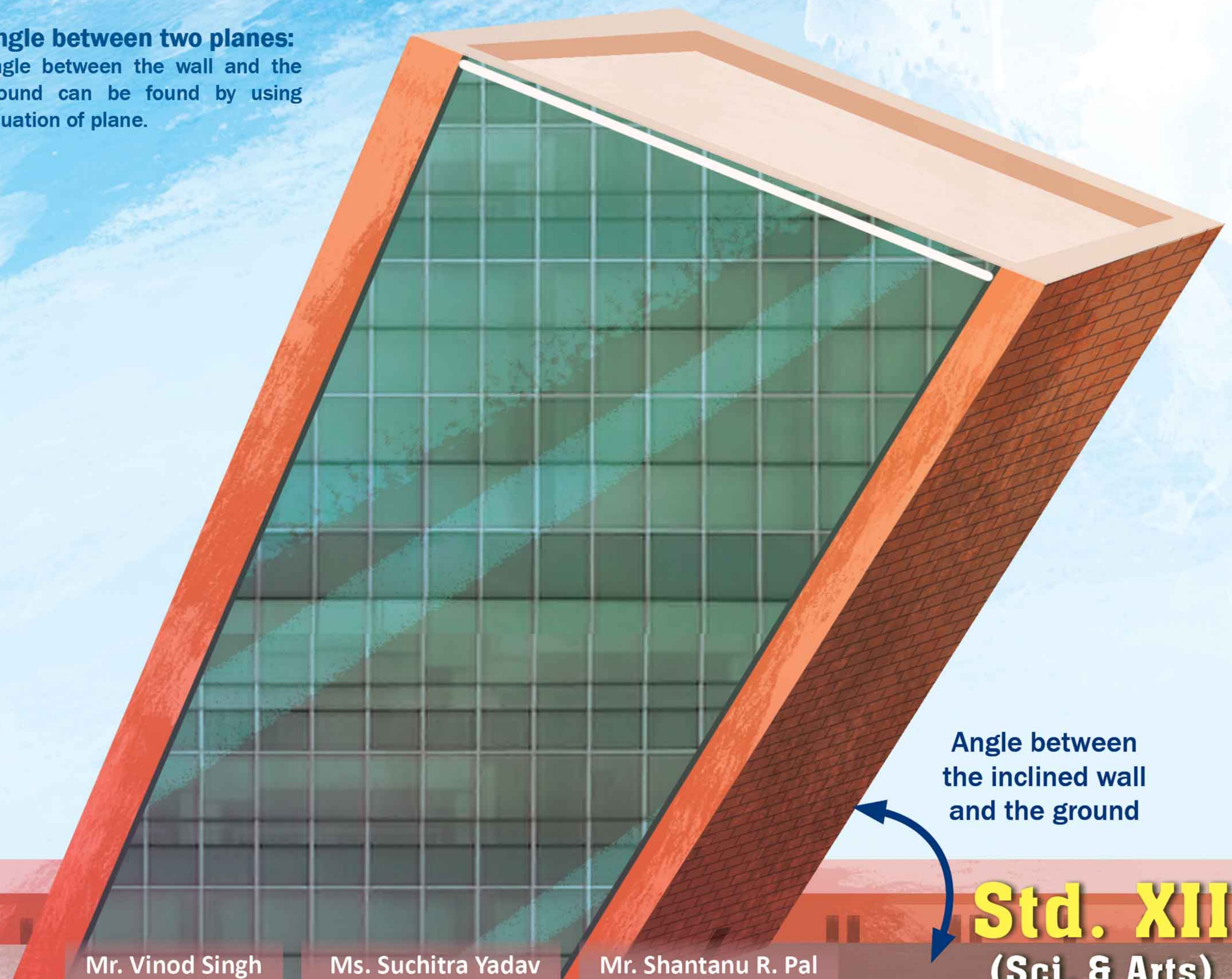
PERFECT

MATHEMATICS & STATISTICS

Part I

Based on New Paper Pattern and Latest Textbook

Angle between two planes:
Angle between the wall and the ground can be found by using equation of plane.



Std. XII
(Sci. & Arts)

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PERFECT

MATHEMATICS & STATISTICS Part - I

Std. XII Sci. & Arts

Salient Features

- Written as per Latest Board Paper Pattern
- Exhaustive coverage of entire syllabus
- Precise theory for every topic
- Covers answers to all exercises and miscellaneous exercises given in the textbook.
- Includes relevant board questions from March 1996 to July 2022.
- All derivations and theorems covered
- Includes MCQs for practice
- Illustrative examples for selective problems
- 'Smart Recap' at the end of the book
- Activity Based Questions covered in every chapter
- Smart Check for Answer verification
- Topic Test at the end of each chapter for self-assessment
- 'Competitive Corner' gives idea of MCQs asked in competitive exams
- QR Codes to access the Solutions of the Topic Tests, Model Question Paper along with Solution
- Includes Board Question Paper of March 2023 (Solution in pdf format through QR code)

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PREFACE

Perfect Mathematics & Statistics Part – I, Std. XII Sci. & Arts is intended for every Maharashtra State Board aspirant of Std. XII, Science & Arts. The scope, sequence, and level of the book are designed to match the new textbook issued by the Maharashtra State Board.

At this crucial juncture in their lives, when the students are grappling with the pressures of cracking a career-defining board examination, we wanted to create a book that not only develops the necessary knowledge, tools, and skills required to excel in the examination, but also enables students to appreciate the beauty of the subject and piques their curiosity.

We believe that students respond favourably to meaningful content, if it is presented in a way that is easy to read and understand, rather than being mired down with facts and information. Consequently, we have always placed the highest priority on writing clear and lucid explanations of fundamental concepts. Moreover, special care has been taken to ensure that the topics are presented in a logical order.

The primary purpose of this book is to assist the students in preparing for the board examination. However, this is closely linked to other goals: to exemplify how important and how incredibly interesting mathematics is, and to help the student become an expert thinker and problem solver.

Practice, practice & more practice is the key to score high in mathematics!

To help the students, this book amalgamates problems that are rich in both variety and number which provides the student with ample practice, ensuring mastery of each concept.

The scope of the book extends beyond the State Board examination as it also offers a plethora of Multiple Choice Questions (MCQs) in order to familiarize the students with the pattern of competitive examinations.

In addition, the chapter-test have been carefully crafted to focus on concepts, thus providing the students with a quick opportunity for self-assessment and giving them an increased appreciation of chapter-preparedness.

*Our **Perfect Mathematics & Statistics Part – I, Std. XII Sci. & Arts** adheres to our vision and achieves several goals: **building concepts, developing competence to solve problems, recapitulation, self-study, self-assessment** and **student engagement**—all while encouraging students toward cognitive thinking.*

\- Publisher

Edition: Fifth

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.
Please write to us on: mail@targetpublications.org

KEY FEATURES

Illustrative Example

Illustrative Example provides a detailed approach towards solving a problem.

Smart Check

Smart Check is a technique to verify the answers. This is our attempt to cross-check the accuracy of the answer.
Smart check is indicated by ✓ symbol

Activities for Practice

In this section we have provided multiple activities for practice which will help students understand the concepts.

One Mark Questions

These questions require very short solutions with direct application of mathematical concepts.

Topic Test

Topic Test covers questions from the chapter for self-evaluation purpose.
This is our attempt to provide the students with revision and help them assess their knowledge of chapter.

Competitive Corner

Competitive Corner presents questions from prominent [JEE (Main), MHT CET] competitive exams based entirely on the syllabus covered in the chapter.
This is our attempt to introduce students to MCQs asked in competitive exams.

Smart Recap

Smart Recap given at the end of the book includes important and relevant concepts and formulae in the chapters.
This is our attempt to offer students a handy tool to solve problems and ace the last minute revision.

QR Codes

QR code provides:
i. The solutions of the Topic tests
ii. Solution to Board Question Paper of March 2023
iii. Model Question Paper with Solution

Board Questions

Includes relevant Board questions from March 1996 to July 2022

PAPER PATTERN

- There will be a single paper of 80 Marks in Mathematics and Statistics.
- Duration of the paper will be 3 hours.

Section A: **(20 Marks)**

This section will contain Multiple Choice Questions and Very Short Answer(VSA) type of questions.
There will be 8 MCQs, each carrying two marks
and 4 VSA type of questions, each carrying one mark.
Students will have to attempt all these questions.

Section B: **(16 Marks)**

This section will contain 12 Short Answer (SA-I) type of questions, each carrying 2 marks.
Students will have to attempt any 8 questions.

Section C: **(24 Marks)**

This section will contain 12 Short Answer (SA-II) type of questions, each carrying 3 marks.
Students will have to attempt any 8 questions.

Section D: **(20 Marks)**

This section will contain 8 Long Answer (LA) type of questions, each carrying 4 marks.
Students will have to attempt any 5 questions.

Distribution of Marks According to the Type of Questions

Type of questions		No of questions	Total Marks (with option)
MCQ	2 Marks each	Q.No. 1 (i to viii)	16 Marks
VSA	1 Mark each	Q.No. 2 (i to iv)	4 Marks
SA - I	2 Marks each	Q.No. 3 to 14	24 Marks
SA - II	3 Marks each	Q.No. 15 to 26	36 Marks
LA	4 Marks each	Q.No. 27 to 34	32 Marks

CONTENTS

[Reference: Maharashtra State Board of Secondary and Higher Secondary Education, Pune - 04**]**

1 Mathematical Logic

Syllabus

- Statement and its truth value
- Logical connectives, Compound statements
- Truth tables, negation of statements and compound statements
- Statement pattern, Logical equivalence
- Tautology, contradiction and contingency
- Quantifiers and quantified statements, Duality
- Application of logic to switching circuits, Switching table

Let's Study

Introduction:

Mathematics is an exact science. Every mathematical statement must be precise. Hence, there has to be proper reasoning in every mathematical proof.

Proper reasoning involves logic. The study of logic helps in increasing one's ability of systematic and logical reasoning. It also helps to develop the skills of understanding various statements and their validity.

Logic has a wide scale application in circuit designing, computer programming etc. Hence, the study of logic becomes essential.

Statement and its truth value

There are various means of communication viz., verbal, written etc. Most of the communication involves the use of language whereby, the ideas are conveyed through sentences.

There are various types of sentences such as:

i. Declarative (Assertive)
ii. Imperative (A command or a request)
iii. Exclamatory (Emotions, excitement)
iv. Interrogative (Question)

Statement:

A statement is a declarative sentence which is either true or false but not both simultaneously. Statements are denoted by the letters p, q, r….

For example:

i. 3 is an odd number.
ii. 5 is a perfect square.
iii. Sun rises in the east.
iv. $x + 3 = 6$, when $x = 3$.

