297}$

8. If the d.r.v. X has the following probability distribution:

X	–2	–1	0	1	2	3
P(X = x)	0·1	k	0·2	2k	0·3	k

then P (X = −1) =

(A) $\dfrac{1}{10}$ (B) $\dfrac{2}{10}$ (C) $\dfrac{3}{10}$ (D) $\dfrac{4}{10}$

9. If the d.r.v. X has the following probability distribution:

X	1	2	3	4	5	6	7
P(X = x)	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

then k =

(A) $\dfrac{1}{7}$ (B) $\dfrac{1}{8}$
(C) $\dfrac{1}{9}$ (D) $\dfrac{1}{10}$

10. The expected value of X for the following p.m.f. is

X	–2	–1	0	1	2
P(X)	0·3	0·3	0·1	0·05	0·25

(A) 0·85 (B) – 0·35
(C) 0·15 (D) – 0·15

Answers:

1.	(B)	2.	(C)	3.	(A)	4.	(B)
5.	(C)	6.	(B)	7.	(B)	8.	(A)
9.	(D)	10.	(B)				

Hints:

1. Given P(X < a) = P(X > a)

∴ $\int_0^a f(x)\,dx = \int_a^1 f(x)\,dx$

∴ $6\int_0^a (x - x^2)\,dx = 6\int_a^1 (x - x^2)\,dx$

∴ $\frac{1}{2}\left[x^2\right]_0^a - \frac{1}{3}\left[x^3\right]_0^a = \frac{1}{2}\left[x^2\right]_a^1 - \frac{1}{3}\left[x^3\right]_a^1$

∴ $\frac{1}{2}a^2 - \frac{1}{3}a^3 = \frac{1}{2}\left[1 - a^2\right] - \frac{1}{3}\left[1 - a^3\right]$

∴ $\frac{1}{2}a^2 - \frac{1}{3}a^3 = \frac{1}{2} - \frac{1}{2}a^2 - \frac{1}{3} + \frac{1}{3}a^3$

$a^2 - \frac{2}{3}a^3 = \frac{1}{6}$

$6a^2 - 4a^3 = 1$

Option (B) satisfies the given condition

3. P(|X| < 1) = P(–1 < X < 1)

$= \int_{-1}^{1} \frac{x^2}{18}\,dx = \frac{2}{18}\int_0^1 x^2\,dx$

…[∵ f(x) is an even function]

$= \frac{1}{9 \times 3}\left[x^3\right]_0^1 = \frac{1}{27}$

4. p.m.f. of a discrete r.v.X is given as

$P(X = x) = \frac{k(x+1)}{5^x}$, for $x = 0, 1, 2, 3, \ldots$

∴ $\sum_{x=0}^{\infty} k\frac{(x+1)}{5^x} = 1$

∴ $k\sum_{x=0}^{\infty} \frac{x+1}{5^x} = 1$

∴ $\frac{1}{k} = 1 + \frac{2}{5} + \frac{3}{5^2} + \frac{4}{5^3} + \ldots$

∴ $\frac{1}{k} = 1 \times 1 + 2 \times \frac{1}{5} + 3 \times \frac{1}{5^2} + 4 \times \frac{1}{5^3} + \ldots$

Here, sequence 1, 2, 3, 4,… is in A.P. and sequence $1, \frac{1}{5}, \frac{1}{5^2}, \frac{1}{5^3}, \ldots$ is in G.P.

∴ Above sequence is in A.G.P where a = 1, r = $\frac{1}{5}$, d = 1

Here, |r| < 1

∴ Sum to the infinity exists which is $\frac{a}{1-r} + \frac{dr}{(1-r)^2}$

∴ $\frac{1}{k} = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

∴ $\frac{1}{k} = \frac{1}{1-\frac{1}{5}} + \frac{1 \times \frac{1}{5}}{\left(1 - \frac{1}{5}\right)^2} = \frac{5}{4} + \frac{5}{16} = \frac{25}{16}$

∴ $k = \frac{16}{25}$

∴ $P(X = 0) = \frac{k(0+1)}{5^0} = k = \frac{16}{25}$

5. Given that

a = P(X ≤ 2) and b = P(X ≥ 3)

