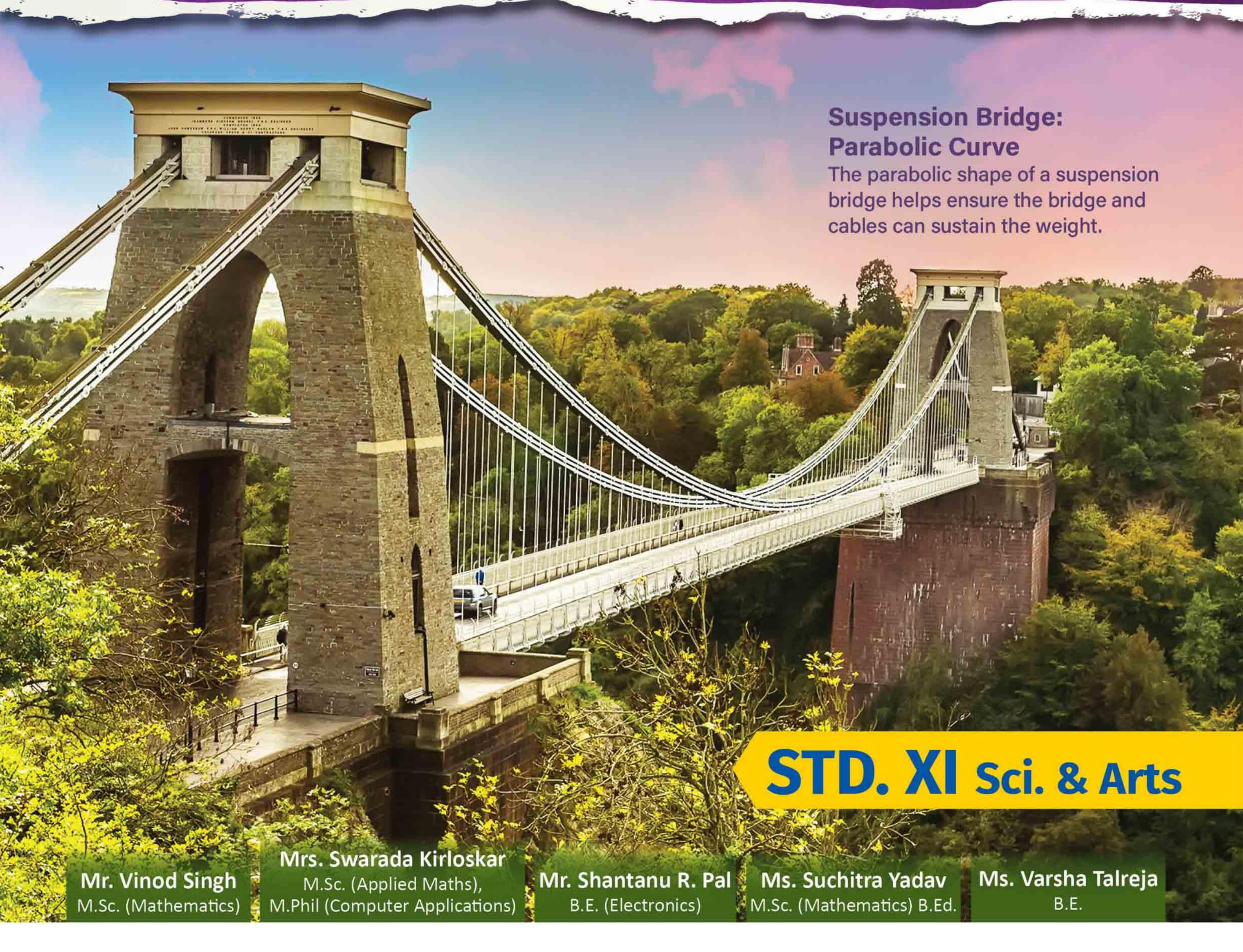
SAMPLE CONTENT

PERFECT

MATHEMATICS & STATISTICS

Part I



Suspension Bridge: Parabolic Curve

The parabolic shape of a suspension bridge helps ensure the bridge and cables can sustain the weight.

STD. XI Sci. & Arts

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Target Publications® Pvt. Ltd.

Written as per the latest textbook prescribed by the Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune

Special Inclusion
- Memory Maps
- Gyan Guru (GG)

PERFECT MATHEMATICS & STATISTICS Part - I

Std. XI Sci. & Arts

Salient Features

- Updated as per the latest textbook
- Exhaustive coverage of entire syllabus
- Covers all derivations and theorems
- Tentative marks allocation for all the problems
- The chapters include:
  - 'Memory Map' at the start of the chapter for quick revision
  - 'Precise Theory' for every topic
  - Solutions to all Exercises and Miscellaneous exercises given in the textbook.
  - 'Additional problems for practice' and 'Multiple choice questions' (MCQs)
  - 'Topic Test' for self-assessment
  - 'Competitive Corner' to give the glimpse of prominent competitive examinations
- Includes Important Features for holistic learning:
  - Gyan Guru (GG)
  - Smart Check
  - Important Formulae
  - Remember This
- Q.R. codes provide solutions to:
  - Additional problems for practice
  - Competitive corner
  - Topic Test

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Preface

"The only way to learn Mathematics is to do Mathematics" – Paul Halmos

"Perfect Mathematics & Statistics Part – I, Std. XI Sci. & Arts" forms a part of 'Target Perfect Notes' prepared as per the Latest Textbook. It is a complete and thorough guide critically analysed and extensively drafted to boost the students' confidence.

The chapters consist of:

- 'Memory Map' to grasp the key concepts of the chapter
- Precise theory for every topic
- Solutions to all textual questions in exercises and miscellaneous exercises
- 'One-mark questions' along with their answers
- 'Additional problems for practice' with ample questions for additional practice and their solutions via QR code
- 'Competitive Corner' to get an idea about the type of questions asked in Competitive exams, with solutions via QR code
- Multiple Choice Questions and Topic Test (as per latest paper pattern) assess the students on their range of preparation and the amount of knowledge of each topic.

We all know that there are certain sums that can be solved by multiple methods. Besides, there are also other ways to check your answer in Maths. 'Smart Check' has been included to help you understand how you can check the correctness of your answer.

A recap of all important formulae has been provided at the end of the book for quick revision.

Our Perfect Mathematics & Statistics Part – I, Std. XI Sci. & Arts *adheres to our vision and achieves several goals:* building concepts, developing competence to solve problems, self-study, self-assessment *and* student engagement—*all while encouraging students toward cognitive thinking.*

The flow chart on the adjacent page will walk you through the key features of the book and elucidate how they have been carefully designed to maximize the student learning.

We hope the book benefits the learner as we have envisioned.

Publisher

Edition: Fifth

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.

Please write to us on: mail@targetpublications.org

Disclaimer

This reference book is transformative work based on latest Textbook of Std. XI Mathematics & Statistics Part – I published by the Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune. We the publishers are making this reference book which constitutes as fair use of textual contents which are transformed by adding and elaborating, with a view to simplify the same to enable the students to understand, memorize and reproduce the same in examinations.

This work is purely inspired upon the course work as prescribed by the Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune. Every care has been taken in the publication of this reference book by the Authors while creating the contents. The Authors and the Publishers shall not be responsible for any loss or damages caused to any person on account of errors or omissions which might have crept in or disagreement of any third party on the point of view expressed in the reference book.

KEY FEATURES

1

Memory Map

Memory Map includes tables/ flow chart to summarize the key points in chapter.

2

GG-Gyan Guru

Gyan Guru illustrates real life applications or examples related to the concept discussed.

3

Smart Check

Smart Check is a technique to verify the answers. This is our attempt to cross-check the accuracy of the answer. Smart check is indicated by symbol

4

One Mark Questions

These questions require very short solutions with direct application of mathematical concepts.

5

Additional Problems for Practice

In this section we have provided ample practice problems for students.
Solved examples from textbook are indicated by "+".

6

Competitive Corner

Competitive Corner presents questions from prominent [JEE (Main), MHT CET] competitive exams based entirely on the syllabus covered in the chapter.
This is our attempt to introduce students to MCQs asked in competitive exams.

7

Topic Test

Topic Test covers questions from the chapter for self-evaluation purpose.
This is our attempt to provide the students with revision and help them assess their knowledge of chapter.

8

QR Codes

Q.R. codes provide solutions to
i Additional problems for practice
ii. Competitive corner
iii. Topic Test

9

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Important Formulae $x^2 + y^2 = r^2$

Important Formulae given at the end of the book include all of the key formulae in the chapter.
This is our attempt to offer students a handy tool to solve problems and ace the last minute revision.

Contents

[Reference: Maharashtra State Board of Secondary and Higher Secondary Education, Pune - 04**]**

Solved examples from textbook are indicated by "+".

Smart check is indicated by ✓ symbol.