Truth Value:

A statement is either True or False. The Truth value of a 'true' statement is defined to be T (TRUE) and that of a 'false' statement is defined to be F (FALSE).

Note: 1 and 0 can also be used for T and F respectively.

Consider the following statements:

i. There is no prime number between 23 and 29.
ii. The Sun rises in the west.
iii. The square of a real number is negative.
iv. The sum of the angles of a plane triangle is 180°.

Here, the truth value of statement i. and iv. is T and that of ii. and iii. is F.

Note:

The sentences like exclamatory, interrogative, imperative etc., are not considered as statements as the truth value for these statements cannot be determined.

Open sentence:

An open sentence is a sentence whose truth can vary according to some conditions, which are not stated in the sentence.

Note:

Open sentence is not considered as statement in logic.

For example:

i. $x \times 5 = 20$
This is an open sentence as its truth depends on value of x (if $x = 4$, it is true and if $x \neq 4$, it is false).

ii. Chinese food is very tasty.
This is an open sentence as its truth varies from individual to individual.

Logical Connectives, Compound Statements and Truth Tables

Logical Connectives:
The words or group of words such as "and, or, if …. then, if and only if, not" are used to join or connect two or more simple sentences. These connecting words are called logical connectives.

Compound Statements:
The new statement that is formed by combining two or more simple statements by using logical connectives are called compound statements.

For example:
Consider the following simple statements,

i. e is a vowel
ii. b is a consonant

These two component statements can be joined by using the logical connective 'or' as shown below:

'e is a vowel or b is a consonant'

The above statement is called **compound statement** formed by using logical connective **'or'.**

Truth Table:

A table that shows the relationship between truth values of simple statements and the truth values of compounds statements formed by using these simple statements is called truth table.

Note:
The truth value of a compound statement depends upon the truth values of its simple statements.

Logical Connectives

1. AND [$\wedge$] (Conjunction):

If p and q are any two statements connected by the word 'and', then the resulting compound statement 'p and q' is called conjunction of p and q which is written in the symbolic form as '$p \wedge q$'.

For example:
p: Today is a pleasant day.
q: I want to go for shopping.
The conjunction of above two statements is '$p \wedge q$', i.e., 'Today is a pleasant day and I want to go for shopping'.
A conjunction is true if and only if both p and q are true.
Truth table for conjunction of p and q is as shown below:

p	q	$p \wedge q$
T	**T**	**T**
T	F	F
F	T	F
F	F	F

Note:
The words such as 'but', 'yet', 'still', 'inspite', 'though', 'moreover', 'while', 'neither…nor' are also use to connect the simple statements.
These words are generally used by replacing 'and'.

2. OR [$\vee$] (Disjunction):

If p and q are any two statements connected by the word 'or', then the resulting compound statement 'p or q' is called disjunction of p and q which is written in the symbolic form as '$p \vee q$'.
The word 'or' is used in English language in two distinct senses, exclusive and inclusive.
For example:

i. Rahul will pass or fail in the exam.
ii. Candidate must be graduate or post-graduate.

In eg. (i), 'or' indicates that only one of the two possibilities exists but not both which is called exclusive sense of 'or'. In eg. (ii), 'or' indicates that first or second or both the possibilities may exist which is called inclusive sense of 'or'.
A disjunction is false only when both p and q are false.
Truth table for disjunction of p and q is as shown below:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	**F**	**F**

3. Not [~] (Negation):

If p is any statement then negation of p, i.e., 'not p' is denoted by ~p. Negation of any simple statement p can also be formed by writing 'It is not true that' or 'It is false that', before p.
For example:
p : Mango is a fruit.
~p : Mango is not a fruit.
Truth table for negation is as shown below:

p	~p
T	F
F	T

Note: If a statement is true its negation is false and vice-versa.

4. If….then (Implication, $\longrightarrow$) (Conditional):

If p and q are any two simple statements, then the compound statement, 'if p then q', meaning "statement p implies statement q or statement q is implied by statement p", is called a conditional statement and is denoted by $p \to q$ or $p \Rightarrow q$.

Here, p is called the antecedent (hypothesis) and q is called the consequent (conclusion).

For example:

Let p: I travel by train.

q: My journey will be cheaper.

Here, the conditional statement is

'$p \to q$: If I travel by train then my journey will be cheaper.'

Conditional statement is false if and only if antecedent is true and consequent is false.

Truth table for conditional is as shown below:

p	q	$p \to q$
T	T	T
T	**F**	**F**
F	T	T
F	F	T

Note: Equivalent forms of the conditional statement $p \to q$:

a. p is sufficient for q.
b. q is necessary for p.
c. p implies q.
d. p only if q.
e. q follows from p.
f. q provided that p.
g. q whenever p.

5. If and only if (Double Implication, $\leftrightarrow$) (Biconditional):

If p and q are any two statements, then 'p if and only if q' or 'p iff q' is called the biconditional statement and is denoted by $p \leftrightarrow q$. Here, both p and q are called implicants.

For example:

Let p : price increases.

q : demand falls.

Here, the Biconditional statement is

'$p \leftrightarrow q$: Price increases if and only if demand falls'.

A biconditional statement is true if and only if both the implicants have same truth value.

Truth table for biconditional is as shown below:

p	q	$p \leftrightarrow q$
T	**T**	**T**
T	F	F
F	T	F
F	**F**	**T**

Note: Equivalent forms of the biconditional statement $p \leftrightarrow q$:

a. p is necessary and sufficient condition for q.
b. q is necessary for p and p is necessary for q.
c. p is sufficient for q and q is sufficient for p.
d. biconditional statement is conjunction of $p \to q$ and $q \to p$.

Exercise 1.1

1. State which of the following are statements. Justify. In case of statement, state its truth value.

i. **$5 + 4 = 13$.**
ii. **$x - 3 = 14$.**
iii. **Close the door.**
iv. **Zero is a complex number.**
v. **Please get me breakfast.**
vi. **Congruent triangles are similar.**
vii. **$x^2 = x$.**
viii. **A quadratic equation cannot have more than two roots.**
ix. **Do you like Mathematics?**
x. **The sun sets in the west.**
xi. **All real numbers are whole numbers.**
xii. **Can you speak in Marathi?**
xiii. **$x^2 - 6x - 7 = 0$, when $x = 7$.**
xiv. **The sum of cube roots of unity is zero.**
xv. **It rains heavily.**

***Solution*:**

i. It is a statement which is false. Hence, its truth value is F.
ii. It is an open sentence. Hence, it is not a statement.
iii. It is an order. Hence, it is not a statement.
iv. It is a statement which is true. Hence, its truth value is T.
v. It is a request. Hence, it is not a statement.
vi. It is a statement which is true. Hence, its truth value is T.
vii. It is an open sentence. Hence, it is not a statement.
viii. It is a statement which is true. Hence, its truth value is T.
ix. It is an interrogative sentence. Hence, it is not a statement.
x. It is a statement which is true. Hence, its truth value is T.
xi. It is a statement which is false. Hence, its truth value is F.
xii. It is an interrogative sentence. Hence, it is not a statement.
xiii. It is a statement which is true. Hence, its truth value is T.
xiv. It is a statement which is true. Hence, its truth value is T.
xv. It is an open sentence. Hence, it is not a statement.

2. Write the following compound statements symbolically.

i. **Nagpur is in Maharashtra and Chennai is in Tamilnadu.**
ii. **Triangle is equilateral or isosceles.**

iii. **The angle is right angle if and only if it is of measure 90º.**
iv. **Angle is neither acute nor obtuse.**
v. **If ΔABC is right angled at B, then $m\angle A + m\angle C = 90^\circ$.**
vi. **Hima Das wins gold medal if and only if she runs fast.**
vii. ***x* is not irrational number but is a square of an integer.**

***Solution*:**

i. Let p : Nagpur is in Maharashtra.
q : Chennai is in Tamilnadu.
The symbolic form is $p \wedge q$.

ii. Let p : Triangle is equilateral.
q : Triangle is isosceles.
The symbolic form is $p \vee q$.

iii. Let p : The angle is right angle.
q : The angle is of measure 90°.
The symbolic form is $p \leftrightarrow q$.

iv. Let p : Angle is acute.
q : Angle is obtuse.
The symbolic form is $\sim p \wedge \sim q$.

v. Let p : ΔABC is right angled at B.
q : $m\angle A + m\angle C = 90^\circ$
The symbolic form is $p \to q$.

vi. Let p : Hima Das wins gold medal.
q : Hima Das runs fast.
The symbolic form is $p \leftrightarrow q$.

vii. Let p : x is an irrational number.
q : x is a square of an integer.
The symbolic form is $\sim p \wedge q$.

3. **Write the truth values of the following.**
i. **4 is odd or 1 is prime.**
ii. **64 is a perfect square and 46 is a prime number.**
iii. **5 is a prime number and 7 divides 94.**
iv. **It is not true that 5 – 3i is a real number.**
v. **If $3 \times 5 = 8$ then $3 + 5 = 15$.**
vi. **Milk is white if and only if sky is blue.**
vii. **24 is a composite number or 17 is a prime number.**

***Solution*:**

i. Let p : 4 is odd.
q : 1 is prime.
Truth values of p and q are F and F respectively.
The given statement in symbolic form is $p \vee q$.
∴ $p \vee q \equiv F \vee F \equiv F$
∴ Truth value of the given statement is F.

ii. Let p : 64 is a perfect square.
q : 46 is a prime number.
Truth values of p and q are T and F respectively.
The given statement in symbolic form is $p \wedge q$.
∴ $p \wedge q \equiv T \wedge F \equiv F$
∴ Truth value of the given statement is F.

iii. Let p : 5 is a prime number.
q : 7 divides 94.
Truth values of p and q are T and F respectively.
The given statement in symbolic form is $p \wedge q$.
∴ $p \wedge q \equiv T \wedge F \equiv F$
∴ Truth value of the given statement is F.

iv. Let p : 5 – 3i is a real number.
Truth values of p is F.
The given statement in symbolic form is $\sim p$.
∴ $\sim p \equiv \sim F \equiv T$
∴ Truth value of the given statement is T.

v. Let p : $3 \times 5 = 8$
q : $3 + 5 = 15$
Truth values of p and q are F and F respectively.
The given statement in symbolic form is $p \to q$.
∴ $p \to q \equiv F \to F \equiv T$
∴ Truth value of the given statement is T.

vi. Let p : Milk is white.
q : Sky is blue.
Truth values of p and q are T and T respectively.
The given statement in symbolic form is $p \leftrightarrow q$.
∴ $p \leftrightarrow q \equiv T \leftrightarrow T \equiv T$
∴ Truth value of the given statement is T.

vii. Let p : 24 is a composite number.
q : 17 is a prime number.
Truth values of p and q are T and T respectively.
The given statement in symbolic form is $p \vee q$.
∴ $p \vee q \equiv T \vee T \equiv T$
∴ Truth value of the given statement is T.