∴ a = P(X = 0) + P(X = 1) + P(X = 2) and

b = P(X = 3) + P(X = 4) + P(X = 5)

∴ $a = \frac{{}^5C_0 + {}^5C_1 + {}^5C_2}{2^5} = \frac{1+5+10}{2^5} = \frac{16}{2^5}$

$b = \frac{{}^5C_3 + {}^5C_4 + {}^5C_5}{2^5} = \frac{10+5+1}{2^5} = \frac{16}{2^5}$

∴ a = b

6. $E(X) = \sum_{x=1}^{n} x.P(X = x)$

$= \sum_{x=1}^{n} x \times \frac{x}{n(n+1)}$

$= \frac{1}{n(n+1)}\sum_{x=1}^{n} x^2$

$= \frac{1}{n(n+1)} \times \frac{n(n+1)(2n+1)}{6}$

$= \frac{2n+1}{6}$

$= \frac{n}{3} + \frac{1}{6}$

7. p.m.f. or d.r.v. X is given

∴ P(1) + P(2) + P(3) = 1

∴ $\frac{c}{(1)^3} + \frac{c}{2^3} + \frac{c}{3^3} = 1$

∴ $c = \frac{216}{251}$

∴ $E(X) = \sum_{x=1}^{n} x \cdot P(x) = \frac{216}{251}\left[\frac{1}{(1)^3} + \frac{2}{(2)^3} + \frac{3}{(3)^3}\right]$

$= \frac{294}{251}$

II. Solve the following:

1. Identify the random variable as either discrete or continuous in each of the following. Write down the range of it. **[1 Mark Each]**

i. An economist is interested in the number of unemployed graduates in the town of population 1 lakh.

Solution:

Let X = number of unemployed graduates in a town.

Here, X takes only finite values.

∴ X is a discrete r.v.

Range of X = {0, 1, 2, ..., 100000}

ii. Amount of syrup prescribed by physician.

Solution:

Let X = amount of syrup prescribed by a physician.

Here, X can take any positive or fractional value, i.e, X takes uncountably infinite values.

∴ X is a continuous r.v.

iii. The person on the high protein diet is interested in the gain of weight in a week.

Solution:

Let X = gain in weight in a week.

Here, X takes uncountably infinite values.

∴ X is a continuous r.v.

iv. 20 white rats are available for an experiment. Twelve rats are male. A scientist randomly selects 5 rats, the number of female rats selected on a specific day.

Solution:

There are 12 male rats and 8 female rats, i.e., finite number of female rats.

∴ X is a discrete r.v.

Range of X = {0, 1, 2, 3, 4, 5}

v. A highway safety group is interested in studying the speed (km/hrs) of a car at a check point.

Solution:

Let X = speed of the car in km/hr.

X takes uncountably infinite values.

∴ X is a continuous r.v.

2. The probability distribution of discrete r.v. X is as follows

X = *x*	1	2	3	4	5	6
P(X = *x*)	k	2k	3k	4k	5k	6k

i. Determine the value of k.

ii. Find P(X ≤ 4), P(2 < X < 4), P(X ≥ 3). **[4 Marks]**

Solution:

i. Since P(X) is the probability distribution of X,

$$\sum_{x=1}^{6} P(X=x)=1$$

∴ k + 2k + 3k + 4k + 5k + 6k = 1

∴ 21k = 1

∴ $k = \frac{1}{21}$

ii. a. $P(X \le 4) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$= k + 2k + 3k + 4k$

$= 10k = \frac{10}{21}$

b. $P(2 < X < 4) = P(X = 3) = 3k = \frac{3}{21} = \frac{1}{7}$

c. $P(X \ge 3) = 1 - P(X < 3)$

$= 1 - [P(X = 1) + P(X = 2)]$

$= 1 - (k + 2k) = 1 - 3k$

$= 1 - \frac{3}{21}$

$= 1 - \frac{1}{7} = \frac{6}{7}$

3. The following probability distribution of r.v. X

X = *x*	–3	–2	–1	0	1	2	3
P(X = *x*)	0·05	0·1	0·15	0·20	0·25	0·15	0·1

Find the probability that **[1 Mark Each]**

i. X is positive.

ii. X is non negative.

iii. X is odd.

iv. X is even.

