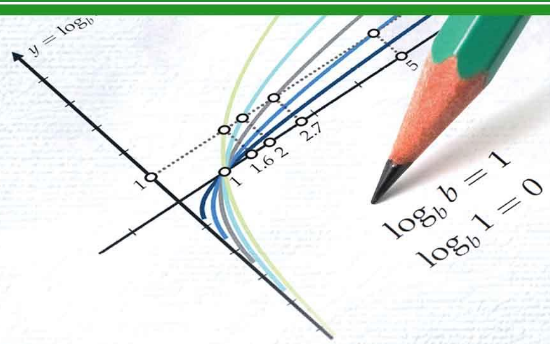

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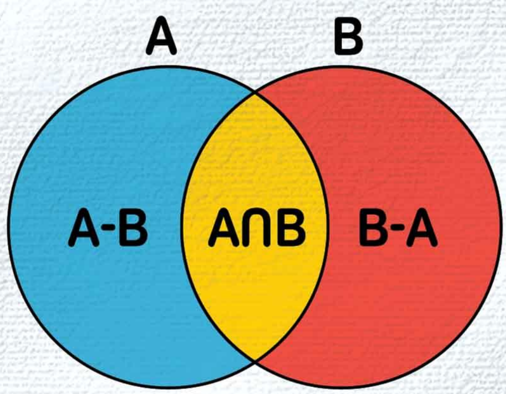

- Memory Map
- Important Formulae
- Smart Check
- Comprehensive coverage of Textual Questions and Selected Solutions
- Additional Problems, One Marks Questions and MCQs for practice
- Aligned with the latest paper pattern

STD XI

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Written as per the latest textbook prescribed by the Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune.

FOCUS

MATHEMATICS & STATISTICS Part - II

Std. XI Sci. & Arts

Printed at: **Print to Print,** Mumbai

Balbharati Registration No.: 2018MH0022

TEID: 3749

P.O. No. 13450

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[Reference: Maharashtra State Board of Secondary and Higher Secondary Education, Pune - 04**]**

Solved examples from textbook are indicated by "+".

Smart check is indicated by ✓ symbol.

Page no.**1** to **85** are purposely left blank.

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Method of Induction and Binomial Theorem

Memory Map

METHOD OF INDUCTION AND BINOMIAL THEOREM

Principle of Mathematical Induction

- Let P(n) be a statement involving the natural number n such that
  i. P(1) is true, i.e., P(n) is true for n = 1 and
  ii. P(k + 1) is true, whenever P(k) is true.
  Then P(n) is true for all $n \in N$.

Binomial Theorem for Positive integral index

- If $a, b \in R$ and $n \in N$, then
  $(a + b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + \ldots + {}^nC_n\, a^0 b^n$
- $(a - b)^n = {}^nC_0 a^n b^0 - {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \ldots + (-1)^n\, {}^nC_n a^0 b^n$

General term in the expansion of $(a + b)^n$

$t_r = {}^nC_{r-1}\, a^{n-r+1}\, b^{r-1}$
$t_{r+1} = {}^nC_r\, a^{n-r}\, b^r$
t_{r+1} is called a general term for all $r \in N$ and $0 \le r \le n$.

Middle term(s) in expansion of $(a + b)^n$

- If n is even, then there are odd number of terms in the expansion of $(a + b)^n$.
  Hence, $\left(\frac{n+2}{2}\right)^{th}$ term is the middle term.
- If n is odd, then there are even number of terms in the expansion of $(a + b)^n$.
  Hence, $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ terms are two middle terms.