Trigonometry – I

Memory Map

TRIGONOMETRY – I

Values of trigonometric functions for the angles 0°, 30°, 45°, 60°, 90°

θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
cot θ	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
cosec θ	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

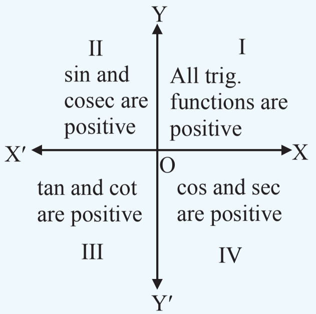
Domain, Range and Periodicity of trigonometric functions

Function	Domain	Range	Period
$y = \sin\theta$	R	[–1, 1]	2π
$y = \cos\theta$	R	[–1, 1]	2π
$y = \tan\theta$	$R - \left\{(2n+1)\frac{\pi}{2} / n \in Z\right\}$	R	π
$y = \operatorname{cosec}\theta$	$R - \{n\pi/n \in Z\}$	R – (–1, 1)	2π
$y = \sec\theta$	$R - \left\{(2n+1)\frac{\pi}{2} / n \in Z\right\}$	R – (–1, 1)	2π
$y = \cot\theta$	$R - \{n\pi/ n \in Z\}$	R	π

Trigonometric functions of negative angle

- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\tan(-\theta) = -\tan\theta$
- $\cot(-\theta) = -\cot\theta$
- $\sec(-\theta) = \sec\theta$
- $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

Signs of trigonometric functions in different quadrants



Fundamental Identities

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta,\ \theta \neq \frac{\pi}{2}$
- $1 + \cot^2\theta = \operatorname{cosec}^2\theta,\ \theta \neq 0$

Polar co-ordinate system

The cartesian co-ordinates of the point P(r, θ) are given by the relations: $x = r\cos\theta$ and $y = r\sin\theta$, where $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$

2 Trigonometry – I

Contents and Concepts

- Trigonometric Functions with the help of Unit Circle
- Extensions of Trigonometric Functions to any Angle
- Range and Signs of Trigonometric Functions in Different Quadrants
- Fundamental Identities and Periodicity of Trigonometric Functions
- Domain, Range and Graph of each Trigonometric Function
- Polar Co-ordinates

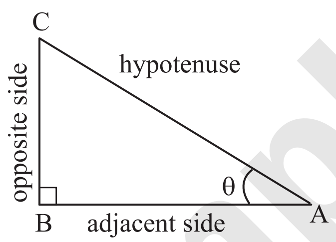
Let's Study

Introduction:

The word trigonometry originated from the two Greek words "trigonon" and "metron" meaning three angle measure. The science of trigonometry is based on certain functions called as trigonometric functions.

In standard X you have studied six trigonometric ratios of an acute angle i.e., sine, cosine, tangent, cotangent, secant, cosecant.

From figure we see that $\angle B = 90°$ and $\angle BAC = \theta$



$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{BC}{AC}$

$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{AB}{AC}$

$\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}} = \dfrac{BC}{AB}$

$\cot\theta = \dfrac{\text{adjacent side}}{\text{opposite side}} = \dfrac{AB}{BC}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent side}} = \dfrac{AC}{AB}$

$\text{cosec}\,\theta = \dfrac{\text{hypotenuse}}{\text{opposite side}} = \dfrac{AC}{BC}$

Trigonometric Functions with the help of Unit Circle

A standard unit circle is a circle with centre at origin and radius 1.

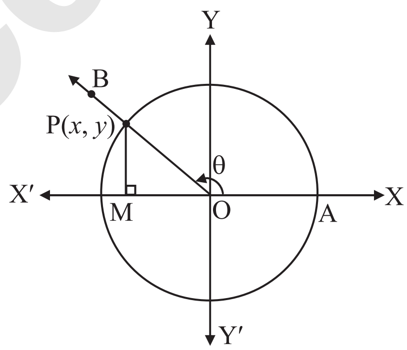
Let θ be the measure of the angle AOB in standard position. Let P(x, y) be any point on the terminal ray OB such that l(OP) = r > 0.

Since P lies on the unit circle, l (OP) = 1

$\therefore \quad \sqrt{x^2 + y^2} = 1$

$\therefore \quad x^2 + y^2 = 1$

Then, we define the trigonometric functions as follows:



i. sine $\theta = \sin\theta = \dfrac{y}{r} = y$

ii. cosine $\theta = \cos\theta = \dfrac{x}{r} = x$

iii. tangent $\theta = \tan\theta = \dfrac{y}{x}$, (if $x \neq 0$)

iv. cosecant $\theta = \text{cosec}\,\theta = \dfrac{1}{y}$, (if $y \neq 0$)

v. secant $\theta = \sec\theta = \dfrac{1}{x}$, (if $x \neq 0$)

vi. cotangent $\theta = \cot\theta = \dfrac{x}{y}$, (if $y \neq 0$)

Interrelation between trigonometric functions

From the above definitions, we have

i. $\text{cosec}\,\theta = \dfrac{1}{y} = \dfrac{1}{\sin\theta}$, (if $\sin\theta \neq 0$)

ii. $\sec\theta = \dfrac{1}{x} = \dfrac{1}{\cos\theta}$, (if $\cos\theta \neq 0$)

iii. $\tan \theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$, (if $\cos \theta \neq 0$)

iv. $\cot \theta = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$, (if $\sin \theta \neq 0$)

Note:

i. The trigonometric functions do not depend on the position of the point P on the terminal ray but they depend on the measure o angle θ.

ii. Co-terminal angles have same trigonometric functions.

iii. The co-ordinates of point P on standard unit circle are given by $P(x, y) \equiv (\cos \theta, \sin \theta)$.

iv. Standard inequalities of trigonometric functions:

a. $-1 \leq \cos \theta \leq 1$

b. $-1 \leq \sin \theta \leq 1$

c. $\sec \theta \leq -1$ or $\sec \theta \geq 1$

d. $\operatorname{cosec} \theta \leq -1$ or $\operatorname{cosec} \theta \geq 1$

GG - GYAN GURU

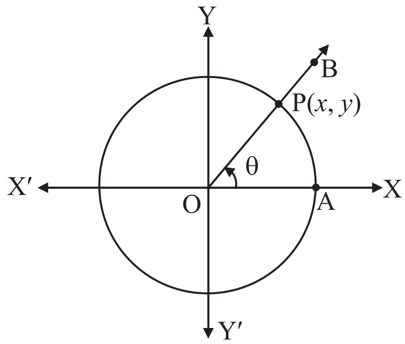
Trigonometry in roof building

When designing the roof of a building, trigonometric principles are applied to calculate angles and distances accurately. By understanding trigonometric functions, architects can ensure structural stability and optimize energy efficiency by positioning solar panels for maximum sunlight exposure.

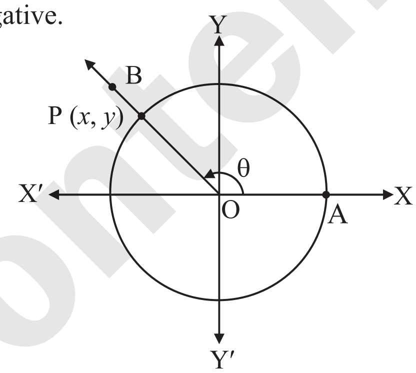
Signs of Trigonometric Functions in Different Quadrants

Since r is always positive, signs of the trigonometric functions depend on the signs of x co-ordinate and y co-ordinate.

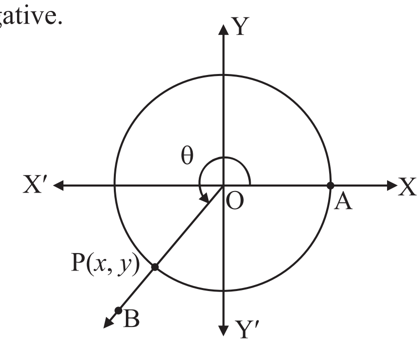
i. If the angle θ is in the first quadrant, then $P(x, y)$ lies in the first quadrant and hence x and y both are positive. Thus, all the trigonometric functions are positive.



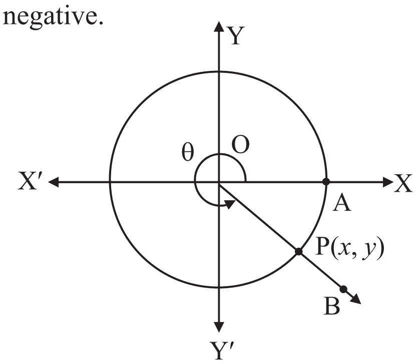
ii. If the angle θ is in the second quadrant, then $P(x, y)$ lies in the second quadrant and hence x is negative and y is positive. Thus only sin θ and cosec θ are positive and rest of the functions are negative.