Solution:

i. P(X is positive)

= P(X = 1 or X = 2 or X = 3)

= P(X = 1) + P(X = 2) + P(X = 3)

= 0.25 + 0.15 + 0.10 = 0.50

ii. P(X is non-negative)

= P(X = 0 or X = 1 or X = 2 or X = 3)

= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)

= 0.20 + 0. 25 + 0.15 + 0.10 = 0.70

iii. P(X is odd)

= P(X = –3 or X = –1 or X = 1 or X = 3)

= P(X = –3) + P(X = –1) + P(X = 1) + P(X = 3)

= 0.05 + 0.15 + 0.25 + 0.10 = 0.55

iv. P(X is even)

= P(X = –2 or X = 0 or X = 2)

= P(X = –2) + P(X = 0) + P(X = 2)

= 0.10 + 0.20 + 0.15 = 0.45

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To see complete chapter buy **Target Notes** or **Target E-Notes**

15. The p.d.f. of r.v. X is given by $f(x) = \frac{k}{\sqrt{x}}$, for $0 < x < 4$ and = 0, otherwise. Determine k . Determine c.d.f. of X and hence find $P(X \le 2)$ and $P(X \le 1)$. **[3 Marks]**

***Solution*:**

Given that $f(x)$ represents p.d.f. of r.v. X.

$\therefore \quad \int_0^4 \frac{k}{\sqrt{x}} dx = 1$

$\therefore \quad k \cdot \left[2\sqrt{x}\right]_0^4 = 1$

$\therefore \quad 2k \left[\sqrt{x}\right]_0^4 = 1$

$\therefore \quad 2k(2-0) = 1$

$\therefore \quad k = \frac{1}{4}$

By definition of c.d.f.,

$F(x) = P(X \le x)$

$= \int_0^x \frac{k}{\sqrt{x}} dx$

$= k\left[2\sqrt{x}\right]_0^x$

$= \frac{1}{4}\left[2\sqrt{x}\right]_0^x \qquad \ldots\left[\because k = \frac{1}{4}\right]$

$= \frac{\sqrt{x}}{2}$

$P(X \le 2) = F(2) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$P(X \le 1) = F(1) = \frac{\sqrt{1}}{2} = \frac{1}{2}$

ONE MARK QUESTIONS

1. For following probability distribution find k.

$X = x$	1	2	3
$P(X = x)$	k	2k	3k

2. The following is c.d.f. of r.v.X.

$X = x$	1	2	3	4
F(X)	0.1	0.3	0.75	1

Find $P(X = 2)$

3. If r.v. X denote the number of prime numbers appear on the upper most face of a fair die when it rolled twice. Find the possible values of X.

4. The following is p.d.f. of continuous r.v. X.

$f(x) = \frac{x^3}{64}$, for $0 < x < 4$

$= 0$ otherwise

Find $F(x)$ at $x = -1$

5. If the p.d.f. of a continuous r.v.X is given by

$f(x) = kx, \quad 0 < x < 4$

$= 0 \quad$ otherwise

then what is the value of F(4)?

MULTIPLE CHOICE QUESTIONS

[2 Marks Each]

1. A p.m.f. of r.v. X is given below:

$X = x$	2	4	6
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$

Then E(X) =

(A) 5.25 (B) 4.25
(C) 4.75 (D) 5.75

2. A r.v. X has the following probability distribution

X	−1	−2	1	2
P(X)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

Then E(X) =

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{6}$

3. Number of stars in the sky is an example of
(A) continuous r.v.
(B) discrete r.v.
(C) both continuous and discrete r.v.
(D) neither continuous nor discrete.

4. The total probability of all the exhaustive events is
(A) 0 (B) 1 (C) −1 (D) ±1

5. Atmospheric pressure at a certain place in $\frac{N}{M^2}$ is
(A) continuous r.v.
(B) discrete r.v.
(C) both continuous and discrete r.v.
(D) neither continuous nor discrete.