4. **If the statements p, q are true statements and r, s are false statements, then determine the truth values of the following.**
i. $p \vee (q \wedge r)$
ii. $(p \to q) \vee (r \to s)$
iii. $(q \wedge r) \vee (\sim p \wedge s)$
iv. $(p \to q) \wedge \sim r$
v. $(\sim r \leftrightarrow p) \to \sim q$
vi. $[\sim p \wedge (\sim q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)]$
vii. $[(\sim p \wedge q) \wedge \sim r] \vee [(q \to p) \to (\sim s \vee r)]$
viii. $\sim[(\sim p \wedge r) \vee (s \to \sim q)] \leftrightarrow (p \wedge r)$

***Solution*:**

i. $p \vee (q \wedge r) \equiv T \vee (T \wedge F) \equiv T \vee F \equiv T$
Hence, truth value is T.

ii. $(p \to q) \vee (r \to s) \equiv (T \to T) \vee (F \to F)$

$\equiv T \vee T$

$\equiv T$

Hence, truth value is T.

iii. $(q \wedge r) \vee (\sim p \wedge s) \equiv (T \wedge F) \vee (\sim T \wedge F)$

$\equiv F \vee (F \wedge F)$

$\equiv F \vee F$

$\equiv F$

Hence, truth value is F.

iv. $(p \to q) \wedge \sim r \equiv (T \to T) \wedge \sim F \equiv T \wedge T \equiv T$

Hence, truth value of T.

v. $(\sim r \leftrightarrow p) \to \sim q \equiv (\sim F \leftrightarrow T) \to \sim T$

$\equiv (T \leftrightarrow T) \to F$

$\equiv T \to F$

$\equiv F$

Hence, truth value is F.

vi. $[\sim p \wedge (\sim q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)]$

$\equiv [\sim T \wedge (\sim T \wedge F)] \vee [(T \wedge F) \vee (T \wedge F)]$

$\equiv [F \wedge (F \wedge F)] \vee [(F \vee F)]$

$\equiv (F \wedge F) \vee F$

$\equiv F \vee F$

$\equiv F$

Hence, truth value is F.

vii. $[(\sim p \wedge q) \wedge \sim r] \vee [(q \to p) \to (\sim s \vee r)]$

$\equiv [(\sim T \wedge T) \wedge \sim F] \vee [(T \to T) \to (\sim F \vee F)]$

$\equiv [(F \wedge T) \wedge T] \vee [T \to (T \vee F)]$

$\equiv (F \wedge T) \vee (T \to T)$

$\equiv F \vee T$

$\equiv T$

Hence, truth value is T.

viii. $\sim[(\sim p \wedge r) \vee (s \to \sim q)] \leftrightarrow (p \wedge r)$

$\equiv \sim[(\sim T \wedge F) \vee (F \to \sim T)] \leftrightarrow (T \wedge F)$

$\equiv \sim[(F \wedge F) \vee (F \to F)] \leftrightarrow F$

$\equiv \sim(F \vee T) \leftrightarrow F$

$\equiv \sim T \leftrightarrow F$

$\equiv F \leftrightarrow F$

$\equiv T$

Hence, truth value is T.

5. Write the negations of the following.

i. Tirupati is in Andhra Pradesh.

ii. 3 is not a root of the equation $x^2 + 3x - 18 = 0$.

iii. $\sqrt{2}$ is a rational number.

iv. Polygon ABCDE is a pentagon.

v. $7 + 3 > 5$

***Solution*:**

i. Tirupati is not in Andhra Pradesh.

ii. 3 is a root of the equation $x^2 + 3x - 18 = 0$.

iii. $\sqrt{2}$ is not a rational number.

iv. Polygon ABCDE is not a pentagon.

v. $7 + 3 \not> 5$

Let's Study

Statement Pattern and Logical Equivalence

Statement Pattern

Let, p, q, r,… be simple statements. A compound statement obtained from these simple statements and by using one or more connectives $\wedge$, $\vee$, $\sim$, $\to$, $\leftrightarrow$ is called a statement pattern.

Following points must be noted while preparing truth tables of the statement patterns:

i. Parentheses must be introduced wherever necessary.

For example:

$\sim(p \wedge q)$ and $\sim p \wedge q$ are not the same.

ii. If a statement pattern consists of 'n' statements and 'm' connectives, then truth table consists of 2^n rows and $(m + n)$ columns.

Logical equivalence

Two logical statements are said to be equivalent if and only if the truth values in their respective columns in the joint truth table are identical.

If S_1 and S_2 are logically equivalent statementpatterns, we write $S_1 \equiv S_2$.

For example:

To prove: $p \wedge q \equiv \sim(p \to \sim q)$ **[Mar 08]**

p	q	$p \wedge q$	$\sim q$	$(p \to \sim q)$	$\sim(p \to \sim q)$
T	T	T	F	F	T
T	F	F	T	T	F
F	T	F	F	T	F
F	F	F	T	T	F

In the above truth table, all the entries in the columns of $p \wedge q$ and $\sim(p \to \sim q)$ are identical.

$\therefore$ **$p \wedge q \equiv \sim(p \to \sim q)$.**

Note:

i. $\sim(p \vee q) \equiv \sim p \wedge \sim q$ (De-Morgan's 1st Law)

[Mar 96; Feb 20]

identical

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

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To see complete chapter buy **Target Notes** or **Target E-Notes**

5. Obtain the simple logical expression of the following. Draw the corresponding switching circuit.

i. $\mathbf{p \vee (q \wedge \sim q)}$

ii. $\mathbf{(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)}$

iii. $\mathbf{[p \vee (\sim q \vee \sim r)] \wedge [p \vee (q \wedge r)]}$

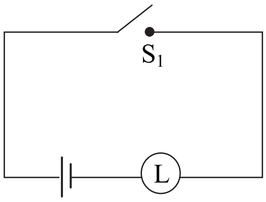
iv. $\mathbf{(p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (p \wedge q \wedge r)}$

***Solution*:**

i. Consider, $p \vee (q \wedge \sim q)$

$\equiv p \vee F$ [Complement law]

$\equiv p$ [Identity law]



ii. Consider, $(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$

$\equiv [(\sim p \wedge q) \vee (\sim p \wedge \sim q)] \vee (p \wedge \sim q)$ [Associative law]

$\equiv [\sim p \wedge (q \vee \sim q)] \vee (p \wedge \sim q)$ [Distributive law]

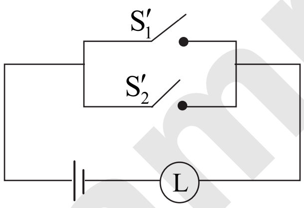
$\equiv (\sim p \wedge T) \vee (p \wedge \sim q)$ [Complement law]

$\equiv \sim p \vee (p \wedge \sim q)$ [Identity law]

$\equiv (\sim p \vee p) \wedge (\sim p \vee \sim q)$ [Distributive law]

$\equiv T \wedge (\sim p \vee \sim q)$ [Complement law]

$\equiv \sim p \vee \sim q$ [Identity law]



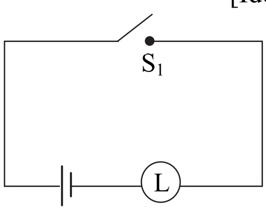
iii. Consider, $[p \vee (\sim q \vee \sim r)] \wedge [p \vee (q \wedge r)]$

$\equiv p \vee [(\sim q \vee \sim r) \wedge (q \wedge r)]$ [Distributive law]

$\equiv p \vee [\sim(q \wedge r) \wedge (q \wedge r)]$ [De Morgan's law]

$\equiv p \vee F$ [Complement law]

$\equiv p$ [Identity law]



iv. $(p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (p \wedge q \wedge r)$

$\equiv (p \wedge \sim p \wedge q) \vee (\sim p \wedge q \wedge r) \vee (\sim q \wedge p \wedge r) \vee (q \wedge p \wedge r)$ [Commutative law]

$\equiv (F \wedge q) \vee (\sim p \wedge q \wedge r) \vee (\sim q \wedge p \wedge r) \vee (q \wedge p \wedge r)$ [Complement law]

$\equiv F \vee (\sim p \wedge q \wedge r) \vee (\sim q \wedge p \wedge r) \vee (q \wedge p \wedge r)$ [Identity law]

$\equiv (\sim p \wedge q \wedge r) \vee (\sim q \wedge p \wedge r) \vee (q \wedge p \wedge r)$ [Identity law]

$\equiv (\sim p \wedge q \wedge r) \vee [(\sim q \vee q) \wedge (p \wedge r)]$ [Distributive law]

$\equiv (\sim p \wedge q \wedge r) \vee [T \wedge (p \wedge r)]$ [Complement law]

$\equiv (\sim p \wedge q \wedge r) \vee (p \wedge r)$ [Identity law]

$\equiv [(\sim p \wedge q) \vee p] \wedge r$ [Distributive law]

$\equiv [(\sim p \vee p) \wedge (q \vee p)] \wedge r$ [Distributive law]

$\equiv [T \wedge (q \vee p)] \wedge r$ [Complement law]

$\equiv (q \vee p) \wedge r$ [Identity law]

[Note: Answer given in the textbook is '$(q \wedge r) \vee p$'. However, we found that it is '$(q \vee p) \wedge r$'.**]**

Miscellaneous Exercise – 1

1. Select and write the correct answer from the given alternatives in each of the following questions:

i. If $p \wedge q$ is false and $p \vee q$ is true, the _______ is not true.

(A) $p \vee q$ (B) $p \leftrightarrow q$
(C) $\sim p \vee \sim q$ (D) $q \vee \sim p$

ii. $(p \wedge q) \rightarrow r$ is logically equivalent to _______.

(A) $p \rightarrow (q \rightarrow r)$ (B) $(p \wedge q) \rightarrow \sim r$
(C) $(\sim p \vee \sim q) \rightarrow \sim r$ (D) $(p \vee q) \rightarrow r$

iii. Inverse of statement pattern $(p \vee q) \rightarrow (p \wedge q)$ is _______.

(A) $(p \wedge q) \rightarrow (p \vee q)$
(B) $\sim(p \vee q) \rightarrow (p \wedge q)$
(C) $(\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$
(D) $(\sim p \vee \sim q) \rightarrow (\sim p \wedge \sim q)$

iv. If $p \wedge q$ is F, $p \rightarrow q$ is F then the truth values of p and q are _______. **[Oct 15]**

(A) T, T (B) T, F
(C) F, T (D) F, F

v. The negation of inverse of $\sim p \rightarrow q$ is _______.