Binomial Theorem for Negative Index or fraction

- $(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$
- $(1 - x)^n = 1 - nx + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$

i. $\frac{1}{1+x} = (1 + x)^{-1} = 1 - x + x^2 - x^3 + x^4 - x^5 + \ldots$

ii. $\frac{1}{1-x} = (1 - x)^{-1} = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots$

iii. $\frac{1}{(1+x)^2} = (1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots$

iv. $\frac{1}{(1-x)^2} = (1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots$

v. $\sqrt{1+x} = (1+x)^{\frac{1}{2}} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \ldots$

vi. $\sqrt{1-x} = (1-x)^{\frac{1}{2}} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \ldots$

Binomial co-efficients

- ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$
- ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = {}^nC_1 + {}^nC_3 + \ldots = 2^{n-1}$

4 Method of Induction and Binomial Theorem

Important Formulae

Bionomial Expansion:

i. $(a + b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + \ldots + {}^nC_n\, a^0 b^n$

ii. $(a - b)^n = {}^nC_0 a^n b^0 - {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \ldots + (-1)^n\, {}^nC_n a^0 b^n$

General term in the expansion of $(a + b)^n$:

$t_{r+1} = {}^nC_r\ a^{n-r}\ b^r$

Middle term(s) in the expansion of $(a + b)^n$:

i. If n is even, then there are odd number of terms in the expansion of $(a + b)^n$.

Hence $\left(\frac{n+2}{2}\right)^{th}$ term is the middle term.

ii. If n is odd, then there are even number of terms in the expansion of $(a + b)^n$.

Hence $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ terms are two middle terms.

Binomial Coefficients:

i. $C_0 + C_1 + C_2 + \ldots + C_n = 2^n$

ii. $C_0 + C_2 + C_4 + \ldots = C_1 + C_3 + \ldots = 2^{n-1}$

Focus on Exercise 4.1

Prove by method of induction, for all n ∈ N.

[4 Marks Each]

1. $2 + 4 + 6 + \ldots + 2n = n(n + 1)$

Solution:

Let $P(n) \equiv 2 + 4 + 6 + \ldots + 2n = n(n + 1)$, for all $n \in N$.

Step I:

Put n = 1

L.H.S. = 2

R.H.S. = 1(1 + 1) = 2

∴ L.H.S. = R.H.S.

∴ P(n) is true for n = 1.

Step II:

Let us assume that P(n) is true for n = k.

∴ $2 + 4 + 6 + \ldots + 2k = k(k + 1)$ …(i)

Step III:

We have to prove that P(n) is true for n = k + 1, i.e., to prove that

$2 + 4 + 6 + \ldots + 2(k + 1) = (k + 1)(k + 2)$

L.H.S. $= 2 + 4 + 6 + \ldots + 2(k + 1)$

$= 2 + 4 + 6 + \ldots + 2k + 2(k + 1)$

$= k(k + 1) + 2(k + 1)$ …[From (i)]

$= (k + 1).(k + 2)$ = R.H.S.

∴ P(n) is true for n = k + 1.

Step IV:

From all the steps above, by the principle of mathematical induction, P(n) is true for all $n \in N$.

∴ $2 + 4 + 6 + \ldots + 2n = n(n + 1)$ for all $n \in N$.

2. $3 + 7 + 11 + \ldots$ to n terms $= n(2n + 1)$

3. $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Solution:

Let $P(n) \equiv 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$, for all $n \in N$.

Step I:

Put n = 1

L.H.S. $= 1^2 = 1$

R.H.S. $= \frac{1(1+1)[2(1)+1]}{6} = \frac{6}{6} = 1$

∴ L.H.S. = R.H.S.

∴ P(n) is true for n = 1.

Step II:

Let us assume that P(n) is true for n = k.

∴ $1^2 + 2^2 + 3^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$ …(i)

Step III:

We have to prove that P(n) is true for n = k + 1, i.e., to prove that

$1^2 + 2^2 + 3^2 + \ldots + (k + 1)^2$

$= \frac{(k+1)[(k+1)+1][2(k+1)+1]}{6}$

$= \frac{(k+1)(k+2)(2k+3)}{6}$

L.H.S. $= 1^2 + 2^2 + 3^2 + \ldots + (k + 1)^2$

$= 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k + 1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$ …[From (i)]

$= (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right]$

$= (k+1)\left(\dfrac{2k^2 + k + 6k + 6}{6}\right)$

$= \dfrac{(k+1)(2k^2 + 7k + 6)}{6}$

$= \dfrac{(k+1)(k+2)(2k+3)}{6}$

= R.H.S.

∴ P(n) is true for n = k + 1.

Step IV:

From all the steps above, by the principle of mathematical induction, P(n) is true for all $n \in N$.