6. If $P(x) = \frac{kx}{3}$, $x = 1, 2, 3$ is p.m.f. of X, then k =

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{1}{2}$

7. A die is thrown at random, what is the expectation of the number on it?
(A) 3.5 (B) 3.6
(C) 4.5 (D) 4.6

8. If a pair of dice is thrown and X denotes the sum of the numbers on them. Find the expectation of X.
(A) 5 (B) 0 (C) 1 (D) 7

9. For the given probability distribution, find E(X)

X	1	2	3
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{5}{8}$

(A) $\frac{11}{8}$ (B) $\frac{17}{8}$ (C) $\frac{19}{8}$ (D) $\frac{21}{8}$

10. In a game, a person is paid Rs. 5 if he gets all heads or all tails when three coins are tossed and he will pay Rs. 3 if either one or two heads show. What can he expect on an average per game ?
(A) gain of ₹ 2 (B) gain of ₹ 1
(C) loss of ₹ 1 (D) No loss no gain

11. A random variable X has the following probability distribution values of X,

X:	0	1	2	3	4	5	6	7
P(X):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find value of $P(X \geq 6)$.
(A) $\frac{13}{100}$ (B) $\frac{17}{100}$
(C) $\frac{21}{100}$ (D) $\frac{19}{100}$

12. The probability distribution of a discrete random variable X is given below:

X	2	3	4	5
P(X)	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

Then k =
(A) 8 (B) 32 (C) 16 (D) 48

13. Let X be a random variable. The probability distribution of X is given below:

X	30	10	–10
P(X)	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{2}$

Then E(X) =
(A) 6 (B) –5 (C) 4 (D) –3

14. Let X be a discrete random variable assuming values x_1, x_2, x_3, ..., x_n with respective probabilities p_1, p_2, p_3, ..., p_n. Then variance of X is given by
(A) $E(X^2)$ (B) $E(X^2) + E(X)$
(C) $E(X^2) - \sqrt{E(X)}$ (D) $E(X^2) - [E(X)]^2$

15. For the following probability distribution

X	1	2	3	4
P(X)	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

$E(X^2)$ =
(A) 3 (B) 5 (C) 7 (D) 10

16. For the following probability distribution

X	–4	–3	–2	–1	0
P(X)	0.1	0.2	0.3	0.2	0.2

E(X) =
(A) –2 (B) –1.8 (C) –1 (D) 0

17. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take value x has the following form, where k is some constant.

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ k(5-x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise.} \end{cases}$$

The probability that you study for at least two hours is
(A) 2k (B) 0.55
(C) 0.75 (D) 5k

18. A random variable has the probability distribution

X	1	2	3	4	5	6	7	8
P(X)	0.15	0.23	0.12	0.1	0.2	0.08	0.07	0.05

For the events E = {X is a prime number} and F = {X < 4}, the probability $P(E \cup F)$ is
(A) 0.87 (B) 0.77
(C) 0.35 (D) 0.5

19. A probability distribution of a discrete random variable X is

X	–1	0	1	2
P(X)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

Then the value of $6E(X^2) - \text{Var}(X)$ is
(A) $\frac{12}{113}$ (B) $\frac{19}{12}$ (C) $\frac{113}{12}$ (D) $\frac{1}{2}$

20. The random variable X has a probability distribution $P(x)$ of the following form, where k is some constant.

$$P(X = x) \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Then value of k and E(X) are
(A) $\frac{1}{6}, \frac{4}{3}$ (B) $\frac{1}{6}, \frac{14}{3}$
(C) $\frac{1}{5}, \frac{4}{3}$ (D) $0, \frac{1}{6}$

21. For the following probability distribution of random variable X find $(P(X \geq 3)$, where k is a constant

X	1	2	3	4
P(X)	k	2k	3k	4k

(A) $\frac{1}{10}$ (B) $\frac{7}{10}$ (C) $\frac{5}{10}$ (D) 1

22. If X is a random variable with probability mass function
$P(x) = kx$, for $x = 1, 2, 3$
$= 0$, otherwise
then k = _______. **[Oct 14]**
(A) $\frac{1}{5}$ (B) $\frac{1}{4}$
(C) $\frac{1}{6}$ (D) $\frac{2}{3}$