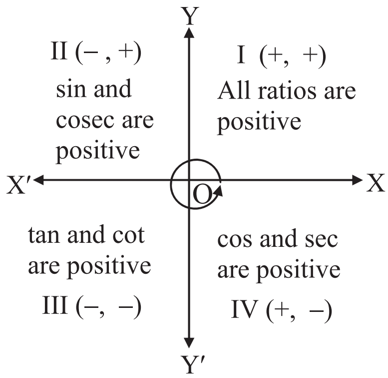
iii. If the angle θ is in the third quadrant, then $P(x, y)$ lies in the third quadrant and hence both x and y are negative. Thus only tan θ and cot θ are positive and rest of the functions are negative.



iv. If the angle θ is in the fourth quadrant, then $P(x, y)$ lies in the fourth quadrant and hence x is positive and y is negative. Thus only cos θ and sec θ are positive and the rest of the functions are negative.



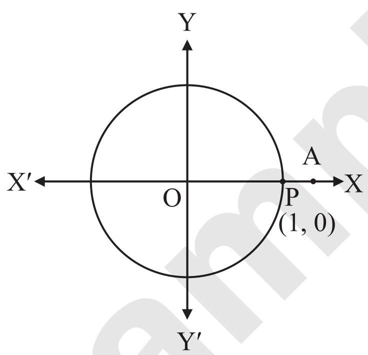
We can express only the positive functions, (with their reciprocals) in the form of diagram as shown below.



θ lies in Quadrant / Trigonometric functions	I	II	III	IV
sin	+ ve	+ ve	– ve	– ve
cos	+ ve	– ve	– ve	+ ve
tan	+ ve	– ve	+ ve	– ve

Trigonometric Functions of Specific Angles

1. Angle of measure 0° or 0^c:



Let m∠XOA = 0° = 0^c

Its terminal arm (ray OA) intersects the standard unit circle in P(1, 0).

Hence, $x = 1$ and $y = 0$

∴ $\sin 0° = y = 0$,

$\cos 0° = x = 1$,

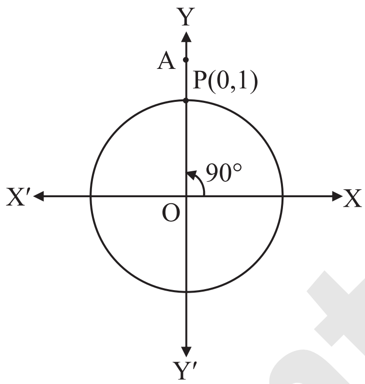
$\tan 0° = \frac{y}{x} = \frac{0}{1} = 0$,

$\cot 0° = \frac{x}{y} = \frac{1}{0}$ which is not defined,

$\sec 0° = \frac{1}{x} = \frac{1}{1} = 1$,

$\text{cosec } 0° = \frac{1}{y} = \frac{1}{0}$ which is not defined.

2. Angle of measure 90° or $\frac{\pi^c}{2}$:



Let m∠XOA = 90° = $\frac{\pi^c}{2}$

Its terminal arm (ray OA) intersects the standard unit circle in P(0, 1).

Hence, $x = 0$ and $y = 1$

∴ $\sin 90° = y = 1$,

$\cos 90° = x = 0$,

$\tan 90° = \frac{y}{x} = \frac{1}{0}$ which is not defined,

$\cot 90° = \frac{x}{y} = \frac{0}{1} = 0$,

$\sec 90° = \frac{1}{x} = \frac{1}{0}$ which is not defined,

$\text{cosec } 90° = \frac{1}{y} = \frac{1}{1} = 1$.

TRY THIS

Find the trigonometric functions of angles 180° and 270°. *(Textbook page no. 17)*

Angle of measure 180° :

Refer Exercise 2.1 Q .1

Angle of measure 270°:

Refer Miscellaneous Exercise 2 Q. II (1)

3. Angle of measure 360° or $2\pi^c$:

Since, 360° and 0° are coterminal angles, the trigonometric functions of 360° are same as those of 0°.

4. Angle of measure 120° or $\left(\frac{2\pi}{3}\right)^c$:

Let m∠XOA = 120°

Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).

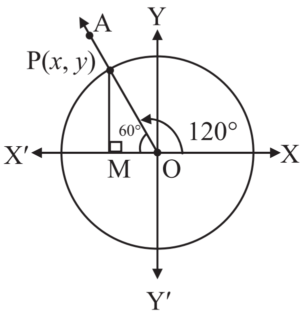
Draw seg PM perpendicular to the X-axis.

∴ ΔOMP is a 30° – 60° – 90° triangle.

OP = 1

$$PM = \frac{\sqrt{3}}{2}\,OP = \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$$

$$OM = \frac{1}{2}\,OP = \frac{1}{2}(1) = \frac{1}{2}$$



Since point P lies in the 2[nd] quadrant,
$x < 0, y > 0$

$\therefore$ $x = -\,OM = -\frac{1}{2}$ and $y = PM = \frac{\sqrt{3}}{2}$

$\therefore$ $P \equiv \left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$

$$\sin 120° = y = \frac{\sqrt{3}}{2}$$

$$\cos 120° = x = -\frac{1}{2}$$

$$\tan 120° = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$

$$\text{cosec } 120° = \frac{1}{y} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$$

$$\sec 120° = \frac{1}{x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\cot 120° = \frac{x}{y} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

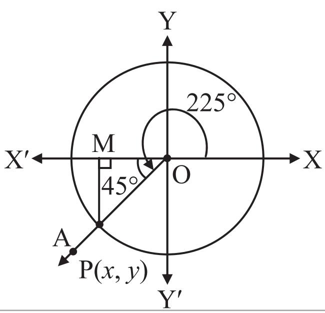
5. Angle of measure 225° or $\left(\frac{5\pi}{4}\right)^c$:

Let m∠XOA = 225°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.

$\therefore$ ΔOMP is a 45° – 45° – 90° triangle.
OP = 1

$$OM = \frac{1}{\sqrt{2}}\,OP = \frac{1}{\sqrt{2}}(1) = \frac{1}{\sqrt{2}}$$

$$PM = \frac{1}{\sqrt{2}}\,OP = \frac{1}{\sqrt{2}}(1) = \frac{1}{\sqrt{2}}$$



Since point P lies in the 3[rd] quadrant,
$x < 0, y < 0$

$\therefore$ $x = -\,OM = -\frac{1}{\sqrt{2}}$ and $y = -\,PM = -\frac{1}{\sqrt{2}}$

$\therefore$ $P \equiv \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$$\sin 225° = y = -\frac{1}{\sqrt{2}}$$

$$\cos 225° = x = -\frac{1}{\sqrt{2}}$$

$$\tan 225° = \frac{y}{x} = \frac{-\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = 1$$

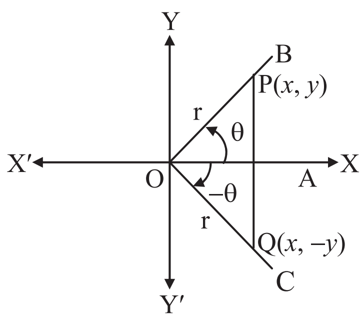
$$\text{cosec } 225° = \frac{1}{y} = \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}$$

$$\sec 225° = \frac{1}{x} = \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}$$

$$\cot 225° = \frac{x}{y} = \frac{-\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = 1$$

Trigonometric Functions of Negative Angle

Let m ∠AOB = θ and m ∠AOC = – θ be in standard position. Let P be any point on the terminal side OB such that l(OP) = r ≠ 0 and Q be any point on side OC such that l(OQ) = r ≠ 0



From the figure, the x co-ordinate of P and Q are same and their y co-ordinate are equal in magnitude but opposite in sign. Thus if co-ordinate of P are (x, y) then co-ordinate of Q are (x, $-y$)

By definitions of trigonometric functions,

$$\sin(-\theta) = \frac{y \text{ co-ordinate of Q}}{r} = \frac{-y}{r}$$

$$= \frac{-(y \text{ co-ordinate of P})}{r} = -\sin\theta$$

$\therefore$ $\sin(-\theta) = -\sin\theta$

$$\cos(-\theta) = \frac{x \text{ co-ordinate of Q}}{r} = \frac{x}{r}$$

$$= \frac{x \text{ co-ordinate of P}}{r} = \cos\theta$$

$\therefore \quad \cos(-\theta) = \cos\theta$

From these functions, the other trigonometric functions can be expressed in terms of θ provided each function exists.

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta$$

$$\text{cosec}(-\theta) = \frac{1}{\sin(-\theta)} = \frac{1}{-\sin\theta} = -\text{cosec}\,\theta$$

$$\sec(-\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos\theta} = \sec\theta$$

$$\cot(-\theta) = \frac{\cos(-\theta)}{\sin(-\theta)} = \frac{\cos\theta}{-\sin\theta} = -\cot\theta$$

6. Angle of measure – 60° or $-\frac{\pi}{3}$:

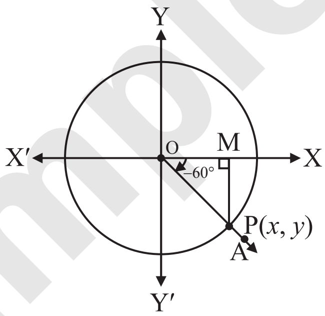
Let m∠XOA = – 60°

Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).