∴ $1^2 + 2^2 + 3^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$ for all $n \in N$.

---

4. $\mathbf{1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \dfrac{n}{3}(2n-1)(2n+1)}$

---

5. $\mathbf{1^3 + 3^3 + 5^3 + \ldots}$ **to n terms** $\mathbf{= n^2(2n^2 - 1)}$

Solution:

Let $P(n) \equiv 1^3 + 3^3 + 5^3 + \ldots$ to n terms

$= n^2(2n^2 - 1)$, for all $n \in N$.

But 1, 3, 5, …… are in A.P.

∴ a = 1, d = 2

Let t_n be the n^{th} term.

∴ $t_n = a + (n-1)d = 1 + (n-1)2 = 2n - 1$

∴ $P(n) \equiv 1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = n^2(2n^2 - 1)$

Step I:

Put n = 1

L.H.S. $= 1^3 = 1$

R.H.S. $= 1^2[2(1)^2 - 1] = 1$

∴ L.H.S. = R.H.S.

∴ P(n) is true for n = 1.

Step II:

Let us assume that P(n) is true for n = k.

∴ $1^3 + 3^3 + 5^3 + \ldots + (2k-1)^3 = k^2(2k^2 - 1)$ …(i)

Step III:

We have to prove that P(n) is true for n = k + 1,

i.e., to prove that

$1^3 + 3^3 + 5^3 + \ldots + [2(k+1) - 1]^3$

$= (k+1)^2[2(k+1)^2 - 1]$

$= (k^2 + 2k + 1)(2k^2 + 4k + 1)$

L.H.S. $= 1^3 + 3^3 + 5^3 + \ldots + [2(k+1) - 1]^3$

$= 1^3 + 3^3 + 5^3 + \ldots + (2k-1)^3 + (2k+1)^3$

$= k^2(2k^2 - 1) + (2k+1)^3$ …[From (i)]

$= 2k^4 - k^2 + 8k^3 + 12k^2 + 6k + 1$

$= 2k^4 + 8k^3 + 11k^2 + 6k + 1$

$= 2k^2(k^2 + 2k + 1) + 4k^3 + 9k^2 + 6k + 1$

$= 2k^2(k^2 + 2k + 1) + 4k(k^2 + 2k + 1) + (k^2 + 2k + 1)$

$= (k^2 + 2k + 1)(2k^2 + 4k + 1)$

= R.H.S.

∴ P(n) is true for n = k + 1.

Step IV:

From all the steps above, by the principle of mathematical induction, P(n) is true for all $n \in N$.

∴ $1^3 + 3^3 + 5^3 + \ldots$ to n terms $= n^2(2n^2 - 1)$ for all $n \in N$.

---

6. $\mathbf{1.2 + 2.3 + 3.4 + \ldots + n(n+1) = \dfrac{n}{3}(n+1)(n+2)}$

---

7. $\mathbf{1.3 + 3.5 + 5.7 + \ldots}$ **to n terms** $\mathbf{= \dfrac{n}{3}(4n^2 + 6n - 1)}$

Solution:

Let $P(n) \equiv 1.3 + 3.5 + 5.7 + \ldots$ to n terms

$= \dfrac{n}{3}(4n^2 + 6n - 1)$, for all $n \in N$.

But first factor in each term, i.e., 1, 3, 5,… are in A.P. with a = 1 and d = 2.

∴ n^{th} term $= a + (n-1)d = 1 + (n-1)2 = (2n-1)$

Also, second factor in each term,

i.e., 3, 5, 7, … are in A.P. with a = 3 and d = 2.

∴ n^{th} term $= a + (n-1)d = 3 + (n-1)2 = (2n+1)$

∴ n^{th} term, $t_n = (2n-1)(2n+1)$

∴ $P(n) \equiv 1.3 + 3.5 + 5.7 + \ldots + (2n-1)(2n+1)$

$= \dfrac{n}{3}(4n^2 + 6n - 1)$

Step I:

Put n = 1

L.H.S. = 1.3 = 3

R.H.S. $= \dfrac{1}{3}[4(1)^2 + 6(1) - 1] = 3$

∴ L.H.S. = R.H.S.