(A) $q \wedge p$ (B) $\sim p \wedge \sim q$
(C) $p \vee q$ (D) $\sim q \rightarrow \sim p$

vi. The negation of $p \wedge (q \rightarrow r)$ is _______.

(A) $\sim p \wedge (\sim q \rightarrow \sim r)$ (B) $p \vee (\sim q \vee r)$
(C) $\sim p \vee (\sim q \rightarrow \sim r)$ (D) $\sim p \vee (q \wedge \sim r)$

[Note: *Option (C) has been modified.*]

vii. If A = {1, 2, 3, 4, 5} then which of the following is not true?
(A) $\exists\, x \in$ A such that $x + 3 = 8$
(B) $\exists\, x \in$ A such that $x + 2 < 9$
(C) $\forall\, x \in$ A, $x + 6 \geq 9$
(D) $\exists\, x \in$ A such that $x + 6 < 10$

Answers:

i.	(B)	ii.	(A)	iii.	(C)	iv.	(B)
v.	(A)	vi.	(D)	vii.	(C)		

Hints:

i. $p \wedge q$ is false and $p \vee q$ is true.
∴ One of the p and q is true and other one is false.
∴ In this case, $p \leftrightarrow q$ cannot be true.

ii. Consider,
$(p \wedge q) \rightarrow r \equiv \sim(p \wedge q) \vee r$ $[p \rightarrow q \equiv \sim p \vee q]$
$\equiv \sim p \vee \sim q \vee r$ [De Morgan's law]
$\equiv \sim p \vee (\sim q \vee r)$ [Associative law]
$\equiv \sim p \vee (q \rightarrow r)$ $[p \rightarrow q \equiv \sim p \vee q]$
$\equiv p \rightarrow (q \rightarrow r)$ $[p \rightarrow q \equiv \sim p \vee q]$

iii. Inverse of $p \rightarrow q$ is $\sim p \rightarrow \sim q$
∴ Inverse of $[(p \vee q) \rightarrow (p \wedge q)]$
$\equiv \sim(p \vee q) \rightarrow \sim(p \wedge q)$
$\equiv (\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$

v. Inverse of $\sim p \rightarrow q \equiv p \rightarrow \sim q$.
∴ Negation of inverse of $\sim p \rightarrow q$
$\equiv \sim(p \rightarrow \sim q)$
$\equiv p \wedge q$ $[\sim(p \rightarrow q) \equiv p \wedge \sim q]$

vi. $\sim[p \wedge (q \rightarrow r)]$
$\equiv \sim p \vee \sim(q \rightarrow r)$
$\equiv \sim p \vee (q \wedge \sim r)$ $[\sim(p \rightarrow q) \equiv p \wedge \sim q]$

2. Which of the following sentences are statements in logic? Justify. Write down the truth value of the statements:
i. 4! = 24.
ii. π is an irrational number.
iii. India is a country and Himalayas is a river.
iv. Please get me a glass of water.
v. $\cos^2\theta - \sin^2\theta = \cos 2\theta$ for all $\theta \in$ R.
vi. If x is a whole number, then $x + 6 = 0$.

***Solution*:**
i. It is a statement which is true. Hence, its truth value is T.

ii. It is a statement which is true. Hence, its truth value is T.

iii. It is a statement which is false. Hence, its truth value is F.

iv. It is a request. Hence, it is not a statement.

v. It is a statement which is true. Hence, its truth value is T.

vi. It is a statement which is false. Hence, its truth value is F.
[Note: Answer given in the textbook is 'T'. However, we found that it is 'F'.**]**

3. Write the truth values of the following statements:
i. $\sqrt{5}$ is an irrational but $3\sqrt{5}$ is a complex number.
ii. $\forall$ n $\in$ N, $n^2 + n$ is even number while $n^2 - n$ is an odd number.
iii. $\exists$ n $\in$ N such that n + 5 > 10. [Oct 14]
iv. The square of any even number is odd or the cube of any odd number is odd.
v. In ΔABC, if all sides are equal, then it's all angles are equal.
vi. $\forall$ n $\in$ N, n + 6 > 8.

***Solution*:**
i. Let p : $\sqrt{5}$ is an irrational number.
q : $3\sqrt{5}$ is a complex number.
Truth values of p and q are T and T respectively.
The given statement in symbolic form is $p \wedge q$.
∴ $p \wedge q \equiv T \wedge T \equiv T$
∴ Truth value of the given statement is T.

ii. For n = 1, $n^2 - n = 1 - 1 = 0$ which is not an odd number.
∴ n = 1 does not satisfies the given statement.
∴ The given statement is wrong.
∴ Its truth value is F.

iii. For n = 6, n + 5 = 6 + 5 = 11 > 10
∴ n = 6 satisfies the equation n + 5 > 10
∴ The given statement is true.
∴ Its truth value is T.

iv. Let p : Square of ony even number is odd.
q : Cube of any odd number is odd.
Truth values of p and q are F and T respectively.
The given statement in symbolic form is $p \vee q$.
∴ $p \vee q \equiv F \vee T \equiv T$
∴ Truth value of the given statement is T.

v. For ΔABC, if all sides are equal, then it is an equilateral triangle.
Equilateral triangle is also an equiangular triangle.
∴ The given statement is true.
∴ Its truth value is T.

vi. For n = 1, n + 6 = 1 + 6 = 7 $\not>$ 8
∴ n = 1 does not satisfies the given statement.
∴ The given statement is wrong.
∴ Its truth value is F.

4. If A = {1, 2, 3, 4, 5, 6, 7, 8, 9}, determine the truth value of each of the following statement:

i. $\exists\, x \in$ **A such that** $x + 8 = 15$.

ii. $\forall\, x \in$ **A,** $x + 5 < 12$.

iii. $\exists\, x \in$ **A, such that** $x + 7 \geq 11$.

iv. $\forall\, x \in$ **A,** $3x \leq 25$.

***Solution*:**

i. For $x = 7$, $x + 8 = 7 + 8 = 15$

$\therefore$ $x = 7$ satisfies the equation $x + 8 = 15$.

$\therefore$ The given statement is true.

$\therefore$ Its truth value is T.

ii. For $x = 7$, $x + 5 = 7 + 5 = 12 \nless 12$

$\therefore$ $x = 7$ does not satisfies the given statement.

$\therefore$ The given statement is wrong.

$\therefore$ Its truth value is F.

iii. For $x = 4$, $x + 7 = 4 + 7 = 11 \geq 11$

$\therefore$ $x = 4$ satisfies the equation $x + 7 \geq 11$.

$\therefore$ The given statement is true.

$\therefore$ Its truth value is T.

iv. For $x = 9$, $3x = 3 \times 9 = 27 \nleq 25$

$\therefore$ $x = 9$ does not satisfies the given statement.

$\therefore$ The given statement is wrong.

$\therefore$ Its truth value is F.

5. Write the negations of the following:

i. $\forall$ **n** $\in$ **A, n + 7 > 6.**

ii. $\exists\, x \in$ **A, such that** $x + 9 \leq 15$.

iii. Some triangles are equilateral triangle.

***Solution*:**

i. $\exists$ n $\in$ A such that n + 7 $\ngtr$ 6.

OR

$\exists$ n $\in$ A such that n + 7 $\leq$ 6.

ii. $\forall\, x \in$ A, $x + 9 \nleq 15$

OR

$\forall\, x \in$ A, $x + 9 > 15$

iii. All triangles are not equilateral triangles.

6. Construct the truth table for each of the following:

i. $p \rightarrow (q \rightarrow p)$

ii. $(\sim p \vee \sim q) \leftrightarrow [\sim(p \wedge q)]$

iii. $\sim(\sim p \wedge \sim q) \vee q$

iv. $[(p \wedge q) \vee r] \wedge [\sim r \vee (p \wedge q)]$

v. $[(\sim p \vee q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

***Solution*:**

i. $p \rightarrow (q \rightarrow p)$

1	2	3	4
p	q	$(q \rightarrow p)$	$p \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

ii. $(\sim p \vee \sim q) \leftrightarrow [\sim (p \wedge q)]$

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \wedge q$	$\sim(p \wedge q)$	$(\sim p \vee \sim q) \leftrightarrow [\sim(p \wedge q)]$
T	T	F	F	F	T	F	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	T	T

iii. *Refer Exercise 1.2: Q.3 (iii)*

[Note: Answer given in the textbook is 'TTTT'. However, we found that it is 'TTTF'.**]**

iv. $[(p \wedge q) \vee r] \wedge [\sim r \vee (p \wedge q)]$

p	q	r	$p \wedge q$	$[(p \wedge q) \vee r]$	$\sim r$	$[\sim r \vee (p \wedge q)]$	$[(p \wedge q) \vee r] \wedge [\sim r \vee (p \wedge q)]$
T	T	T	T	T	F	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	F	F	F
T	F	F	F	F	T	T	F
F	T	T	F	T	F	F	F
F	T	F	F	F	T	T	F
F	F	T	F	T	F	F	F
F	F	F	F	F	T	T	F

v. $[(\sim p \vee q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

p	q	r	$\sim p$	$\sim p \vee q$	$q \rightarrow r$	$(\sim p \vee q) \wedge (q \rightarrow r)$	$p \rightarrow r$	$[(\sim p \vee q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	F	F	T	F	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

7. Determine whether the following statement patterns are tautologies contradictions or contingencies:

i. $[(p \to q) \wedge \sim q)] \to \sim p$
ii. $[(p \vee q) \wedge \sim p] \wedge \sim q$
iii. $(p \to q) \wedge (p \wedge \sim q)$
iv. $[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$
v. $[(p \wedge (p \to q)] \to q$
vi. $(p \wedge q) \vee (\sim p \wedge q) \vee (p \vee \sim q) \vee (\sim p \wedge \sim q)$
vii. $[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$
viii. $(p \to q) \vee (q \to p)$

***Solution*:**

i. *Refer Exercise 1.2: Q 3 (v)*

---

ii. $[(p \vee q) \wedge \sim p] \wedge \sim q$

p	q	~p	~q	$p \vee q$	$[(p \vee q) \wedge \sim p]$	$[(p \vee q) \wedge \sim p] \wedge (\sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	F
F	F	T	T	F	F	F

All the truth values in the last column are F.
Hence, it is contradiction.

---

iii. $(p \to q) \wedge (p \wedge \sim q)$

p	q	$p \to q$	~q	$p \wedge \sim q$	$(p \to q) \wedge (p \wedge \sim q)$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	F	T	T	F	F

All the truth values in the last column are F.
Hence, it is contradiction.