Draw seg PM perpendicular to the X-axis.

∴ ΔOMP is a 30° – 60° – 90° triangle.

OP = 1,



$$OM = \frac{1}{2}\,OP = \frac{1}{2}(1) = \frac{1}{2}$$

$$PM = \frac{\sqrt{3}}{2}\,OP = \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$$

Since point P lies in the 4th quadrant,

$x > 0, y < 0$

$\therefore \quad x = OM = \frac{1}{2}$ and $y = -PM = -\frac{\sqrt{3}}{2}$

$\therefore \quad P \equiv \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$$\sin(-60°) = y = -\frac{\sqrt{3}}{2}$$

$$\cos(-60°) = x = \frac{1}{2}$$

$$\tan(-60°) = \frac{y}{x} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$$

$$\text{cosec}(-60°) = \frac{1}{y} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\sec(-60°) = \frac{1}{x} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$\cot(-60°) = \frac{x}{y} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

TRY THIS

Find trigonometric functions of angles 150°, 210°, 330°, – 45°, –120°, $-\frac{3\pi}{4}$ and complete the table.

(Textbook page no. 19)

Trig. fun. / θ Angle	sin θ	cos θ	tan θ	cosec θ	sec θ	cot θ
150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	2	$-\frac{2}{\sqrt{3}}$	$-\sqrt{3}$
210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	–2	$-\frac{2}{\sqrt{3}}$	$\sqrt{3}$
330°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	–2	$\frac{2}{\sqrt{3}}$	$-\sqrt{3}$
– 45°	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	–1	$-\sqrt{2}$	$\sqrt{2}$	–1
–120°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	–2	$\frac{1}{\sqrt{3}}$
$-\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1

The values of trigonometric functions for the angles 0°, 30°, 45°, 60°, 90° are shown in the following table.

Angles / Trigono-metric functions	0° (0)	30° ($\frac{\pi^c}{6}$)	45° ($\frac{\pi^c}{4}$)	60° ($\frac{\pi^c}{3}$)	90° ($\frac{\pi^c}{2}$)
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
cot	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
cosec	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Exercise 2.1

1. Find the trigonometric functions of 0°, 30°, 45°, 60°, 150°, 180°, 210°, 300°, 330°, – 30°, – 45°, – 60°, – 90°, – 120°, – 225°, – 240°, – 270°, – 315°

[3 Marks Each]

Solution:

Angle of measure 0°:

Refer page no. 26

Angle of measure 30°:

Let m∠XOA = 30°

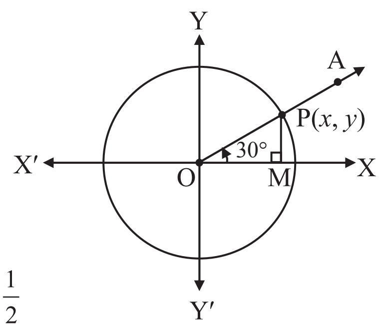
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).

Draw seg PM perpendicular to the X-axis.

∴ ΔOMP is a 30° – 60° – 90° triangle.

OP = 1

$$\text{OM} = \frac{\sqrt{3}}{2}\text{OP} = \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$$



$$\text{PM} = \frac{1}{2}\text{OP} = \frac{1}{2}(1) = \frac{1}{2}$$

Since point P lies in the 1st quadrant,

$x > 0, y > 0$

∴ $x = \text{OM} = \frac{\sqrt{3}}{2}$ and $y = \text{PM} = \frac{1}{2}$

∴ $\text{P} \equiv \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\sin 30° = y = \frac{1}{2}$

$\cos 30° = x = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

$\text{cosec } 30° = \frac{1}{y} = \frac{1}{\left(\frac{1}{2}\right)} = 2$

$\sec 30° = \frac{1}{x} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$

$\cot 30° = \frac{x}{y} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

Angle of measure 45°:

Let m∠XOA = 45°

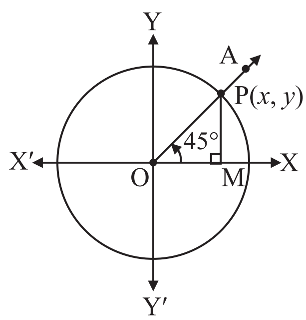
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).

Draw seg PM perpendicular to the X-axis.

∴ ΔOMP is a 45° – 45° – 90° triangle.

OP = 1,

$$\text{OM} = \frac{1}{\sqrt{2}}\text{OP} = \frac{1}{\sqrt{2}}(1) = \frac{1}{\sqrt{2}}$$



$$\text{PM} = \frac{1}{\sqrt{2}}\text{OP} = \frac{1}{\sqrt{2}}(1) = \frac{1}{\sqrt{2}}$$

Since point P lies in the 1st quadrant,

$x > 0, y > 0$

∴ $x = \text{OM} = \frac{1}{\sqrt{2}}$ and

$y = \text{PM} = \frac{1}{\sqrt{2}}$

$\therefore$ $P \equiv \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\sin 45° = y = \frac{1}{\sqrt{2}}$

$\cos 45° = x = \frac{1}{\sqrt{2}}$

$\tan 45° = \frac{y}{x} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

$\text{cosec } 45° = \frac{1}{y} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}$

$\sec 45° = \frac{1}{x} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}$

$\cot 45° = \frac{x}{y} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

Angle of measure 60°:

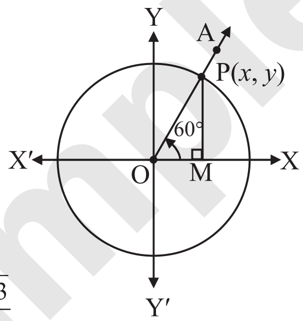
Let m∠XOA = 60°

Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).

Draw seg PM perpendicular to the X-axis.

$\therefore$ ΔOMP is a 30° – 60° – 90° triangle.

OP = 1,



$OM = \frac{1}{2} OP$

$= \frac{1}{2}(1)$

$= \frac{1}{2}$

$PM = \frac{\sqrt{3}}{2} OP$

$= \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$

Since point P lies in the 1st quadrant,

$x > 0, y > 0$

$\therefore$ $x = OM = \frac{1}{2}$ and $y = PM = \frac{\sqrt{3}}{2}$

$\therefore$ $P \equiv \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sin 60° = y = \frac{\sqrt{3}}{2}$

$\cos 60° = x = \frac{1}{2}$

$\tan 60° = \frac{y}{x} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \sqrt{3}$

$\text{cosec } 60° = \frac{1}{y} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$

$\sec 60° = \frac{1}{x} = \frac{1}{\left(\frac{1}{2}\right)} = 2$

$\cot 60° = \frac{x}{y} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}}$

Angle of measure 150°:

Let m∠XOA = 150°

Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).

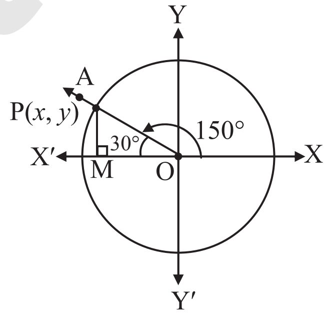
Draw seg PM perpendicular to the X-axis.

$\therefore$ ΔOMP is a 30° – 60° – 90° triangle.

OP = 1,

$OM = \frac{\sqrt{3}}{2} OP$

$= \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$



$PM = \frac{1}{2} OP$

$= \frac{1}{2}(1) = \frac{1}{2}$

Since point P lies in the 2nd quadrant,

$x < 0, y > 0$

$\therefore$ $x = -OM = \frac{-\sqrt{3}}{2}$ and $y = PM = \frac{1}{2}$

$\therefore$ $P \equiv \left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$

$\sin 150° = y = \frac{1}{2}$

$\cos 150° = x = -\frac{\sqrt{3}}{2}$

$\tan 150° = \frac{y}{x} = \frac{\frac{1}{2}}{-\left(\frac{\sqrt{3}}{2}\right)} = -\frac{1}{\sqrt{3}}$

$\text{cosec } 150° = \frac{1}{y} = \frac{1}{\left(\frac{1}{2}\right)} = 2$

$\sec 150° = \frac{1}{x} = \frac{1}{-\left(\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$

$\cot 150° = \frac{x}{y} = \frac{-\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}} = -\sqrt{3}$

Angle of measure 180°:

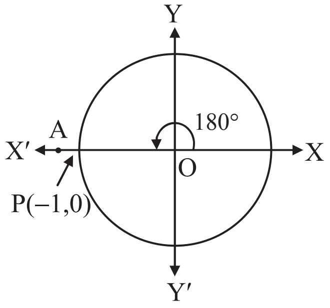
Let m∠XOA = 180°

Its terminal arm (ray OA) intersects the standard unit circle at P(–1, 0).