∴ P(n) is true for n = 1.

Step II:

Let us assume that P(n) is true for n = k.

∴ $1.3 + 3.5 + 5.7 + \ldots + (2k-1)(2k+1)$

$= \dfrac{k}{3}(4k^2 + 6k - 1)$ …(i)

Step III:

We have to prove that P(n) is true for n = k + 1,

i.e., to prove that

$1.3 + 3.5 + 5.7 + \ldots + [2(k+1) - 1][2(k+1) + 1]$

$= \dfrac{(k+1)}{3}[4(k+1)^2 + 6(k+1) - 1]$

$= \dfrac{(k+1)}{3}(4k^2 + 8k + 4 + 6k + 6 - 1)$

$= \dfrac{(k+1)}{3}(4k^2 + 14k + 9)$

L.H.S. $= 1.3 + 3.5 + 5.7 + \ldots + [2(k+1) - 1][2(k+1) + 1]$

$= 1.3 + 3.5 + 5.7 + \ldots + (2k-1)(2k+1) + (2k+1)(2k+3)$

$= \dfrac{k}{3}(4k^2 + 6k - 1) + (2k+1)(2k+3)$

…[From (i)]

$= \frac{1}{3}[4k^3 + 6k^2 - k + 3(2k + 1)(2k + 3)]$

$= \frac{1}{3}(4k^3 + 6k^2 - k + 12k^2 + 24k + 9)$

$= \frac{1}{3}(4k^3 + 18k^2 + 23k + 9)$

$= \frac{1}{3}(k + 1)(4k^2 + 14k + 9)$

= R.H.S.

∴ P(n) is true for n = k + 1.

Step IV:

From all the steps above, by the principle of mathematical induction, P(n) is true for all $n \in N$.

∴ $1.3 + 3.5 + 5.7 + \ldots$ to n terms $= \frac{n}{3}(4n^2 + 6n - 1)$ for all $n \in N$.

---

8. $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$

---

9. $\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \ldots$ **to n terms** $= \frac{n}{3(2n+3)}$

---

10. $(2^{3n} - 1)$ is divisible by 7.

Solution:

$(2^{3n} - 1)$ is divisible by 7 if and only if $(2^{3n} - 1)$ is a multiple of 7.

Let $P(n) \equiv (2^{3n} - 1) = 7m$, where $m \in N$.

Step I:

Put n = 1

∴ $2^{3n} - 1 = 2^{3(1)} - 1 = 2^3 - 1 = 8 - 1 = 7$

∴ $(2^{3n} - 1)$ is a multiple of 7.

∴ P(n) is true for n = 1.

Step II:

Let us assume that P(n) is true for n = k.

i.e., $2^{3k} - 1$ is a multiple of 7.

∴ $2^{3k} - 1 = 7a$, where $a \in N$

∴ $2^{3k} = 7a + 1$ …(i)

Step III:

We have to prove that P(n) is true for n = k + 1, i.e., to prove that

$2^{3(k+1)} - 1 = 7b$, where $b \in N$.

∴ $P(k + 1) = 2^{3(k+1)} - 1 = 2^{3k+3} - 1 = 2^{3k}.(2^3) - 1$

$= (7a + 1)8 - 1$ …[From (i)]

$= 56a + 8 - 1$

$= 56a + 7$

$= 7(8a + 1)$

$= 7b$, where $b = (8a + 1) \in N$

∴ P(n) is true for n = k + 1.

Step IV:

From all the steps above, by the principle of mathematical induction, P(n) is true for all $n \in N$.

∴ $(2^{3n} - 1)$ is divisible by 7, for all $n \in N$.

11. $(2^{4n} - 1)$ is divisible by 15.

Solution:

$(2^{4n} - 1)$ is divisible by 15 if and only if $(2^{4n} - 1)$ is a multiple of 15.

Let $P(n) \equiv (2^{4n} - 1) = 15m$, where $m \in N$.

Step I:

Put n = 1

∴ $2^{4(1)} - 1 = 16 - 1 = 15$

∴ $(2^{4n} - 1)$ is a multiple of 15.

∴ P(n) is true for n = 1.

Step II:

Let us assume that P(n) is true for n = k.