---

iv. $[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$

p	q	r	$q \to r$	$[p \to (q \to r)]$	$(p \wedge q)$	$[(p\wedge q) \to r]$	$[p \to (q \to r)] \leftrightarrow [(p\wedge q) \to r]$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

All the truth vlaues in the last column are T.
Hence, it is tautology.

---

v. $[(p \wedge (p \to q)] \to q$

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

All the truth values in the last column are T.
Hence, it is tautology.

vi. $(p \wedge q) \vee (\sim p \wedge q) \vee (p \vee \sim q) \vee (\sim p \wedge \sim q)$

p	q	~p	~q	$p \wedge q$	$\sim p \wedge q$	$p \vee \sim q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge q) \vee (p \vee \sim q) \vee (\sim p \wedge \sim q)$
T	T	F	F	T	F	T	F	T
T	F	F	T	F	F	T	F	T
F	T	T	F	F	T	F	F	T
F	F	T	T	F	F	T	T	T

All the truth value in the last column are T.
Hence, it is tautology.

---

vii. $[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$

p	q	r	~p	~q	$p \vee \sim q$	$\sim p \wedge q$	$(p \vee \sim q) \vee (\sim p \wedge q)$	$[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$
T	T	T	F	F	T	F	T	T
T	T	F	F	F	T	F	T	F
T	F	T	F	T	T	F	T	T
T	F	F	F	T	T	F	T	F
F	T	T	T	F	F	T	T	T
F	T	F	T	F	F	T	T	F
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	F

Truth values in the last column are not identical.
Hence, it is contingency

---

viii. $(p \to q) \vee (q \to p)$

p	q	$(p \to q)$	$(q \to p)$	$(p \to q) \vee (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

All the truth values in the last column are T.
Hence, it is tautology.

---

8. Determine the truth values of p and q in the following cases:

i. $(p \vee q)$ **is T and** $(p \wedge q)$ **is T**
ii. $(p \vee q)$ **is T and** $(p \vee q) \to q$ **is F**
iii. $(p \wedge q)$ **is F and** $(p \wedge q) \to q$ **is T**

***Solution*:**

i. $(p \vee q)$ is T and $(p \wedge q)$ is T.
Consider the following truth table:

p	q	$p \vee q$	$p \wedge q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

$\therefore$ If $(p \vee q)$ is T and $(p \wedge q)$ is T, then both p and q have to be true, i.e., their truth value must be T.

ii. $(p \vee q)$ is T and $(p \vee q) \to q$ is F.
Consider the following truth table:

p	q	$p \vee q$	$(p \vee q) \to q$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	T

$\therefore$ If $(p \vee q)$ is T and $(p \vee q) \to q$ is F, then p is true and q is false, i.e., truth value of p is T and that of q is F.

---

iii. $(p \wedge q)$ is F and $(p \wedge q) \to q$ is T
Consider the following truth table:

p	q	$p \wedge q$	$(p \wedge q) \to q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$\therefore$ If $(p \wedge q)$ is F and $(p \wedge q) \to q$ is T, then there are three possibilities for the truth value of p and q.

Either, p is T and q is F or
p is F and q is T or
both p and q are F.

---

9. Using truth tables prove the following logical equivalences:

i. $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$
[Oct 15; Mar 18; July 19]

ii. $(p \wedge q) \to r \equiv p \to (q \to r)$

***Solution*:**

i. $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

1	2	3	4	5	6	7	8
p	q	$p \leftrightarrow q$	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
T	T	T	F	F	T	F	T
T	F	F	F	T	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	T	F	T	T

In the above truth table, the entries in the columns 3 and 8 are identical.

$\therefore$ $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

---

ii.

1	2	3	4	5	6	7
p	q	r	$p \wedge q$	$q \to r$	$(p \wedge q) \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	T	T	T
F	T	F	F	F	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

In the above truth table, the entries in the columns 6 and 7 are identical.

$\therefore$ $(p \wedge q) \to r \equiv p \to (q \to r)$

---

10. Using rules in logic, prove the following:

i. $p \leftrightarrow q \equiv \sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)$

ii. $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

iii. $\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$ [Mar 16]

***Solution*:**

i. RHS

$= \sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)$

$\equiv \sim[(p \wedge \sim q) \vee (q \wedge \sim p)]$ [De Morgan's law]

$\equiv \sim[\sim(p \leftrightarrow q)]$

[$\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$]

$\equiv p \leftrightarrow q$ [Negation of negation]

$=$ LHS

---

ii. RHS

$= (p \vee q) \wedge \sim p$

$\equiv (p \wedge \sim p) \vee (q \wedge \sim p)$

[Distributive law]

$\equiv (p \wedge \sim p) \vee (\sim p \wedge q)$ [Commutative law]

$\equiv F \vee (\sim p \wedge q)$ [Complement law]

$\equiv \sim p \wedge q$ [Identity law]

$=$ LHS

$\therefore$ $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

---

iii. LHS

$= \sim(p \vee q) \vee (\sim p \wedge q)$

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ [De Morgan's law]

$\equiv \sim p \wedge (\sim q \vee q)$ [Distributive law]

$\equiv \sim p \wedge T$ [Complement law]

$\equiv \sim p$ [Identity law]

$=$ RHS

---

11. Using the rules in logic, write the negations of the following:

i. $(p \vee q) \wedge (q \vee \sim r)$ **ii. $p \wedge (q \vee r)$**

iii. $(p \to q) \wedge r$

iv. $(\sim p \wedge q) \vee (p \wedge \sim q)$

***Solution*:**

i. $\sim[(p \vee q) \wedge (q \vee \sim r)]$

$\equiv \sim(p \vee q) \vee \sim(q \vee \sim r)$ [De Morgan's law]

$\equiv (\sim p \wedge \sim q) \vee (\sim q \wedge r)$ [De Morgan's law]

$\equiv (\sim q \wedge \sim p) \vee (\sim q \wedge r)$ [Commutative law]

$\equiv \sim q \wedge (\sim p \vee r)$ [Distributive law]

ii. $\sim[p \wedge (q \vee r)]$

$\equiv \sim p \vee \sim(q \vee r)$ [De Morgan's law]

$\equiv \sim p \vee (\sim q \wedge \sim r)$ [De Morgan's law]

iii. $\sim[(p \rightarrow q) \wedge r]$

$\equiv \sim(p \rightarrow q) \vee \sim r$ [De Morgan's law]

$\equiv (p \wedge \sim q) \vee \sim r$ [$\sim(p \rightarrow q) \equiv p \wedge \sim q$]

[Note: Answer given in the textbook is '$(p \wedge \sim q) \vee r$'. However, we found that it is '$(p \wedge \sim q) \vee \sim r$'.**]**

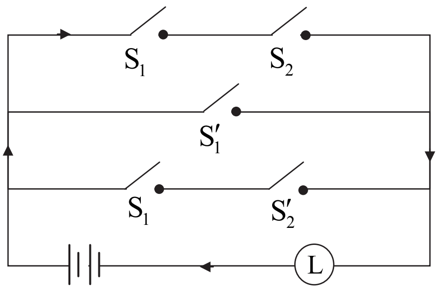
iv. $\sim[(\sim p \wedge q) \vee (p \wedge \sim q)]$

$\equiv \sim(\sim p \wedge q) \wedge \sim(p \wedge \sim q)$ [De Morgan's law

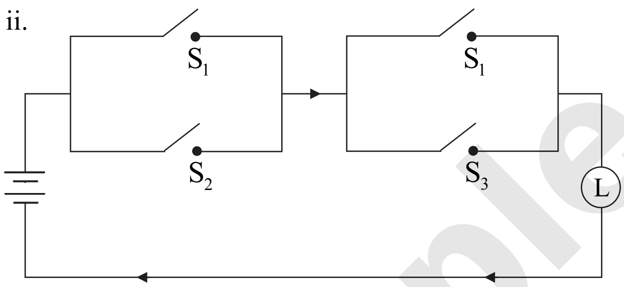
$\equiv (p \vee \sim q) \wedge (\sim p \vee q)$

12. Express the following circuits in the symbolic form. Prepare the switching table:

i.



ii.



***Solution*:**

i. Let p : The switch S_1 is closed.

q : The switch S_2 is closed.

$\sim p$: The switch S_1' is closed.

$\sim q$: The switch S_2' is closed.

Symbolic form of the given circuit is $(p \wedge q) \vee (\sim p) \vee (p \wedge \sim q)$

The switching table:

p	q	$p \wedge q$	$\sim p$	$\sim q$	$(p \wedge q) \vee (\sim p)$	$p \wedge \sim q$	$(p \wedge q) \vee (\sim p) \vee (p \wedge \sim q)$
1	1	1	0	0	1	0	1
1	0	0	0	1	0	1	1
0	1	0	1	0	1	0	1
0	0	0	1	1	1	0	1

ii. Let p : The switch S_1 is closed.

q : The switch S_2 is closed.

r : The switch S_3 is closed.

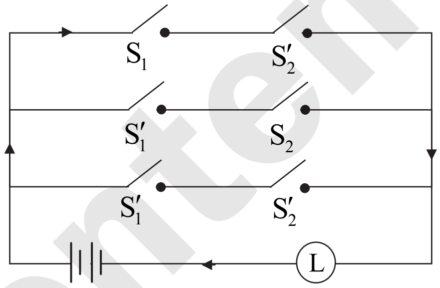
Symbolic form of the given circuit is $(p \vee q) \wedge (p \vee r)$

The switching table:

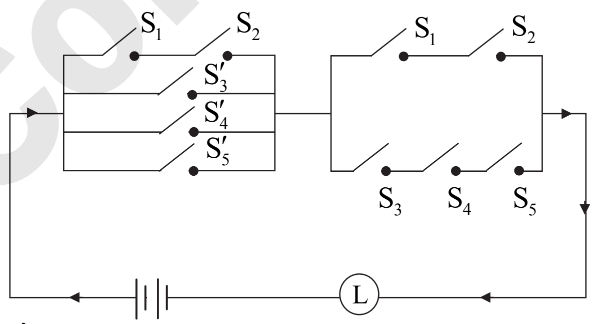
p	q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
1	1	1	1	1	1
1	1	0	1	1	1
1	0	1	1	1	1
1	0	0	1	1	1
0	1	1	1	1	1
0	1	0	1	0	0
0	0	1	0	1	0
0	0	0	0	0	0

13. Simplify the following so that the new circuit has minimum number of switches. Also, draw the simplified circuit.

i.



ii.



***Solution*:**

i. Let p : The switch S_1 is closed.

q : The switch S_2 is closed.

$\sim p$: The switch S_1' is closed.

$\sim q$: The switch S_2' is closed.