∴ $x = -1$ and $y = 0$

$\sin 180° = y = 0$

$\cos 180° = x = -1$



$\tan 180° = \frac{y}{x}$

$= \frac{0}{-1} = 0$

$\operatorname{cosec} 180° = \frac{1}{y}$

$= \frac{1}{0}$, which is not defined.

$\sec 180° = \frac{1}{x} = \frac{1}{-1} = -1$

$\cot 180° = \frac{x}{y} = \frac{-1}{0}$, which is not defined.

Angle of measure 210°:

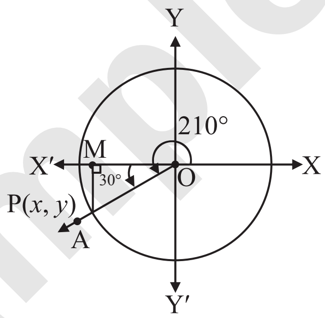
Let m∠XOA = 210°

Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).

Draw seg PM perpendicular to the X-axis.

∴ ΔOMP is a 30° – 60° – 90° triangle.

OP = 1,



$\text{OM} = \frac{\sqrt{3}}{2}\text{OP}$

$= \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$

$\text{PM} = \frac{1}{2}\text{OP}$

$= \frac{1}{2}(1) = \frac{1}{2}$

Since point P lies in the 3rd quadrant,

$x < 0, y < 0$

∴ $x = -\text{OM} = \frac{-\sqrt{3}}{2}$ and $y = -\text{PM} = \frac{-1}{2}$

∴ $\text{P} \equiv \left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$

$\sin 210° = y = \frac{-1}{2}$

$\cos 210° = x = \frac{-\sqrt{3}}{2}$

$\tan 210° = \frac{y}{x} = \frac{\left(-\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}}$

$\operatorname{cosec} 210° = \frac{1}{y} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$

$\sec 210° = \frac{1}{x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$

$\cot 210° = \frac{x}{y} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$

Angle of measure 300°:

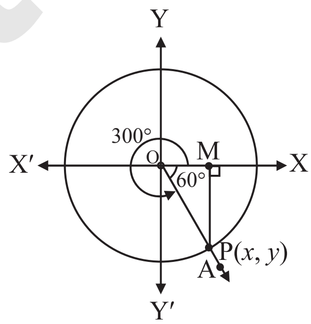
Let m∠XOA = 300°

Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).

Draw seg PM perpendicular to the X-axis.

∴ ΔOMP is a 30° – 60° – 90° triangle.

OP = 1,



$\text{OM} = \frac{1}{2}\text{OP}$

$= \frac{1}{2}(1) = \frac{1}{2}$

$\text{PM} = \frac{\sqrt{3}}{2}\text{OP}$

$= \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$

Since point P lies in the 4th quadrant,

$x > 0, y < 0$

∴ $x = \text{OM} = \frac{1}{2}$ and $y = -\text{PM} = \frac{-\sqrt{3}}{2}$

∴ $\text{P} \equiv \left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

$\sin 300° = y = \frac{-\sqrt{3}}{2}$

$\cos 300° = x = \frac{1}{2}$

$\tan 300° = \frac{y}{x} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

$\operatorname{cosec} 300° = \frac{1}{y} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$

$\sec 300° = \frac{1}{x} = \frac{1}{\left(\frac{1}{2}\right)} = 2$

$\cot 300° = \frac{x}{y} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$

Page no. **33** to **50** are purposely left blank.

To see complete chapter buy **Target Notes** or **Target E-Notes**

xi. L.H.S. $= 1 + 3\operatorname{cosec}^2\theta\cot^2\theta + \cot^6\theta$

$= 1 + 3\operatorname{cosec}^2\theta\cot^2\theta + (\cot^2\theta)^3$

$= 1 + 3\operatorname{cosec}^2\theta(\operatorname{cosec}^2\theta - 1) + (\operatorname{cosec}^2\theta - 1)^3$

$= 1 + 3\operatorname{cosec}^4\theta - 3\operatorname{cosec}^2\theta + (\operatorname{cosec}^6\theta - 3\operatorname{cosec}^4\theta + 3\operatorname{cosec}^2\theta - 1)$

$= \operatorname{cosec}^6\theta$ = R.H.S.

xii. We know that, $\sec^2\theta - \tan^2\theta = 1$

$\therefore$ $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\therefore$ $\dfrac{\sec\theta - \tan\theta}{1} = \dfrac{1}{\sec\theta + \tan\theta}$

By componendo-dividendo, we get

$\dfrac{1-(\sec\theta - \tan\theta)}{1+(\sec\theta - \tan\theta)} = \dfrac{\sec\theta + \tan\theta - 1}{\sec\theta + \tan\theta + 1}$

$\therefore$ $\dfrac{1-\sec\theta+\tan\theta}{1+\sec\theta-\tan\theta} = \dfrac{\sec\theta+\tan\theta-1}{\sec\theta+\tan\theta+1}$

Miscellaneous Exercise – 2

I. Select the correct option from the given alternatives. **[2 Marks Each]**

1. The value of the expression $\cos1°.\cos2°.\cos3° \ldots \cos179° =$

(A) -1 (B) 0
(C) $\dfrac{1}{\sqrt{2}}$ (D) 1

2. $\dfrac{\tan A}{1+\sec A} + \dfrac{1+\sec A}{\tan A}$ is equal to

(A) 2cosec A (B) 2sec A
(C) 2sin A (D) 2cos A

3. If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$, $\dfrac{\pi}{2} < \alpha < \pi$, then sin 2$\alpha$ is equal to

(A) $-\dfrac{24}{25}$ (B) $-\dfrac{13}{18}$
(C) $\dfrac{13}{18}$ (D) $\dfrac{24}{25}$

4. If $\theta = 60°$, then $\dfrac{1+\tan^2\theta}{2\tan\theta}$ is equal to

(A) $\dfrac{\sqrt{3}}{2}$ (B) $\dfrac{2}{\sqrt{3}}$
(C) $\dfrac{1}{\sqrt{3}}$ (D) $\sqrt{3}$

5. If $\sec\theta = m$ and $\tan\theta = n$, then $\dfrac{1}{m}\left\{(m+n)+\dfrac{1}{(m+n)}\right\}$ is equal to

(A) 2 (B) mn (C) 2m (D) 2n

6. If $\operatorname{cosec}\theta + \cot\theta = \dfrac{5}{2}$, then the value of $\tan\theta$ is

(A) $\dfrac{14}{25}$ (B) $\dfrac{20}{21}$
(B) $\dfrac{21}{20}$ (D) $\dfrac{15}{16}$

7. $1-\dfrac{\sin^2\theta}{1+\cos\theta}+\dfrac{1+\cos\theta}{\sin\theta}-\dfrac{\sin\theta}{1-\cos\theta}$ equals

(A) 0 (B) 1
(C) $\sin\theta$ (D) $\cos\theta$

8. If $\operatorname{cosec}\theta - \cot\theta = q$, then the value of $\cot\theta$ is

(A) $\dfrac{2q}{1+q^2}$ (B) $\dfrac{2q}{1-q^2}$
(C) $\dfrac{1-q^2}{2q}$ (D) $\dfrac{1+q^2}{2q}$

9. The cotangent of the angles $\dfrac{\pi}{3}, \dfrac{\pi}{4}$ and $\dfrac{\pi}{6}$ are in

(A) A.P.
(B) G.P.
(C) H.P.
(D) Not in progression

10. The value of $\tan 1°.\tan 2° \tan 3° \ldots \tan 89°$ is equal to

(A) -1 (B) 1
(C) $\dfrac{\pi}{2}$ (D) 2

Answers:

1.	(B)	2.	(A)	3.	(A)	4.	(B)
5.	(A)	6.	(B)	7.	(D)	8.	(C)
9.	(B)	10.	(B)				

Hints:

1. $\cos 1° \cos 2° \cos 3° \ldots \cos 179°$

$= \cos 1° \cos 2° \cos 3° \ldots \cos 90° \ldots \cos 179°$

$= 0$ …[$\because \cos 90° = 0$]

2. $\dfrac{\tan A}{1+\sec A} + \dfrac{1+\sec A}{\tan A}$

$= \dfrac{\tan^2 A + 1 + \sec^2 A + 2\sec A}{(1+\sec A)\tan A}$

$= \dfrac{\sec^2 A + \sec^2 A + 2\sec A}{(1+\sec A)\tan A}$ …[$\because 1 + \tan^2 A = \sec^2 A$]

$= \dfrac{2\sec A(\sec A + 1)}{(1+\sec A)\tan A}$

$= \dfrac{2\sec A}{\tan A}$

$= \dfrac{2}{\sin A}$

$= 2\operatorname{cosec} A$

Page no. **51** to **59** are purposely left blank.