∴ $2^{4k} - 1 = 15a$, where $a \in N$

∴ $2^{4k} = 15a + 1$ …(i)

Step III:

We have to prove that P(n) is true for n = k + 1, i.e., to prove that

$2^{4(k+1)} - 1 = 15b$, where $b \in N$

∴ $P(k + 1) = 2^{4(k+1)} - 1 = 2^{4k+4} - 1$

$= 2^{4k}.2^4 - 1$

$= 16.(2^{4k}) - 1$

$= 16(15a + 1) - 1$ …[From (i)]

$= 240a + 16 - 1$

$= 240a + 15$

$= 15(16a + 1)$

$= 15b$, where $b = (16a + 1) \in N$

∴ P(n) is true for n = k + 1.

Step IV:

From all the steps above, by the principle of mathematical induction, P(n) is true for all $n \in N$.

∴ $(2^{4n} - 1)$ is divisible by 15, for all $n \in N$.

---

12. $3^n - 2n - 1$ is divisible by 4.

---

13. $5 + 5^2 + 5^3 + \ldots + 5^n = \frac{5}{4}(5^n - 1)$

---

14. $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

Solution:

Let $P(n) \equiv (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, for all $n \in N$.

Step I:

Put n = 1

L.H.S. $= (\cos\theta + i\sin\theta)^1 = \cos\theta + i\sin\theta$

R.H.S. $= \cos[(1)\theta] + i\sin[(1)\theta] = \cos\theta + i\sin\theta$

∴ L.H.S. = R.H.S.

∴ P(n) is true for n = 1.

Step II:

Let us assume that P(n) is true for n = k.

∴ $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$ …(i)

Step III:

We have to prove that P(n) is true for n = k + 1, i.e., to prove that

$(\cos\theta + i\sin\theta)^{k+1} = \cos(k + 1)\theta + i\sin(k + 1)\theta$

L.H.S. $= (\cos\theta + i\sin\theta)^{k+1}$
$= (\cos\theta + i\sin\theta)^{k}.(\cos\theta + i\sin\theta)$
$= (\cos k\theta + i\sin k\theta).(\cos\theta + i\sin\theta)$ …[From (i)]
$= \cos k\theta\cos\theta + i\sin\theta\cos k\theta + i\sin k\theta\cos\theta - \sin k\theta\sin\theta$ …[$\because i^2 = -1$]
$= (\cos k\theta\cos\theta - \sin k\theta\sin\theta) + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$
$= \cos(k\theta + \theta) + i\sin(k\theta + \theta)$
$= \cos(k+1)\theta + i\sin(k+1)\theta$
= R.H.S.

$\therefore$ P(n) is true for n = k + 1.

Step IV:

From all the steps above, by the principle of mathematical induction, P(n) is true for all $n \in N$.

$\therefore$ $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$, for all $n \in N$.

---

15. Given that $t_{n+1} = 5t_n + 4$, $t_1 = 4$, prove by method of induction that $t_n = 5^n - 1$.

---

16. Prove by method of induction

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix} \forall n \in N$$

Solution:

Let P(n) $\equiv \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$, for all $n \in N$.

Step I:

Put n = 1

L.H.S. $= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^1 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

R.H.S. $= \begin{bmatrix} 1 & 2(1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$\therefore$ L.H.S. = R.H.S.

$\therefore$ P(n) is true for n = 1.

Step II:

Let us assume that P(n) is true for n = k.

$\therefore$ $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^k = \begin{bmatrix} 1 & 2k \\ 0 & 1 \end{bmatrix}$ …(i)

Step III:

We have to prove that P(n) is true for n = k + 1,

i.e., to prove that $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^{k+1} = \begin{bmatrix} 1 & 2(k+1) \\ 0 & 1 \end{bmatrix}$

L.H.S. $= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^{k+1}$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^k \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ …[From (i)]

$= \begin{bmatrix} 1 + 2k(0) & 1(2) + 2k(1) \\ 0(1) + 1(0) & 0(2) + 1(1) \end{bmatrix}$

$= \begin{bmatrix} 1 & 2k + 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2(k+1) \\ 0 & 1 \end{bmatrix}$

= R.H.S.