23. A random variable X has the following probability distribution:

$X = x$	–2	–1	0	1	2	3
$P(x)$	0.1	0.1	0.2	0.2	0.3	0.1

Then $E(x) =$ **[Mar 15]**
(A) 0.8 (B) 0.9
(C) 0.7 (D) 1.1

24. If the p.d.f. of a continuous random variable X is given as
$f(x) = \frac{x^2}{3}$ for $-1 < x < 2 = 0$ otherwise.
then c.d.f. fo X is **[July 16]**
(A) $\frac{x^3}{9}+\frac{1}{9}$ (B) $\frac{x^3}{9}-\frac{1}{9}$
(C) $\frac{x^2}{4}+\frac{1}{4}$ (D) $\frac{1}{9x^3}+\frac{1}{9}$

25. The expected value of the number of heads obtained when three fair coins are tossed simultaneously is _______. **[Mar 17]**
(A) 1 (B) 1.5
(C) 0 (D) –1

26. Let the p.m.f. of a random variable X be
$P(x) = \frac{3-x}{10}$ for $x = -1, 0, 1, 2$
$= 0$ otherwise
Then E(X) is _______. **[Mar 18]**
(A) 1 (B) 2
(C) 0 (D) –1

27. If $f(x) = kx^2(1-x)$, for $0 < x < 1$
$= 0$, otherwise,
is the probability distribution function of a random variable X, then the value of k is _______. **[July 23]**
(A) 12 (B) 10
(C) –9 (D) –12

Time: 1 Hour	TOPIC TEST	Total Marks: 20

SECTION A

Q.1. Select and write the correct answer. **[4]**

i. For a random variable X, if $E(X) = 5$ and $Var(X) = 6$, then $E(X^2)$ is equal to
(A) 19 (B) 31
(C) 61 (D) 11

ii. The p.m.f. of a r.v. X is given by
$P(X = x) = \frac{^5C_x}{2^5}$, $x = 0, 1, 2, \ldots, 5$
$= 0$, otherwise
Then, $P(X \le 2) =$
(A) $\frac{3}{32}$ (B) $\frac{7}{32}$
(C) $\frac{11}{32}$ (D) $\frac{16}{32}$

Q.2. Answer the following. **[2]**

i. The following is c.d.f. of r.v. X.

$X = x$	1	2	3	4
F(X)	0.1	0.3	0.75	1

Find $P(X = 2)$

ii. If the p.d.f. of a continuous r.v.X is given by
$f(x) = kx$, $0 < x < 4$
$= 0$ otherwise
then what is the value of F(4)?

SECTION B

Attempt any two of the following: **[4]**

Q.3. State if the following is the probability mass function of a random variable. Give reasons for your answer.

X	0	–1	–2
P(X)	0·3	0·4	0·3

Q.4. Find the probability distribution of number of heads in two tosses of a coin.

Q.5. Verify if the following is p.d.f. of r.v. X:

$f(x) = \sin x, \qquad \text{for } 0 \leq x \leq \frac{\pi}{2}$

SECTION C

Attempt any two of the following: **[6]**

Q.6. Find k if the following function represent p.d.f. of r.v. X.

$f(x) = kx$, for $0 < x < 2$ and $= 0$ otherwise, Also find $P\left(\frac{1}{4} < x < \frac{3}{2}\right)$.

Q.7. The p.d.f. of r.v. X is given by $f(x) = \frac{1}{2a}$, for $0 < x < 2a$ and $= 0$, otherwise.

Show that $P\left(X < \frac{a}{2}\right) = P\left(X > \frac{3a}{2}\right)$.