To see complete chapter buy **Target Notes** or **Target E-Notes**

Alternate Method:

L.H.S. = $\dfrac{\operatorname{cosec}\theta + \cot\theta - 1}{\operatorname{cosec}\theta + \cot\theta + 1}$

$= \dfrac{1+\cos\theta - \sin\theta}{\sin\theta} \times \dfrac{\sin\theta}{1+\cos\theta+\sin\theta}$

$= \dfrac{1+\cos\theta - \sin\theta}{1+\cos\theta+\sin\theta} \times \dfrac{(1+\cos\theta+\sin\theta)}{1+\cos\theta+\sin\theta}$

$= \dfrac{1+\cos^2\theta + 2\cos\theta - \sin^2\theta}{1+\cos^2\theta+\sin^2\theta+2\cos\theta+2\sin\theta+2\sin\theta\cos\theta}$

$= \dfrac{2\cos\theta(\cos\theta+1)}{2(1+\cos\theta)\,(1+\sin\theta)} = \dfrac{\cos\theta}{1+\sin\theta}$

$= \dfrac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta} = \dfrac{(1-\sin\theta)\cos\theta}{\cos^2\theta}$

$= \dfrac{1-\sin\theta}{\cos\theta}$ = R.H.S.

xviii. We know that,

$\cot^2\theta = \operatorname{cosec}^2\theta - 1$

$\therefore$ $\cot\theta \cdot \cot\theta = (\operatorname{cosec}\theta + 1)(\operatorname{cosec}\theta - 1)$

$\therefore$ $\dfrac{\cot\theta}{\operatorname{cosec}\theta - 1} = \dfrac{\operatorname{cosec}\theta + 1}{\cot\theta}$

By the theorem on equal ratios, we get

$\dfrac{\cot\theta}{\operatorname{cosec}\theta - 1} = \dfrac{\operatorname{cosec}\theta + 1}{\cot\theta} = \dfrac{\cot\theta + \operatorname{cosec}\theta + 1}{\operatorname{cosec}\theta - 1 + \cot\theta}$

$\therefore$ $\dfrac{\operatorname{cosec}\theta + \cot\theta + 1}{\cot\theta + \operatorname{cosec}\theta - 1} = \dfrac{\cot\theta}{\operatorname{cosec}\theta - 1}$

One Mark Questions

1. Determine the quadrant in which θ lies if $\sec\theta > 0$ and $\sin\theta < 0$.
2. State the sign of cosec 230°.
3. Evaluate : cos 60° + sin 45° + tan 360°.
4. If $\tan\theta = \dfrac{-4}{3}$, $270° < \theta < 360°$, find sec θ.
5. Find the value of $\sin\dfrac{25\pi^c}{6}$.

Additional Problems For Practice

Based on Exercise 2.1

1. Find the trigonometric functions of **[3 Marks Each]**

i. 135° ii. – 135°

iii. – 630°

2. State the signs of trigonometric functions. **[1 Mark Each]**

i. sin (159°) ii. $\cos\left(\dfrac{11\pi}{9}\right)^c$

iii. tan (610°) iv. $\sec\left(\dfrac{3\pi}{5}\right)^c$

+3. Find the signs of the following: **[1 Mark Each]**

i. sin 300° ii. cos 400°

iii. cot (–206)°

4. Determine the quadrant in which θ lies, if **[1 Mark Each]**

i. $\sin\theta > 0$, $\sec\theta < 0$

ii. $\cos\theta < 0$, $\cot\theta > 0$

iii. $\sec\theta > 0$, $\operatorname{cosec}\theta < 0$

+5. For θ = 30°, Verify that sin2θ = 2sinθ cosθ **[2 Marks]**

6. Evaluate the following: **[2 Marks Each]**

i. $\sin 0 + 2.\cos 0 + 3.\sin\left(\dfrac{\pi}{2}\right)^c + 4.\cos\left(\dfrac{\pi}{2}\right)^c + 5.\sec 0 + 6.\operatorname{cosec}\left(\dfrac{\pi}{2}\right)^c$

ii. $4\cot 45° - \sec^2 60° + \sin^2 30°$

+iii. cos30° × cos60° + sin30° × sin60°

+iv. $4\cos^3 45° - 3\cos 45° + \sin 45°$

+v. $\sin^2 0 + \sin^2\dfrac{\pi}{6} + \sin^2\dfrac{\pi}{3} + \sin^2\dfrac{\pi}{2}$

+vi. $\sin\pi + 2\cos\pi + 3\sin\dfrac{3\pi}{2} + 4\cos\dfrac{3\pi}{2} - 5\sec\pi - 6\operatorname{cosec}\dfrac{3\pi}{2}$

7. Verify that.

i. $\cot^2 60° + \sin^2 45° + \sin^2 30° + \cos^2 90° = \dfrac{13}{12}$ **[2 Marks]**

ii. $\sin^2 30° + \cos^2 60° + \tan^2 45° + \sec^2 60° - \operatorname{cosec}^2 30° = \dfrac{3}{2}$ **[2 Marks]**

iii. $4.\cot^2 30° + 9.\sin^2 60° - 6.\operatorname{cosec}^2 60° - \dfrac{9}{4}.\tan^2 60° = 4$ **[2 Marks]**

iv. $\dfrac{\tan^2\left(\frac{\pi}{6}\right)^c + \sin^2\left(\frac{\pi}{6}\right)^c + \cos^2\left(\frac{\pi}{3}\right)^c}{\sec^2\left(\frac{\pi}{4}\right)^c - \cos^2\pi^c} = \dfrac{1}{\sqrt{3}}\sec\left(\dfrac{\pi}{6}\right)^c + \dfrac{1}{3}\cos\left(\dfrac{\pi}{3}\right)^c$ **[3 Marks]**

8. Find the trigonometric functions of angles in standard position whose terminal arms pass through **[3 Marks Each]**
i. (5, – 12) ii. (– 7, – 24).

+9. Find all trigonometric functions of the angle made by OP with X-axis where P is (– 5, 12). **[3 Marks]**

10. i. If cosec θ = –2 and $\pi < \theta < \frac{3\pi}{2}$, then find the values of other trigonometric functions. **[3 Marks]**

ii. If $\cos\theta = -\frac{12}{13}$ and $\pi < \theta < \frac{3\pi}{2}$, then find the value of 12 tan θ – 5 cosec θ. **[2 Marks]**

+11. i. secθ = – 3 and $\pi < \theta < \frac{3\pi}{2}$, then find the values of other trigonometric functions. **[3 Marks]**

ii. If $\sec x = \frac{13}{5}$, x lies in the fourth quadrant, find the values of other trigonometric functions. **[3 Marks]**

iii. If $\sin\theta = -\frac{3}{5}$ and 180° < θ < 270° then find all trigonometric functions of θ. **[3 Marks]**

+12. If $\tan A = \frac{4}{3}$, find the value of $\frac{2\sin A - 3\cos A}{2\sin A + 3\cos A}$ **[3 Marks]**

Based on Exercise 2.2

+1. If $\tan\theta + \frac{1}{\tan\theta} = 2$, then find the value of $\tan^2\theta + \frac{1}{\tan^2\theta}$. **[2 Marks]**

+2. If $\tan\theta = \frac{1}{\sqrt{7}}$, then evaluate $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$. **[2 Marks]**

+3. If $5\tan A = \sqrt{2}$, $\pi < A < \frac{3\pi}{2}$ and $\sec B = \sqrt{11}$, $\frac{3\pi}{2} < B < 2\pi$, then find the value of cosec A – tan B. **[3 Marks]**

+4. Eliminate θ from the following: **[2 Marks Each]**
i. $x = a\cos\theta$, $y = b\sin\theta$
ii. $x = a\cos^3\theta$, $y = b\sin^3\theta$
iii. $x = 2 + 3\cos\theta$, $y = 5 + 3\sin\theta$

5. Eliminate θ, if **[2 Marks Each]**
i. $x = a.\sec\theta$, $y = b.\tan\theta$
ii. $x = 2.\cos\theta - 3.\sin\theta$, $y = \cos\theta + 2.\sin\theta$
iii. $x.\cos\theta + y.\sin\theta = a$, $x.\sin\theta - y.\cos\theta = b$
iv. $x = 2.\sec\theta + 3.\tan\theta$, $y = 3.\sec\theta - 2.\tan\theta$.