$\therefore$ P(n) is true for n = k + 1.

Step IV:

From all the steps above, by the principle of mathematical induction, P(n) is true for all $n \in N$.

$\therefore$ $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$, for all $n \in N$.

Practice Based On Exercise 4.1

By method of induction, prove that, for all $n \in N$

[4 Marks Each]

+1. $1.3 + 2.5 + 3.7 + \ldots + n(2n+1) = \frac{n}{6}(n+1)(4n+5)$

2. i. $1.2 + 3.4 + 5.6 + \ldots + (2n-1)(2n) = \frac{n(n+1)(4n-1)}{3}$

   ii. $1.2 + 2.5 + 3.8 + \ldots + n(3n-1) = n^2(n+1)$

3. i. $2 + 6 + 10 + \ldots + (4n-2) = 2n^2$

   ii. $1 + 4 + 7 \ldots + (3n-2) = \frac{n(3n-1)}{2}$

   iii. $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

   iv. $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$

4. $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

5. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right) \ldots \left(1-\frac{1}{n}\right) = \frac{1}{n+1}$

6. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$

+7. $\sum_{r=1}^{n} ax^{r-1} = a\left(\frac{1-x^n}{1-x}\right), x \neq 1$

+8. 5^{2n-1} is divisible by 6

9. $3^n - 1$ is divisible by 2
10. $n^2 - 3n + 4$ is even.
11. $n^3 + 2n$ is divisible by 3.
12. $3^n > n^2$
+13. $n! \geq 2^n$; $\forall$ n $\in$ N, n $\geq$ 4
+14. Given that (recurrence relation) $t_{n+1} = 3t + 4$, t = 1
+15. $2^n > n$

Focus on Exercise 4.2

1. Expand: **[2 Marks Each]**

i. $\left(\sqrt{3}+\sqrt{2}\right)^4$ **ii.** $\left(\sqrt{5}-\sqrt{2}\right)^5$

Solution:

i. Here, a = $\sqrt{3}$, b = $\sqrt{2}$ and n = 4.

Using binomial theorem,

$\left(\sqrt{3}+\sqrt{2}\right)^4$

$= {}^4C_0\left(\sqrt{3}\right)^4\left(\sqrt{2}\right)^0 + {}^4C_1\left(\sqrt{3}\right)^3\left(\sqrt{2}\right)^1 + {}^4C_2\left(\sqrt{3}\right)^2\left(\sqrt{2}\right)^2 + {}^4C_3\left(\sqrt{3}\right)^1\left(\sqrt{2}\right)^3 + {}^4C_4\left(\sqrt{3}\right)^0\left(\sqrt{2}\right)^4$

Now, ${}^4C_0 = {}^4C_4 = 1$, ${}^4C_1 = {}^4C_3 = 4$,

${}^4C_2 = \frac{4\times3}{2\times1} = 6$

$\therefore$ $\left(\sqrt{3}+\sqrt{2}\right)^4$

$= 1(9)(1) + 4\left(3\sqrt{3}\right)\left(\sqrt{2}\right) + 6(3)(2) + 4\left(\sqrt{3}\right)\left(2\sqrt{2}\right) + 1(1)(4)$

$= 9 + 12\sqrt{6} + 36 + 8\sqrt{6} + 4$

$= 49 + 20\sqrt{6}$

2. Expand: **[2 Marks Each]**

i. $(2x^2 + 3)^4$ **ii.** $\left(2x - \frac{1}{x}\right)^6$

Solution:

ii. Here, a = $2x$, b = $\frac{1}{x}$ and n = 6.