Q.8. The following is the c.d.f. of r.v. X

X	–3	–2	–1	0	1	2	3	4
F(X)	0·1	0·3	0·5	0·65	0·75	0·85	0·9	1

Find p.m.f. of X.
i. $P(-1 \leq X \leq 2)$
ii. $P(X \leq 3/ X > 0)$

SECTION D

Attempt any one of the following: **[4]**

Q.9. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings drawn.

Q.10. Suppose error involved in making a certain measurement is continuous r.v.X with p.d.f.
$f(x) = k(4 - x^2)$, for $-2 \leq x \leq 2$ and $= 0$ otherwise.
Compute: i. $P(X > 0)$
ii. $P(-1 < X < 1)$
iii. $P(X < -0.5 \text{ or } X > 0{\cdot}5)$

ANSWERS

ONE MARK QUESTIONS

1. $\frac{1}{6}$
2. 0.2
3. {0, 1, 2}
4. 0
5. 1

MULTIPLE CHOICE QUESTIONS

1. (C) 2. (D) 3. (B) 4. (B)
5. (A) 6. (D) 7. (A) 8. (D)
9. (C) 10. (C) 11. (D) 12. (B)
13. (C) 14. (D) 15. (D) 16. (B)
17. (C) 18. (B) 19. (C) 20. (A)
21. (B) 22. (C) 23. (A) 24. (A)
25. (B) 26. (C) 27. (A)

TOPIC TEST

1. i. (B) ii. (D)
2. i. 0.2 ii. 1
3. Given distribution is p.m.f.
4.

X	0	1	2
P(X = x)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

5. Given function is p.d.f.
6. $\frac{1}{2}, \frac{35}{64}$
8. i. 0.55 ii. $\frac{5}{7}$
9. $\frac{34}{221}$, 0.1392, $\sqrt{0.1392}$
10. i. $\frac{1}{2}$ ii. $\frac{11}{16}$ iii. 0.6328

COMPETITIVE CORNER

1. A player tosses 2 fair coins. He wins ₹ 5 if 2 heads appear, ₹ 2 if 1 head appear and ₹ 1 if no head appears, then variance of his winning amount is **[MHT CET 2019]**
   (A) 1.6 (B) $\frac{9}{4}$
   (C) $\frac{17}{2}$ (D) $\frac{5}{2}$

2. The p.d.f. of a continuous random variable X is given by
   $f(x) = \frac{1}{2}$ if $0 < x < 2$
   $= 0$ otherwise
   and if $a = P\left(X < \frac{1}{2}\right)$, $b = P\left(X > \frac{3}{2}\right)$, then relation between a and b is **[MH CET 2020]**
   (A) $a - b = 0$ (B) $2a - b = 0$
   (C) $3a - b = 0$ (D) $a - 2b = 0$

3. A random variable X has the following probability distribution.

X = x	0	1	2	3	4	5	6	7
P [X = x]	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

   Then F(4) = **[MHT CET 2021]**
   (A) $\frac{3}{10}$ (B) $\frac{1}{10}$
   (C) $\frac{7}{10}$ (D) $\frac{4}{5}$

4. Two numbers are selected at random from the first six positive integers. If X denotes the larger of two numbers, then Var (X) = **[MHT CET 2022]**
   (A) $\frac{14}{3}$ (B) $\frac{14}{9}$
   (C) $\frac{1}{3}$ (D) $\frac{70}{3}$

5. A random variable X has the following probability distribution **[MHT CET 2022]**

X	0	1	2	3	4	5	6
P (X)	k	3k	5k	7k	9k	11k	13k

   then P (X ≥ 2) =
   (A) $\frac{1}{49}$ (B) $\frac{45}{49}$
   (C) $\frac{40}{49}$ (D) $\frac{15}{49}$

6. A random variable X has the following probability distribution

X	0	1	2	3	4
P(X)	K	2K	4K	6K	8K

   The value of $P(1 < X < 4 \mid X \le 2)$ is equal to **[JEE (Main) 2022]**
   (A) $\frac{4}{7}$ (B) $\frac{2}{3}$
   (C) $\frac{3}{7}$ (D) $\frac{4}{5}$