6. Find the possible value of sin x, if $8.\sin x - \cos x = 4$. **[3 Marks]**

+7. If $2\sin^2\theta + 7\cos\theta = 5$, then find the permissible values of cosθ. **[3 Marks]**

+8. Solve for θ, if
$4\sin^2\theta - 2(\sqrt{3} + 1)\sin\theta + \sqrt{3} = 0$ **[4 Marks]**

+9. If tanθ + secθ = 1.5, then find tanθ, sinθ and secθ. **[3 Marks]**

+10. Which of the following is true?
i. $2\cos^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ **[2 Marks]**
ii. $\frac{\cot A - \tan B}{\cot B - \tan A} = \cot A \tan B$ **[2 Marks]**
iii. $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} = \sin\theta + \cos\theta$ **[3 Marks]**

+11. Prove that $\cos^6\theta + \sin^6\theta = 1 - 3\sin^2\theta\cos^2\theta$ **[2 Marks]**

+12. Prove that
i. $\frac{\sin\theta}{1-\cos\theta} + \frac{\tan\theta}{1+\cos\theta} = \sec\theta\,\text{cosec}\theta + \cot\theta$ **[3 Marks]**
ii. $\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = 1 - 2\sec\theta\tan\theta + 2\tan^2\theta$ **[2 Marks]**
iii. $(\sec A - \tan A)^2 = \frac{1-\sin A}{1+\sin A}$ **[2 Marks]**

13. Prove the following:
i. $\sqrt{\frac{1-\cos A}{1+\cos A}} = \text{cosec}\,A - \cot A$ **[2 Marks]**
ii. $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$ **[2 Marks]**
iii. $(1 - \tan x)^2 + (1 - \cot x)^2 = (\sec x - \text{cosec}\,x)^2$ **[2 Marks]**
iv. $(\text{cosec}\,x - \sin x)(\sec x - \cos x)(\tan x + \cot x) = 1$ **[2 Marks]**
v. $\frac{1+\sin\theta}{1+\cos\theta} + \frac{1-\sin\theta}{1-\cos\theta} = 2.(\text{cosec}^2\theta - \cot\theta)$ **[3 Marks]**

+14. i. Find the value of $\sin\frac{41\pi}{4}$. [1 Mark]

ii. Find the value of cos 765°. [1 Mark]

15. Find the cartesian co-ordinates of the points whose polar co-ordinates are [2 Marks Each]

i. $\left(\sqrt{2}, \frac{\pi}{4}\right)$ ii. $\left(4, \frac{\pi}{2}\right)$

16. Find the polar co-ordinates of the point whose Cartesian co-ordinates are [2 Marks Each]

+i. (3, 3) ii. $\left(\sqrt{2}, \sqrt{2}\right)$

Based on Miscellaneous Exercise – 2

1. If $6.\sin^2\theta - 11.\sin\theta + 4 = 0$, then find sin θ. [2 Marks]

2. Which is greater?
cos (1110°) or cos (930°) [2 Marks]

3. Find the trigonometric functions of angles in standard position whose terminal arm passes through (–6, 8). [3 Marks]

4. i. If $\cos\theta = -\frac{3}{5}$, $\pi < \theta < \frac{3\pi}{2}$, find the value of $\frac{\text{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta}$. [2 Marks]

ii. If $\tan\theta = -\frac{4}{3}$, $\frac{3\pi}{2} < \theta < 2\pi$, find the value of 3.sec θ + 5.tan θ [2 Marks]

5. Prove that

i. $\frac{1+2\cos^2 A}{1+3\cot^2 A} = \sin^2 A$ [2 Marks]

ii. $\frac{\tan\theta}{\left(1+\tan^2\theta\right)^2} + \frac{\cot\theta}{\left(1+\cot^2\theta\right)^2} = \sin\theta.\cos\theta$ [3 Marks]

6. i. If x = r.cos θ.cos ϕ, y = r.cos θ.sin ϕ, z = r.sin θ, then show that $x^2 + y^2 + z^2 = r^2$ [2 Marks]

ii. If x = a.sin θ + b.cos θ, y = a.cos θ – b.sin θ, then show that $(ax - by)^2 + (bx + ay)^2 = (a^2 + b^2)^2$. [3 Marks]

7. If sin x + cosec x = 5, then show that $\sin^4 x + \text{cosec}^4 x = 527$. [3 Marks]

8. i. If tan x + cot x = 3, then show that $\tan^4 x + \cot^4 x = 47$. [3 Marks]

ii. If sec x + tan x = k, then show that $\sin x = \frac{k^2 - 1}{k^2 + 1}$. [3 Marks]

Multiple Choice Questions

[2 Marks Each]

1. The incorrect statement is
(A) $\sin\theta = -\frac{1}{5}$ (B) cos θ =1
(C) $\sec\theta = \frac{1}{2}$ (D) tan θ = 20

2. If $\sin\theta = -\frac{1}{\sqrt{2}}$ and tan θ = 1, then θ lies in which quadrant?
(A) first (B) second
(C) third (D) fourth

3. If sin θ + cosec θ = 2, the value of $\sin^{10}\theta + \text{cosec}^{10}\theta$ is
(A) 10 (B) 2^{10}
(C) 2^9 (D) 2

4. If sin θ + cos θ = m and sec θ + cosec θ = n, then n(m + 1)(m – 1) =
(A) m (B) n
(C) 2m (D) 2n

5. If sin θ + cos θ = 1, then sin θ cos θ =
(A) 0 (B) 1
(C) 2 (D) $\frac{1}{2}$

6. If 5 tan θ = 4, then $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} =$
(A) 0 (B) 1
(C) $\frac{1}{6}$ (D) 6

7. If $\sin x = \frac{-24}{25}$, $\frac{3\pi}{2} < x < 2\pi$, then the value of tan x is
(A) $\frac{24}{25}$ (B) $\frac{-24}{7}$
(C) $\frac{25}{24}$ (D) none of these

8. If $\sin(\alpha - \beta) = \frac{1}{2}$ and $\cos(\alpha + \beta) = \frac{1}{2}$, where α and β are positive acute angles, then
(A) α = 45°, β = 15°
(B) α = 15°, β = 45°
(C) α = 60°, β = 15°
(D) none of these

9. If θ lies in the second quadrant, then the value of $\sqrt{\left(\frac{1-\sin\theta}{1+\sin\theta}\right)}+\sqrt{\left(\frac{1+\sin\theta}{1-\sin\theta}\right)}$ is
(A) 2 sec θ
(B) –2 sec θ
(C) 2 cosec θ
(D) none of these

10. $\frac{2\sin\theta\tan\theta(1-\tan\theta)+2\sin\theta\sec^2\theta}{(1+\tan\theta)^2}=$
(A) $\frac{\sin\theta}{1+\tan\theta}$
(B) $\frac{2\sin\theta}{1+\tan\theta}$
(C) $\frac{2\sin\theta}{(1+\tan\theta)^2}$
(D) none of these

11. If tan A + cot A = 4, then $\tan^4$ A + $\cot^4$ A is equal to
(A) 110 (B) 191
(C) 80 (D) 194

12. If acos θ + bsin θ = m and asin θ – bcos θ = n, then $a^2 + b^2 =$
(A) m + n (B) $m^2 - n^2$
(C) $m^2 + n^2$ (D) none of these

13. If $\cos x + \cos^2 x = 1$, then the value of $\sin^2 x + \sin^4 x$ is
(A) 1 (B) – 1
(C) 0 (D) 2

14. The value of
$6(\sin^6\theta + \cos^6\theta) - 9(\sin^4\theta + \cos^4\theta) + 4$ is
(A) –3 (B) 0
(C) 1 (D) 3

15. If $x\sin45°\cos^2 60° = \frac{\tan^2 60°\text{cosec}30°}{\sec45°\cot^2 30°}$, then $x =$
(A) 2 (B) 4
(C) 8 (D) 16

16. (secA + tanA –1)(secA – tanA + 1) – 2 tanA =
(A) sec A (B) 2 sec A
(C) 0 (D) 1

17. The value of k, for which $(\cos x + \sin x)^2 + k\sin x\cos x - 1 = 0$ is an identity, is
(A) – 1 (B) – 2
(C) 0 (D) 1

18. If $\sin A = \frac{3}{5}$ and $\tan B = \frac{1}{2}, \frac{\pi}{2} < A < \pi, \pi < B < \frac{3\pi}{2}$.
Then the value of $8\tan A - \sqrt{5}\sec B$ is
(A) $\frac{-3}{4}$ (B) $\frac{4}{5}$
(C) $\frac{7}{2}$ (D) $\frac{-7}{2}$

19. If $x = a\sin^2\theta$, $y = a\cos^2\theta$, then $x + y =$
(A) $a\sin^2\theta$
(B) a
(C) 1
(D) $a\cos^2\theta$

Competitive Corner

1. cos 1°.cos 2°. cos 3° … cos 179° =
[MHT CET 2018]
(A) 0 (B) 1
(C) $-\frac{1}{2}$ (D) – 1

2. If cos θ + sec θ = 2, then $\sec^2\theta - \sin^2\theta =$
[MHT CET 2019]
(A) 3 (B) 4
(C) 1 (D) 2

3. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression
$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals
[JEE (Main) 2019]
(A) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$
(B) $13 - 4\cos^2\theta + 6\cos^4\theta$
(C) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$
(D) $13 - 4\cos^6\theta$

4. If cosec θ + cot θ = 5, then sin θ =
[MHT CET 2020]
(A) $\frac{5}{13}$ (B) $\frac{5}{26}$
(C) $\frac{1}{13}$ (D) $\frac{1}{5}$

5. The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \le x \le 4\pi$ is:
[JEE (Main) 2022]
(A) 4 (B) 6
(C) 8 (D) 12

Topic Test

Time: 1 Hour **Total Marks: 20**

SECTION A

Q.1. Select and write the correct answer. **[4]**

i. If 5 tan θ = 4, then $\dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} =$

(A) 0 (B) 1 (C) $\dfrac{1}{6}$ (D) 6

ii. The value of the expression cos1°. cos2°. cos3° … cos179° =

(A) –1 (B) 0 (C) $\dfrac{1}{\sqrt{2}}$ (D) 1

Q.2. Answer the following. **[2]**

i. State the sign of sec 230°.

ii. Find the value of $\sin\dfrac{19\pi^c}{3}$.