Using binomial theorem,

$\left(2x-\frac{1}{x}\right)^6$

$= {}^6C_0(2x)^6\left(\frac{1}{x}\right)^0 - {}^6C_1(2x)^5\left(\frac{1}{x}\right)^1 + {}^6C_2(2x)^4\left(\frac{1}{x}\right)^2 - {}^6C_3(2x)^3\left(\frac{1}{x}\right)^3 + {}^6C_4(2x)^2\left(\frac{1}{x}\right)^4 - {}^6C_5(2x)^1\left(\frac{1}{x}\right)^5 + {}^6C_6(2x)^0\left(\frac{1}{x}\right)^6$

Now, ${}^6C_0 = {}^6C_6 = 1$, ${}^6C_1 = {}^6C_5 = 6$,

${}^6C_2 = {}^6C_4 = \frac{6\times5}{2\times1} = 15$, ${}^6C_3 = \frac{6\times5\times4}{3\times2\times1} = 20$

$\therefore$ $\left(2x-\frac{1}{x}\right)^6$

$= 1(64x^6)(1) - 6(32x^4) + 15(16x^2) - 20(8) + 15\left(\frac{4}{x^2}\right) - 6\left(\frac{2}{x^4}\right) + 1(1)\left(\frac{1}{x^6}\right)$

$= 64x^6 - 192x^4 + 240x^2 - 160 + \frac{60}{x^2} - \frac{12}{x^4} + \frac{1}{x^6}$

3. Find the value of **[4 Marks Each]**

i. $\left(\sqrt{3}+1\right)^4 - \left(\sqrt{3}-1\right)^4$

ii. $\left(2+\sqrt{5}\right)^5 + \left(2-\sqrt{5}\right)^5$

Solution:

i. $\left(\sqrt{3}+1\right)^4$

$= {}^4C_0\left(\sqrt{3}\right)^4(1)^0 + {}^4C_1\left(\sqrt{3}\right)^3(1)^1 + {}^4C_2\left(\sqrt{3}\right)^2(1)^2 + {}^4C_3\left(\sqrt{3}\right)^1(1)^3 + {}^4C_4\left(\sqrt{3}\right)^0(1)^4$

Now, ${}^4C_0 = {}^4C_4 = 1$, ${}^4C_1 = {}^4C_3 = 4$,

${}^4C_2 = \frac{4\times3}{2\times1} = 6$

$\therefore$ $\left(\sqrt{3}+1\right)^4 = 1(9)(1) + 4\left(3\sqrt{3}\right)(1) + 6(3)(1) + 4\left(\sqrt{3}\right)(1) + 1(1)(1)$

$\therefore$ $\left(\sqrt{3}+1\right)^4 = 9 + 12\sqrt{3} + 18 + 4\sqrt{3} + 1$ …(i)

Also, $\left(\sqrt{3}-1\right)^4$

$= {}^4C_0\left(\sqrt{3}\right)^4(1)^0 - {}^4C_1\left(\sqrt{3}\right)^3(1)^1 + {}^4C_2\left(\sqrt{3}\right)^2(1)^2 - {}^4C_3\left(\sqrt{3}\right)^1(1)^3 + {}^4C_4\left(\sqrt{3}\right)^0(1)^4$

$= 1(9)(1) - 4\left(3\sqrt{3}\right)(1) + 6(3)(1) - 4\left(\sqrt{3}\right)(1) + 1(1)(1)$

$\therefore$ $\left(\sqrt{3}-1\right)^4 = 9 - 12\sqrt{3} + 18 - 4\sqrt{3} + 1$ …(ii)

Subtracting (ii) from (i), we get

$\left(\sqrt{3}+1\right)^4 - \left(\sqrt{3}-1\right)^4$

$= (9 + 12\sqrt{3} + 18 + 4\sqrt{3} + 1) - (9 - 12\sqrt{3} + 18 - 4\sqrt{3} + 1)$

$= 24\sqrt{3} + 8\sqrt{3}$

$= 32\sqrt{3}$

4. Prove that: **[4 Marks Each]**

i. $\left(\sqrt{3}+\sqrt{2}\right)^6 + \left(\sqrt{3}-\sqrt{2}\right)^6 = 970$

ii. $\left(\sqrt{5}+1\right)^5 - \left(\sqrt{5}-1\right)^5 = 352$

Solution:

i. $\left(\sqrt{3}+\sqrt{2}\right)^6$

$= {}^6C_0\left(\sqrt{3}\right)^6\left(\sqrt{2}\right)^0 + {}^6C_1\left(\sqrt{3}\right)^5\left(\sqrt{2}\right)^1$
$+ {}^6C_2\left(\sqrt{3}\right)^4\left(\sqrt{2}\right)^2 + {}^6C_3\left(\sqrt{3}\right)^3\left(\sqrt{2}\right)^3$
$+ {}^6C_4\left(\sqrt{3}\right)^2\left(\sqrt{2}\right)^4 + {}^6C_5\left(\sqrt{3}\right)^1\left(\sqrt{2}\right)^5$
$+ {}^6C_6\left(\sqrt{3}\right)^0\left(\sqrt{2}\right)^6$