7. A random variable X assumes values 1, 2, 3, …., n with equal probabilities, if var(X) = E(X), then n is **[MHT CET 2023]**
   (A) 4 (B) 5
   (C) 7 (D) 9

8. Three fair coins with faces numbered 1 and 0 are tossed simultaneously. Then variance (X) of the probability distribution of random variable X, where X is the sum of numbers on the upper most faces, is **[MHT CET 2023]**
   (A) 0.7 (B) 0.75
   (C) 0.65 (D) 0.6

9. A random variable X has the probability distribution

X = x	1	2	3	4	5	6	7	8
P(X = x)	0.15	0.23	0.12	0.20	0.08	0.10	0.05	0.07

   For the events E = {X is a prime number} and F = {x < 5}, P(E ∪ F) is **[MHT CET 2023]**
   (A) 0.63 (B) 0.75
   (C) 0.83 (D) 0.90

Answers:

1.	(B)	2.	(A)	3.	(D)	4.	(B)
5.	(B)	6.	(A)	7.	(C)	8.	(B)
9.	(C)						

Hints:

1. Let X denote winning amount.

∴ Possible value of X are 1, 2, 5

Let P(head appears) = p = $\frac{1}{2}$

∴ $q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

∴ P(X = 1) = P(no heads appear) = qq = $\frac{1}{4}$

P(X = 2) = P(one head appears) = pq + qp = $\frac{2}{4}$

P(X = 5) = P(two heads appear) = pp = $\frac{1}{4}$

∴ $E(X) = 1 \times \frac{1}{4} + 2 \times \frac{2}{4} + 5 \times \frac{1}{4} = \frac{1+4+5}{4} = \frac{5}{2}$

$E(X^2) = 1 \times \frac{1}{4} + 4 \times \frac{2}{4} + 25 \times \frac{1}{4} = \frac{1+8+25}{4} = \frac{17}{2}$

$Var(X) = E(X^2) - [E(X)]^2 = \frac{9}{4}$

2. $a = P\left(X < \frac{1}{2}\right)$, $b = P\left(X > \frac{3}{2}\right)$

∴ $a = \int_0^{\frac{1}{2}} \frac{1}{2}\, dx = \frac{1}{4}$

$b = \int_{\frac{3}{2}}^{2} \frac{1}{2}\, dx = \frac{1}{4}$

∴ a = b, i.e., a – b = 0

3. $\sum_{x=0}^{7} P(x) = 1$

∴ $k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

∴ $9k + 10k^2 = 1$

∴ $10k^2 + 9k - 1 = 0$

∴ $10k^2 + 10k - k - 1 = 0$

∴ $10k(k + 1) - 1(k + 1) = 0$

∴ $(10k - 1)(k + 1)$

$\Rightarrow k = \frac{1}{10}$

∴ $F(4) = k + 2k + 2k + 3k = 8k = 8 \times \frac{1}{10} = \frac{4}{5}$

4. Two numbers are selected from {1, 2, 3, 4, 5, 6}.
Let S = sample space

∴ $n(S) = {}^6C_2 = \frac{6!}{2! \times 4!} = 15$

X denotes the larger of the two numbers obtained.

∴ Possible values of X are 2, 3, 4, 5, 6.

When X = 2,
one of the two numbers is 2 and remaining one is smaller than 2, i.e., 1.

∴ Remaining number can be selected in 1 way only

∴ $P(X = 2) = \frac{1}{15}$

When X = 3,
one of the two numbers is 3 and remaining one is smaller than 3, i.e., 1 or 2.

∴ Remaining number can be selected in ${}^2C_1 = 2$ ways.