$\therefore$ Symbolic form of the given circuit is

$(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

$\equiv (p \wedge \sim q) \vee [\sim p \wedge (q \vee \sim q)]$

[Associative and Distributive law]

$\equiv (p \wedge \sim q) \vee (\sim p \wedge T)$ [Complement law]

$\equiv (p \wedge \sim q) \vee \sim p$ [Identity law]

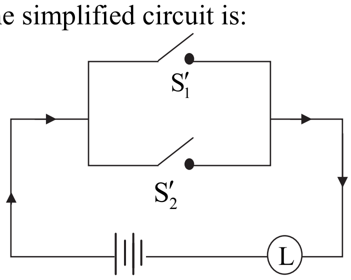
$\equiv (p \vee \sim p) \wedge (\sim q \vee \sim p)$ [Distributive law]

$\equiv (p \vee \sim p) \wedge (\sim p \vee \sim q)$ [Commutative law]

$\equiv T \wedge (\sim p \vee \sim q)$ [Complement law]

$\equiv \sim p \vee \sim q$ [Identity law]

The simplified circuit is:



ii. Let p : The switch S_1 is closed.
q : The switch S_2 closed.
r : The switch S_3 is closed.
s : The switch S_4 is closed.
t : The switch S_5 is closed.
~r : The switch S_3' is closed.
~s : The switch S_4' is closed.
~t : The switch S_5' is closed.

Symbolic form of the given circuit is

$[(p \wedge q) \vee (\sim r \vee \sim s \vee \sim t)] \wedge [(p \wedge q) \vee (r \wedge s \wedge t)]$

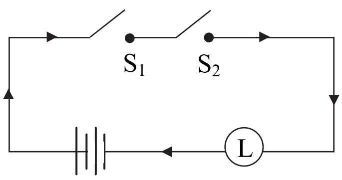
$\equiv [(p \wedge q) \vee \sim(r \wedge s \wedge t)] \wedge [(p \wedge q) \vee (r \wedge s \wedge t)]$ [De Morgan's law]

$\equiv (p \wedge q) \vee [\sim(r \wedge s \wedge t) \wedge (r \wedge s \wedge t)]$ [Distributive law]

$\equiv (p \wedge q) \vee F$ [Complement law]

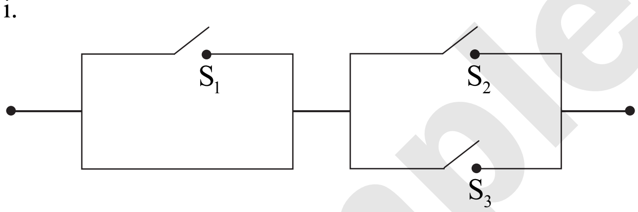
$\equiv p \wedge q$ [Identity law]

The simplified circuit is:

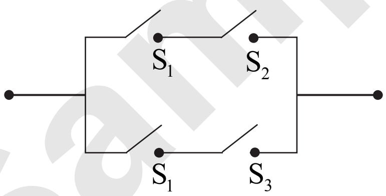


14. Check whether the following switching circuits are logically equivalent - Justify.

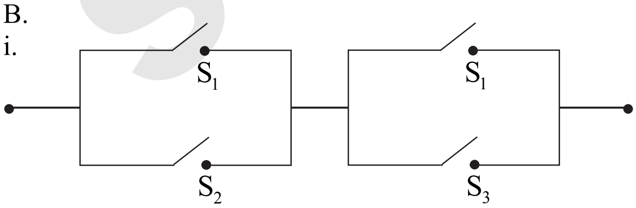
A.
i.



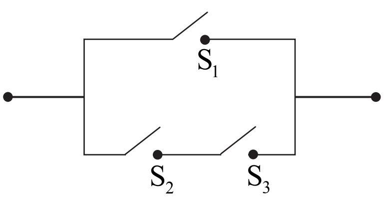
ii.



B.
i.



ii.



***Solution*:**

Let p : The switch S_1 is closed.
q : The switch S_2 is closed.
r : The switch S_3 is closed.

A. i. Symbolic form of given circuit is
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ [Distributive law]

ii. Symbolic form of given circuit is,
$(p \wedge q) \vee (p \wedge r)$

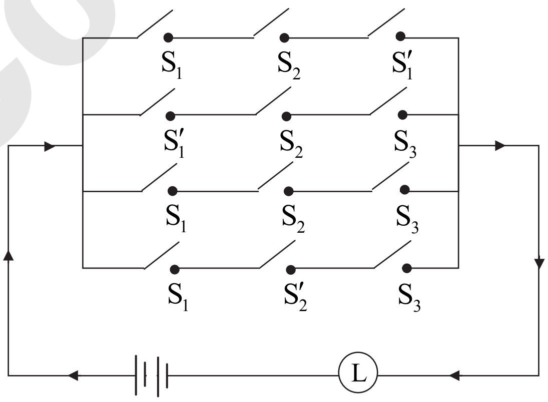
$\therefore$ The given circuits are logically equivalent. …[From (i) and (ii)]

B. i. The symbolic form of the given circuit is
$(p \vee q) \wedge (p \vee r)$

ii. The symbolic form of the given circuit is
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ [Distributive law]

$\therefore$ The given circuits are logically equivalent. …[From (i) and (ii)]

15. Give alternative arrangement of the switching following circuit, has minimum switches.



***Solution*:**

Let p : The switch S_1 is closed.
q : The switch S_2 is closed.
r : The switch S_3 is closed.
~p : The switch S_1' is closed.
~q : The switch S_2' is closed.

Symbolic form of given circuit is

$(p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$

$\equiv (p \wedge \sim p \wedge q) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$ [Commutative law]

$\equiv (F \wedge q) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$ [Complement law]

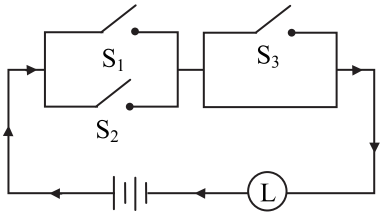
$\equiv F \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$ [Identity law]

$\equiv (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$ [Identity Law]

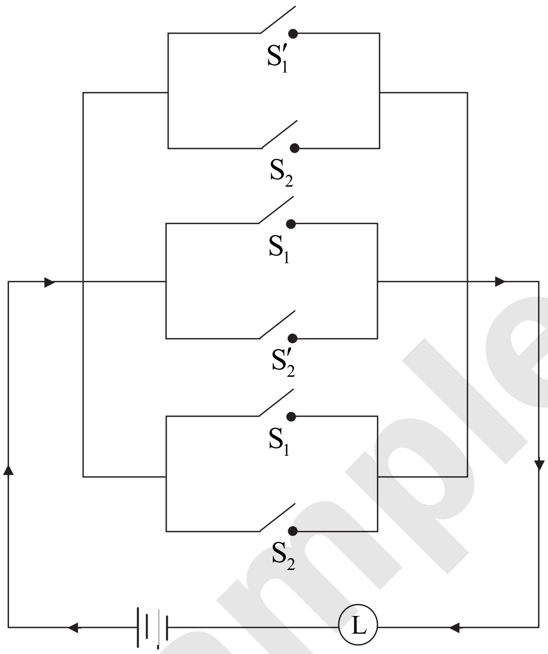
$\equiv [(\sim p \vee p) \wedge (q \wedge r)] \vee (p \wedge \sim q \wedge r)$ [Distributive law]

$\equiv [T \wedge (q \wedge r)] \vee (p \wedge \sim q \wedge r)$ [Complement law]
$\equiv (q \wedge r) \vee (p \wedge \sim q \wedge r)$ [Identity law]
$\equiv [q \vee (p \wedge \sim q)] \wedge r$ [Distributive law]
$\equiv [(q \vee p) \wedge (q \vee \sim q)] \wedge r$ [Distributive law]
$\equiv [(q \vee p) \wedge T] \wedge r$ [Complement law]
$\equiv (q \vee p) \wedge r$ [Identity law]
$\equiv (p \vee q) \wedge r$ [Commutative law]

$\therefore$ The switching circuit corresponding to the given statement is:



16. Simplify the following so that the new circuit has minimum switches.



***Solution*:**

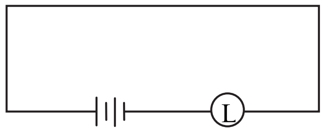
Let p : The switch S_1 is closed.
q : The switch S_2 is closed.
~p : The switch S_1' is closed.
~q : The switch S_2' is closed.

Symbolic form of the given circuit is

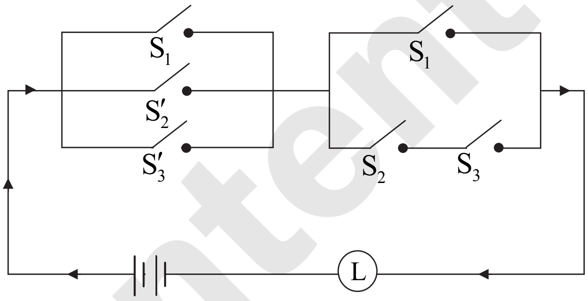
$(\sim p \vee q) \vee (p \vee \sim q) \vee (p \vee q)$

$\equiv (\sim p \vee q) \vee [p \vee (q \vee \sim q)]$
…[Commutative and Distributive law]
$\equiv (\sim p \vee q) \vee (p \vee T)$ [Complement law]
$\equiv (\sim p \vee q) \vee T$ [Identity law]
$\equiv \sim p \vee (q \vee T)$ [Commutative law]
$\equiv \sim p \vee T$ [Identity law]
$\equiv T$ [Identity law]

$\therefore$ The symbolic form of the given circuit is a tautology. Hence, the current will always flow through the circuit irrespective of whether the switches are open or closed.



17. Represent the following switching circuit in symbolic form and construct its switching table. Write your conclusion from the switching table.



***Solution*:**

Let p : The switch S_1 is closed.
q : The switch S_2 is closed.
r : The switch S_3 is closed.
~p : The switch S_1' is closed.
~q : The switch S_2' is closed.
~r : The switch S_3' is closed.

Symbolic form of the given circuit is

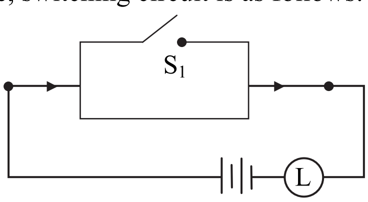
$(p \vee \sim q \vee \sim r) \wedge [p \vee (q \wedge r)]$

$\therefore$ The switching table corresponding to the given statements is :

p	q	r	~q	~r	$p \vee (\sim q) \vee (\sim r)$	$q \wedge r$	$p \vee (q \wedge r)$	$[p \vee (\sim q) \vee (\sim r)] \wedge [p \vee (q \wedge r)]$
1	1	1	0	0	1	1	1	1
1	1	0	0	1	1	0	1	1
1	0	1	1	0	1	0	1	1
1	0	0	1	1	1	0	1	1
0	1	1	0	0	0	1	1	0
0	1	0	0	1	1	0	0	0
0	0	1	1	0	1	0	0	0
0	0	0	1	1	1	0	0	0

The final column of the above table is equivalent to the column of 'p', i.e., column corresponding to switch S_1. Hence, the given circuit is equivalent to the circuit where only switch S_1 is present.