SECTION B

Attempt any two of the following: **[4]**

Q.3. Eliminate θ from the following: $x = 6\operatorname{cosec}\theta$, $y = 8\cot\theta$

Q.4. Find the polar co-ordinates of the point whose cartesian co-ordinates are $\left(1, \sqrt{3}\right)$.

Q.5. Evaluate the following:

$$\sin\pi + 2\cos\pi + 3\sin\frac{3\pi}{2} + 4\cos\frac{3\pi}{2} - 5\sec\pi - 6\operatorname{cosec}\frac{3\pi}{2}$$

SECTION C

Attempt any two of the following: **[6]**

Q.6. Find all trigonometric functions of angle in standard position whose terminal arm passes through point (3, – 4).

Q.7. Prove the following:

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$$

Q.8. If $\sin\theta = \dfrac{x^2 - y^2}{x^2 + y^2}$, then find the values of cos θ, tan θ in terms of x and y.

SECTION D

Attempt any one of the following: **[4]**

Q.9. If $2\sin A = 1 = \sqrt{2}\cos B$ and $\dfrac{\pi}{2} < A < \pi$, $\dfrac{3\pi}{2} < B < 2\pi$, then find the value of $\dfrac{\tan A + \tan B}{\cos A - \cos B}$.

Q.10. Solve for θ, if $4\sin^2\theta - 2\left(\sqrt{3}+1\right)\sin\theta + \sqrt{3} = 0$.

ANSWERS

One Mark Questions

1. 4^{th} quadrant
2. Negative
3. $\dfrac{\sqrt{2}+1}{2}$
4. $\dfrac{5}{3}$
5. $\dfrac{1}{2}$

Additional Problems For Practice

Based on Exercise 2.1

1. i. $\sin 135° = \dfrac{1}{\sqrt{2}}$

   $\text{cosec } 135° = \sqrt{2}$

   $\cos 135° = -\dfrac{1}{\sqrt{2}}$

   $\sec 135° = -\sqrt{2}$

   $\tan 135° = -1$

   $\cot 135° = -1$

   ii. $\sin(-135°) = -\dfrac{1}{\sqrt{2}}$

   $\text{cosec }(-135°) = -\sqrt{2}$

   $\cos(-135°) = -\dfrac{1}{\sqrt{2}}$

   $\sec(-135°) = -\sqrt{2}$

   $\tan(-135°) = 1$

   $\cot(-135°) = 1$

   iii. $\sin(-630°) = 1$,

   $\cos(-630°) = 0$,

   $\tan(-630°)$: not defined,

   $\cot(-630°) = 0$,

   $\sec(-630°)$: not defined,

   $\text{cosec }(-630°) = 1$

2. i. positive ii. negative
   iii. positive iv. negative

3. i. negative ii. positive
   iii. negative

4. i. 2^{nd} quadrant ii. 3^{rd} quadrant
   iii. 4^{th} quadrant

6. i. 16 ii. $\dfrac{1}{4}$
   iii. $\dfrac{\sqrt{3}}{2}$ iv. 0
   v. 2 vi. 6

8. i. $\sin\theta = \dfrac{-12}{13}$, $\text{cosec }\theta = \dfrac{-13}{12}$,

   $\cos\theta = \dfrac{5}{13}$, $\sec\theta = \dfrac{13}{5}$,

   $\tan\theta = \dfrac{-12}{5}$, $\cot\theta = \dfrac{-5}{12}$.

   ii. $\sin\theta = \dfrac{-24}{25}$, $\text{cosec }\theta = \dfrac{-25}{24}$,

   $\cos\theta = \dfrac{-7}{25}$, $\sec\theta = \dfrac{-25}{7}$,

   $\tan\theta = \dfrac{-24}{-7} = \dfrac{24}{7}$, $\cot\theta = \dfrac{7}{24}$.

9. $\sin\theta = \dfrac{12}{13}$, $\text{cosec }\theta = \dfrac{13}{12}$, $\cos\theta = \dfrac{-5}{13}$, $\sec\theta = \dfrac{-13}{12}$,

   $\tan\theta = \dfrac{-12}{5}$, $\cot\theta = \dfrac{-5}{12}$.

10. i. $\sin\theta = -\dfrac{1}{2}$, $\cos\theta = -\dfrac{\sqrt{3}}{2}$, $\tan\theta = \dfrac{1}{\sqrt{3}}$,

    $\cot\theta = \sqrt{3}$, $\sec\theta = -\dfrac{2}{\sqrt{3}}$

    ii. 18

11. i. $\sin\theta = -\dfrac{2\sqrt{2}}{3}$, $\cos\theta = -\dfrac{1}{3}$, $\tan\theta = 2\sqrt{2}$,

    $\cot\theta = \dfrac{1}{2\sqrt{2}}$, $\text{cosec }\theta = -\dfrac{3}{2\sqrt{2}}$

    ii. $\sin x = -\dfrac{12}{13}$, $\cos x = \dfrac{5}{13}$, $\tan x = -\dfrac{12}{5}$,

    $\cot x = -\dfrac{5}{12}$, $\text{cosec } x = -\dfrac{13}{12}$

    iii. $\cos\theta = -\dfrac{4}{5}$, $\tan\theta = \dfrac{3}{4}$, $\cot\theta = \dfrac{4}{3}$,

    $\sec\theta = -\dfrac{5}{4}$, $\text{cosec }\theta = -\dfrac{5}{3}$

12. $\dfrac{-1}{17}$

Based on Exercise 2.2

1. 2
2. $\dfrac{3}{4}$
3. $\dfrac{\sqrt{20}-\sqrt{27}}{\sqrt{2}}$
4. i. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ ii. $\left(\dfrac{x}{a}\right)^{\frac{2}{3}} + \left(\dfrac{y}{a}\right)^{\frac{2}{3}} = 1$

   iii. $(x-2)^2 + (y-5)^2 = 9$

5. i. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
ii. $(x - 2y)^2 + (2x + 3y)^2 = 49$
iii. $x^2 + y^2 = a^2 + b^2$
iv. $(2x + 3y)^2 - (3x - 2y)^2 = 169$

6. $\sin x = \frac{3}{5}$ or $\sin x = \frac{5}{13}$.

7. $\frac{1}{2}$

8. $\theta = \frac{\pi}{6}$ or $\frac{\pi}{3}$

9. $\sec\theta = \frac{13}{12}$, $\tan\theta = \frac{5}{12}$ and $\sin\theta = \frac{5}{13}$

10. i. not true ii. not true
iii. true

14. i. $\frac{1}{\sqrt{2}}$ ii. $\frac{1}{\sqrt{2}}$

15. i. (1, 1) ii. (0, 4)

16. i. $(3\sqrt{2}, 45°)$ ii. $\left(2, \frac{\pi}{4}\right)$

Based on Miscellaneous Exercise – 2

1. $\frac{1}{2}$

2. cos (1110°) is greater.

3. $\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$

$\text{cosec}\,\theta = \frac{5}{4}$, $\sec\theta = -\frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

4. i. $\frac{1}{6}$ ii. $\frac{-5}{3}$

Multiple Choice Questions

1.	(C)	2.	(C)	3.	(D)	4.	(C)
5.	(A)	6.	(C)	7.	(B)	8.	(A)
9.	(B)	10.	(B)	11.	(D)	12.	(C)
13.	(A)	14.	(C)	15.	(C)	16.	(C)
17.	(B)	18.	(D)	19.	(B)		

Competitive Corner

1.	(A)	2.	(C)	3.	(D)	4.	(A)
5.	(C)						

Topic Test

1. i. (C) ii. (B)

2. i. negative ii. $\frac{\sqrt{3}}{2}$

3. $16x^2 - 9y^2 = 576$

4. (2, 60°)

5. 6

6. $\sin\theta = -\frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = -\frac{4}{3}$,

$\text{cosec}\,\theta = -\frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

8. $\cos\theta = \pm\frac{2xy}{x^2 + y^2}$ and $\tan\theta = \pm\frac{x^2 - y^2}{2xy}$

9. $\frac{2\left(1+\sqrt{3}\right)}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}\right)}$

10. $\frac{\pi}{3}$ or $\frac{\pi}{6}$

Scan the given Q. R. Code in *Quill - The Padhai App* to view solutions of:
i. Additional Problems for Practice
ii. Competitive Corner
iii. Topic Test