∴ $P(X = 3) = \frac{2}{15}$

Similarly, $P(X = 4) = \frac{3}{15}$, $P(X = 5) = \frac{4}{15}$,

$P(X = 6) = \frac{5}{15}$

∴ $E(X) = \sum_{i=1}^{5} x_i . P(x_i)$

$= 2 \times \frac{1}{15} + 3 \times \frac{2}{15} + 4 \times \frac{3}{15} + 5 \times \frac{4}{15} + 6 \times \frac{5}{15}$

$= \frac{14}{3}$

$Var(X) = E(X^2) - [E(X)]^2$

$= 2^2\left(\frac{1}{15}\right) + 3^2\left(\frac{2}{15}\right) + 4^2\left(\frac{3}{15}\right) + 5^2\left(\frac{4}{15}\right) + 6^2\left(\frac{5}{15}\right) - \left(\frac{14}{3}\right)^2$

$= \frac{70}{3} - \left(\frac{14}{3}\right)^2$

$= \frac{14}{9}$

5. Since $\sum_{x=0}^{6} P(X = x) = 1$,

$k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

$\Rightarrow 49k = 1 \Rightarrow k = \frac{1}{49}$

$P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$

$= 5k + 7k + 9k + 11k + 13k$

$= 45k$

$= \frac{45}{49}$

6. The given probability distribution is

X	0	1	2	3	4
P(X)	K	2K	4K	6K	8K

$P(1 < X < 4 \mid X \leq 2)$

$= \frac{P(1 < X \leq 2)}{P(X \leq 2)}$ $\quad \ldots \left[\because P(A \mid B) = \frac{P(A \cap B)}{P(B)}\right]$

$= \frac{P(X=2)}{P(X=0)+P(X=1)+P(X=2)}$

$= \frac{4K}{K+2K+4K}$

$= \frac{4K}{7K}$

$= \frac{4}{7}$

7. $X = 1, 2, 3, \ldots.n$

$P(X) = \frac{1}{n}$

$E(X) = \sum_{i=1}^{n} x_i P(x_i)$

$= \frac{(1+2+3+\ldots+n)}{n}$

$= \frac{n(n+1)}{2n}$

$\therefore$ $E(X) = \frac{n+1}{2}$

$\text{Var}(X) = \sum_{i=1}^{n} x_i^2 P(x_i) - [E(X)]^2$

$= \frac{1^2+2^2+3^2+\ldots+n^2}{n} - \left(\frac{n+1}{2}\right)^2$

$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2$

$= \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$

$\text{Var}(X) = E(X)$ …[Given]

$\therefore$ $\frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = \frac{n+1}{2}$

$\therefore$ $\frac{2n^2+n+2n+1}{6} - \left(\frac{n^2+2n+1}{4}\right) = \frac{n+1}{2}$

$\therefore$ $\frac{4n^2+6n+2-3n^2-6n-3}{12} = \frac{n+1}{2}$

$\therefore$ $n^2 - 1 = 6(n + 1)$

$\therefore$ $n^2 - 1 = 6n + 6$

$\therefore$ $n^2 - 6n - 7 = 0$

$\therefore$ $n = -1$ or $n = 7$

But $n \neq -1$

$\therefore$ $n = 7$

8. Possible value of X are 0, 1, 2, 3

Here,

S = {000, 001, 010, 100, 111, 110, 101, 011}

$\therefore$ n(S) = 8

$\therefore$

x_i	0	1	2	3
p_i	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$\therefore$ $E(X) = \sum x_i p_i = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8}$

$E(X^2) = \sum x_i^2 p_i^2 = 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8} = \frac{24}{8}$

$\therefore$ Variance $= E(X^2) - [E(X)]^2$

$= 3 - \left(\frac{3}{2}\right)^2$

$= 3 - \frac{9}{4}$

$= \frac{3}{4}$

$= 0.75$

9. $P(E) = P(X = 2 \text{ or } X = 3 \text{ or } X = 5 \text{ or } X = 7)$

$= P(X = 2) + P(X = 3) + P(X = 5) + P(X = 7)$

$= 0.23 + 0.12 + 0.08 + 0.05$

$= 0.48$

$P(F) = P(X < 5)$

$= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$= 0.15 + 0.23 + 0.12 + 0.20$

$= 0.7$

$P(E \cap F) = P(X \text{ is a prime number less than } 5)$

$= P(X= 2) + P(X = 3)$

$= 0.23 + 0.12$

$= 0.35$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= 0.48 + 0.7 - 0.35$

$= 0.83$

Scan the given Q. R. Code in *Quill - The Padhai App* to view the solutions of the Topic Test.