Hence, switching circuit is as follows:



Activities for Practice

1. If p and q are true and r and s are false statements, find the truth value of $\sim(p \wedge \sim r) \vee (\sim q \vee s)$. Fill in the blanks.

$\sim(p \wedge \sim r) \vee (\sim q \vee s)$

$\equiv \sim(T \wedge \square) \vee (F \vee F)$

$\equiv \sim \square \vee F$

$\equiv \square \vee F$

$\equiv \square$

2. Prepare the truth table of

$(p \wedge q) \rightarrow (\sim p)$

p	q	$p \wedge q$	$\sim p$	$(p \wedge q) \rightarrow (\sim p)$
T	T	$\square$	F	F
T	F	F	F	$\square$
F	T	$\square$	T	T
F	F	F	T	$\square$

3. Examine whether the following statement patterns are tautology, contradiction or contingency. $[(p \vee q) \vee r] \rightarrow [p \vee (q \vee r)]$. Fill in the boxes.

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$	$[(p\vee q)\vee r] \rightarrow [p\vee(q \vee r)]$
T	T	T	T	T	T	T	T
T	T	$\square$	T	T	T	T	T
T	F	T	T	$\square$	T	$\square$	T
T	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	$\square$	T	T	T	T
F	F	F	F	F	F	F	$\square$

In the above truth table, all the entries in the last column are T.

$\therefore$ $[(p \vee q) \vee r] \leftrightarrow [p \vee (q \vee r)]$ is a $\square$.

4. By filling in the blanks prove that $[(p \vee q) \wedge \sim p] \rightarrow q$ is a tautology.

$[(p \vee q) \wedge \sim p] \rightarrow q$

$\equiv [(p \wedge \sim p) \vee (q \wedge \sim p)] \rightarrow q$ ...[$\square$]

$\equiv [\square \vee (q \wedge \sim p)] \rightarrow q$ ...[Complement law]

$\equiv (q \wedge \sim p) \rightarrow q$ ...[$\square$]

$\equiv \sim(q \wedge \sim p) \; \square \; q$ ...[Conditional law]

$\equiv (\sim q \vee p) \vee q$ ...[De Morgan's law]

$\equiv (p \vee \sim q) \vee q$ ...[Commutative law]

$\equiv p \vee (\sim q \vee q)$ ... $\square$

$\equiv p \vee \square$ ...[Complement law]

$\equiv T$ ...[Identity law]

Since the truth value of the given statement pattern is T, it is a tautology.

5. By filling in the boxes write the negations of the following statement $(p \wedge q) \vee (q \wedge \sim r)$

$\sim[(p \wedge q) \vee (q \wedge \sim r)]$

$\equiv \sim(p \wedge q) \; \square \; \sim(q \wedge \sim r)$ ...[De Morgan's law]

$\equiv (\sim p \vee \sim q) \; \square \; (\sim q \vee \square)$ ...[De Morgan's law]

$\equiv (\sim p \vee \sim q) \; \square \; (r \vee \sim q)$ ...[$\square$]

$\equiv (\square \wedge r) \vee \sim q$ ...[Distributive law]

One Mark Questions

1. Write the negation of the given statement
$\exists\, x \in R$ such that $x^2 = -1$

2. Using statements
p : Kiran passed the examination.
q : Kiran is sad.
Write the statement 'If Kiran passed the examination, then he is not sad' in symbolic form.

3. Assuming p : She is beautiful.
q : She is clever.
Write the verbal form of $\sim p \wedge \sim q$.

4. If p, q, r have truth values T, F, T respectively, then what is the truth value of $[(\sim p \vee q) \wedge \sim r)]$?

5. Write the converse of the given statement:
'If a man is rich then he does not work'.

6. Write the dual of the following statement:
$\sim p \wedge (q \vee c)$ **[July 18]**

7. Write the dual of $p \wedge \sim p \equiv F$. **[Feb 20]**

Multiple Choice Questions

1. Which of the following is a statement?
(A) Stand up!
(B) Will you help me?
(C) Do you like social studies?
(D) 27 is a perfect cube.

2. Which of the following is not a statement?
(A) Please do me a favour.
(B) 2 is an even integer.
(C) 2 + 1 = 3.
(D) The number 17 is prime.

3. Which of the following is an open statement?
(A) x is a natural number.
(B) Give me a glass of water.
(C) Wish you best of luck.
(D) Good morning to all.

4. Which of the following is not a proposition in logic.
(A) $\sqrt{3}$ is a prime.
(B) $\sqrt{2}$ is a irrational.
(C) Mathematics is interesting.
(D) 5 is an even integer.

5. If p: The sun has set
q: The moon has risen,
then the statement 'The sun has not set or the moon has not risen' in symbolic form is written as
(A) $\sim p \vee \sim q$ (B) $\sim p \wedge q$
(C) $p \wedge \sim q$ (D) $p \vee \sim q$

6. Let p: 'It is hot' and q: 'It is raining'. The verbal statement for $(p \wedge \sim q) \rightarrow p$ is
(A) If it is hot and not raining, then it is hot.
(B) If it is hot and raining, then it is hot.
(C) If it is hot or raining, then it is not hot.
(D) If it is hot and raining, then it is not hot.

7. Using the statements
p: Kiran passed the examination,
s : Kiran is sad.
the statement 'It is not true that Kiran passes therefore he is sad' in symbolic form is
(A) $\sim p \rightarrow s$ (B) $\sim (p \rightarrow \sim s)$
(C) $\sim p \rightarrow \sim s$ (D) $\sim (p \rightarrow s)$

8. The converse of the statement 'If it is raining then it is cool' is
(A) If it is cool then it is raining.
(B) If it is not cool then it is raining.
(C) If it is not cool then it is not raining.
(D) If it is not raining then it is not cool.

9. If p and q are simple propositions, then $p \wedge q$ is true when
(A) p is true and q is false.
(B) p is false and q is true.
(C) p is true and q is true.
(D) p is false q is false.

10. Which of the following is logically equivalent to $\sim[\sim p \rightarrow q]$
(A) $p \vee \sim q$ (B) $\sim p \wedge q$
(C) $\sim p \wedge q$ (D) $\sim p \wedge \sim q$

11. The logically equivalent statement of $p \rightarrow q$ is
(A) $\sim p \vee q$ (B) $q \rightarrow \sim p$
(C) $\sim q \vee p$ (D) $\sim q \vee \sim p$

12. The logically equivalent statement of $\sim p \vee \sim q$ is
(A) $\sim p \wedge \sim q$ (B) $\sim(p \wedge q)$
(C) $\sim(p \vee q)$ (D) $p \wedge q$

13. The contrapositive of $(p \vee q) \rightarrow r$ is
(A) $\sim r \rightarrow \sim p \wedge \sim q$ (B) $\sim r \rightarrow (p \vee q)$
(C) $r \rightarrow (p \vee q)$ (D) $p \rightarrow (q \vee r)$

14. Which of the following propositions is true?
(A) $p \rightarrow q \equiv \sim p \rightarrow \sim q$
(B) $\sim(p \rightarrow \sim q) \equiv \sim p \wedge q$
(C) $\sim(p \leftrightarrow q) \equiv [\sim (p \rightarrow q) \wedge \sim (q \rightarrow p)]$
(D) $\sim(\sim p \rightarrow \sim q) \equiv \sim p \wedge q$

15. When two statements are connected by the connective 'if and only if' then the compound statement is called
(A) conjunction of the statements.
(B) disjunction of the statements.
(C) biconditional statement.
(D) conditional statement.

16. If p and q be two statements then the conjunction of the statements, $p \wedge q$ is false when
(A) both p and q are true.
(B) either p or q are true
(C) either p or q or both are false.
(D) both p and q are false.

17. The negation of the statement, "The question paper is not easy and we shall not pass" is
(A) The question paper is not easy or we shall not pass.
(B) The question paper is not easy implies we shall not pass.
(C) The question paper is easy or we shall pass.
(D) We shall pass implies the question paper is not easy.

18. The statement $(p \wedge q) \wedge (\sim p \vee \sim q)$ is
    (A) a contradiction.
    (B) a tautology.
    (C) neither a contradiction nor a tautology.
    (D) equivalent to $p \vee q$.

19. The proposition $p \wedge \sim p$ is a
    (A) tautology and contradiction.
    (B) contingency.
    (C) tautology.
    (D) contradiction.

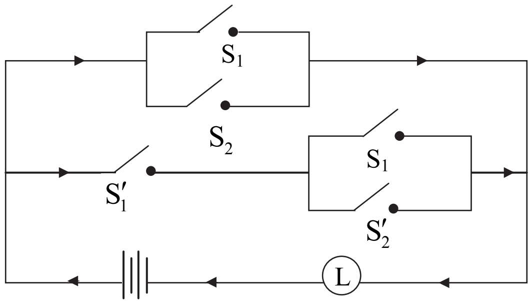
20. The proposition $p \rightarrow \sim (p \wedge q)$ is a
    (A) tautology (B) contradiction
    (C) contingency (D) either (A) or (B)

21. The false statement in the following is
    (A) $p \wedge (\sim p)$ is a contradiction.
    (B) $(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$ is a contradiction.
    (C) $\sim(\sim p) \rightarrow p$ is a tautology.
    (D) $p \vee (\sim p)$ is a tautology.

22. Negation of $(p \vee q)$ is
    (A) $\sim p \vee \sim q$ (B) $\sim p \wedge \sim q$
    (C) $p \wedge \sim q$ (D) $p \vee \sim q$

23. The dual of $\sim(p \vee q) \vee [p \vee (q \wedge \sim r)]$ is,
    (A) $\sim(p \wedge q) \wedge [p \vee (q \wedge \sim r)]$
    (B) $(p \wedge q) \wedge [p \wedge (q \vee \sim r)]$
    (C) $\sim(p \wedge q) \wedge [p \wedge (q \wedge r)]$
    (D) $\sim(p \wedge q) \wedge [p \wedge (q \vee \sim r)]$

24. The symbolic form of the following circuit, where p: switch S_1 is closed.and q: switch S_2 is closed, is-
    (A) $(p \vee q) \wedge [\sim p \vee (p \wedge \sim q)]$
    (B) $(\sim p \wedge q) \vee [\sim p \vee (p \wedge \sim q)]$
    (C) $(p \vee q) \vee [\sim p \wedge (p \vee \sim q)]$
    (D) $(p \wedge q) \vee [\sim p \wedge (p \wedge \sim q)]$



25. If A = {2, 3, 4, 5, 6}, then which of the following is not true? **[Oct 13]**
    (A) $\exists\, x \in A$ such that $x + 3 = 8$
    (B) $\exists\, x \in A$ such that $x + 2 < 5$
    (C) $\exists\, x \in A$ such that $x + 2 < 9$
    (D) $\forall\, x \in A$ such that $x + 6 \geq 9$

26. The negation of $p \wedge (q \rightarrow r)$ is **[Mar 16]**
    (A) $p \vee (\sim q \vee r)$ (B) $\sim p \wedge (q \rightarrow r)$
    (C) $\sim p \wedge (\sim q \rightarrow \sim r)$ (D) $\sim p \vee (q \wedge \sim r)$

27. Inverse of the statement pattern $(p \vee q) \rightarrow (p \wedge q)$ is **[July 16]**
    (A) $(p \wedge q) \rightarrow (p \vee q)$
    (B) $\sim (p \vee q) \rightarrow (p \wedge q)$
    (C) $(\sim p \vee \sim q) \rightarrow (\sim p \wedge \sim q)$
    (D) $(\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$

28. The negation of $p \wedge (q \rightarrow r)$ is ________. **[Mar 22]**
    (A) $\sim p \wedge (\sim q \rightarrow \sim r)$ (B) $p \vee (\sim q \vee r)$
    (C) $\sim p \wedge (\sim q \rightarrow r)$ (D) $p \rightarrow (q \wedge \sim r)$

29. The negation of $(p \vee \sim q) \wedge r$ is ________. **[July 22]**
    (A) $(\sim p \wedge q) \wedge r$ (B) $(\sim p \wedge q) \vee r$
    (C) $(\sim p \wedge q) \vee \sim r$ (D) $(\sim p \vee q) \wedge \sim r$

Time: 1 Hour	TOPIC TEST	Total Marks: 20

SECTION A

Q.1. Select and write the correct answer. **[4]**

i. $(p \wedge q) \rightarrow r$ is logically equivalent to ________.
   (A) $p \rightarrow (q \rightarrow r)$ (B) $(p \wedge q) \rightarrow \sim r$
   (C) $(\sim p \vee \sim q) \rightarrow \sim r$ (D) $(p \vee q) \rightarrow r$

ii. The negation of $p \wedge (q \rightarrow r)$ is ________.
   (A) $\sim p \wedge (\sim q \rightarrow \sim r)$ (B) $p \vee (\sim q \vee r)$
   (C) $\sim p \wedge (\sim q \rightarrow \sim r)$ (D) $\sim p \vee (q \wedge \sim r)$

Q.2. Answer the following. **[2]**

i. Assuming p : She is beautiful.
   q : She is clever.
   Write the verbal form of $\sim p \wedge \sim q$.

ii. Write the dual of the following.
   $[\sim(p \vee q)] \wedge [p \vee \sim(q \wedge \sim s)]$

SECTION B

Attempt any two of the following: [4]

Q.3. Determine the truth values of p and q in the following cases:
$(p \vee q)$ is T and $(p \wedge q)$ is T

Q.4. Construct the truth table for the following statement patterns.
$p \to (q \to p)$

Q.5. Rewrite the following statements without using if ... then.
If a man is a judge, then he is honest.

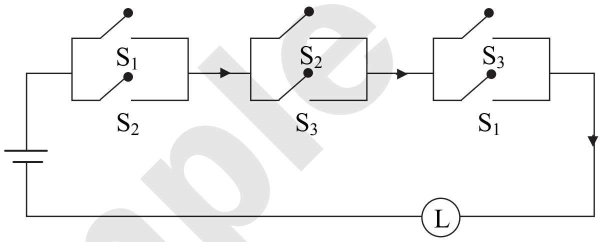
SECTION C

Attempt any two of the following: [6]

Q.6. Write converse, inverse and contrapositive of the following statements.
A family becomes literate if the woman in it is literate.

Q.7. Examine whether each of the following statement patterns is a tautology or a contradiction or a contingency.
$[p \to (\sim q \vee r)] \leftrightarrow \sim [p \to (q \to r)]$

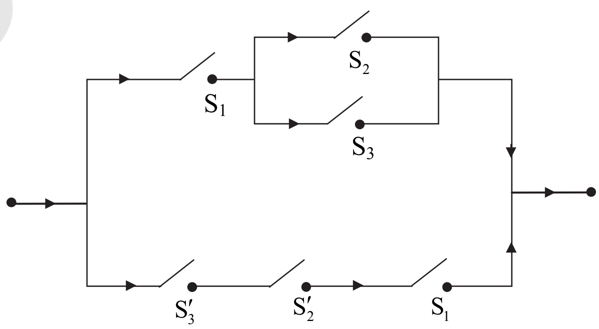
Q.8. Express the following circuits in the symbolic form of logic and write the input-output table.



SECTION D

Attempt any one of the following: [4]

Q.9. Give an alternative equivalent simple circuits for the following circuits:



Q.10. Obtain the simple logical expression of the following. Draw the corresponding switching circuit.
$(p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$

Answers

Activities for Practice

1. i. T ii. T
   iii. F iv. F
2. i. T ii. T
   iii. F iv. T
3. i. F ii. T
   iii. T iv. F
   v. T vi. Tautology
4. i. Distributive law ii. F
   iii. Identity law iv. $\vee$
   v. Associative law vi. T
5. i. $\wedge$ ii. $\wedge$
   iii. r iv. $\wedge$
   v. Commutative law vi. ~p

One Mark Questions

1. $\forall\, x \in R, x^2 \neq -1$ 2. $p \rightarrow \sim q$
3. She is not beautiful and not clever.
4. F
5. If a man does not work then he is rich.
6. The dual of this statement is $\sim p \vee (q \wedge t)$
7. The dual of this statement is $p \vee \sim p \equiv T$

Multiple Choice Questions

1.	(D)	2.	(A)	3.	(A)	4.	(C)
5.	(A)	6.	(A)	7.	(D)	8.	(A)
9.	(C)	10.	(D)	11.	(A)	12.	(B)
13.	(A)	14.	(D)	15.	(C)	16.	(C)
17.	(C)	18.	(A)	19.	(D)	20.	(C)
21.	(B)	22.	(B)	23.	(D)	24.	(C)
25.	(D)	26.	(D)	27.	(D)	28.	(D)
29.	(C)						

Topic Test

1. i. (A) ii. (D)
2. i. She is not beautiful and not clever.
   ii. $[\sim(p \wedge q)] \vee [p \wedge \sim(q \vee \sim s)]$
3. T
4. TTTT
5. A man is not a judge or he is honest.
6. Converse: If a family becomes literate then the woman in it is literate.
   Inverse : If the woman in the family is not literate then the family does not become literate.
   Contrapositive: If a family does not become literate then the woman in the family is not literate.
7. Contradiction
8. $(p \vee q) \wedge (q \vee r) \wedge (r \vee p)$ $\boxed{11101000}$
10. $r \wedge [(\sim p \wedge q) \vee (p \wedge \sim q)]$

Competitive Corner

1. The logical statement
   $[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$ is equivalent to:
   [JEE (Main) 2019]
   (A) $(\sim p \wedge \sim q) \wedge r$ (B) $(p \wedge \sim q) \vee r$
   (C) $\sim p \vee r$ (D) $(p \wedge r) \wedge \sim q$
2. The contrapositive of the statement 'If Raju is courageous, then he will join Indian Army', is
   [MHT CET 2020]
   (A) If Raju does not join Indian Army, then he is not courageous.
   (B) If Raju join Indian Army, then he is not courageous
   (C) If Raju join Indian Army, then he is courageous.
   (D) If Raju does not join Indian Army, then he is courageous.
3. The negation of a statement '$x \in A \cap B \rightarrow (x \in A \text{ and } x \in B)$' is
   [MHT CET 2021]
   (A) $x \in A \cap B \rightarrow (x \in A \text{ or } x \in B)$
   (B) $x \in A \cap B \text{ or} (x \in A \text{ and } x \in B)$
   (C) $x \in A \cap B \text{ and } (x \notin A \text{ or } x \notin B)$
   (D) $x \notin A \cap B \text{ and } (x \in A \text{ and } x \in B)$
4. The statement among the following that is a tautology is : **[JEE (Main) 2021]**
   (A) $A \wedge (A \vee B)$
   (B) $A \vee (A \wedge B)$
   (C) $[A \wedge (A \rightarrow B)] \rightarrow B$
   (D) $B \rightarrow [A \wedge (A \rightarrow B)]$
5. For three simple statements p, q, and r, $p \rightarrow (q \vee r)$ is logically equivalent to
   [MHT CET 2022]
   (A) $(p \vee q) \rightarrow r$
   (B) $(p \rightarrow \sim q) \wedge (p \rightarrow r)$
   (C) $(p \rightarrow q) \vee (p \rightarrow r)$
   (D) $(p \rightarrow q) \wedge (p \rightarrow \sim r)$

Answers:

1. (D) 2. (A) 3. (C) 4. (C)
5. (C)

Hints:

1. $[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$
   $\equiv [(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$ ...[De Morgan's law]
   $\equiv p \wedge (\sim q \vee r) \wedge (\sim q \wedge r)$ ...[Distributive law]
   $\equiv p \wedge [(\sim q \vee r) \wedge \sim q] \wedge r$ ...[Associative law]
   $\equiv p \wedge (\sim q) \wedge r$ ...[Absorption law]
   $\equiv (p \wedge r) \wedge \sim q$ ...[Commutative law]

2. p: Raju is courageous
q: Raju will join Indian Army
Given statement is: $p \to q$
It's contrapositive is: $\sim q \to \sim p$
If Raju does not join Indian army, then he is not courageous.

3. $p \to q \equiv \sim p \vee q$
$\therefore$ $\sim (p \to q) \equiv p \wedge \sim q$

4. Consider option (C),

A	B	$A \to B$	$A \wedge (A \to B)$	$[A \wedge (A \to B)] \to B$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

All the entries in the last column of the above truth table is T.
$[A \wedge (A \to B)] \to B$ is a tautology.

5.

1	2	3	4	5	6	7	8
p	q	r	$q \vee r$	$p \to (q \vee r)$	$p \to q$	$p \to r$	$(p \to q) \vee (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

Columns 5 and 8 are identical
$\therefore$ $p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$

Scan the given Q. R. Code in *Quill - The Padhai App* to view the solutions of the Topic Test.