Now, ${}^6C_0 = {}^6C_6 = 1$, ${}^6C_1 = {}^6C_5 = 6$,

${}^6C_2 = {}^6C_4 = \dfrac{6\times5}{2\times1} = 15$, ${}^6C_3 = \dfrac{6\times5\times4}{3\times2\times1} = 20$

$\therefore$ $\left(\sqrt{3}+\sqrt{2}\right)^6$

$= 1(27)(1) + 6\left(9\sqrt{3}\right)\left(\sqrt{2}\right) + 15(9)(2)$
$+ 20\left(3\sqrt{3}\right)\left(2\sqrt{2}\right) + 15(3)(4) + 6\left(\sqrt{3}\right)\left(4\sqrt{2}\right)$
$+ 1(1)(8)$

$\therefore$ $\left(\sqrt{3}+\sqrt{2}\right)^6$

$= 27 + 54\sqrt{6} + 270 + 120\sqrt{6} + 180 + 24\sqrt{6} + 8$ …(i)

Also, $\left(\sqrt{3}-\sqrt{2}\right)^6$

$= {}^6C_0\left(\sqrt{3}\right)^6\left(\sqrt{2}\right)^0 - {}^6C_1\left(\sqrt{3}\right)^5\left(\sqrt{2}\right)^1$
$+ {}^6C_2\left(\sqrt{3}\right)^4\left(\sqrt{2}\right)^2 - {}^6C_3\left(\sqrt{3}\right)^3\left(\sqrt{2}\right)^3$
$+ {}^6C_4\left(\sqrt{3}\right)^2\left(\sqrt{2}\right)^4 - {}^6C_5\left(\sqrt{3}\right)^1\left(\sqrt{2}\right)^5$
$+ {}^6C_6\left(\sqrt{3}\right)^0\left(\sqrt{2}\right)^6$

$= 1(27)(1) - 6\left(9\sqrt{3}\right)\left(\sqrt{2}\right) + 15(9)(2)$
$- 20\left(3\sqrt{3}\right)\left(2\sqrt{2}\right) + 15(3)(4) - 6\left(\sqrt{3}\right)\left(4\sqrt{2}\right)$
$+ 1(1)(8)$

$\therefore$ $\left(\sqrt{3}-\sqrt{2}\right)^6$

$= 27 - 54\sqrt{6} + 270 - 120\sqrt{6} + 180 - 24\sqrt{6} + 8$ …(ii)

Adding (i) and (ii), we get

$\left(\sqrt{3}+\sqrt{2}\right)^6 + \left(\sqrt{3}-\sqrt{2}\right)^6$

$= (27 + 54\sqrt{6} + 270 + 120\sqrt{6} + 180 + 24\sqrt{6} + 8)$
$+ (27 - 54\sqrt{6} + 270 - 120\sqrt{6} + 180 - 24\sqrt{6} + 8)$
$= 54 + 540 + 360 + 16 = 970$

---

5. Using binomial theorem, find the value of **[2 Marks Each]**

i. $(102)^4$ **ii. $(1.1)^5$**

Solution:

i. $(102)^4 = (100 + 2)^4$
$= {}^4C_0(100)^4(2)^0 + {}^4C_1(100)^3(2)^1$
$+ {}^4C_2(100)^2(2)^2 + {}^4C_3(100)^1(2)^3 + {}^4C_4(100)^0(2)^4$

Now, ${}^4C_0 = {}^4C_4 = 1$, ${}^4C_1 = {}^4C_3 = 4$, ${}^4C_2 = 6$

$\therefore$ $(102)^4 = 1(100000000)(1) + 4(1000000)(2)$
$+ 6(10000)(4) + 4(100)(8) + 1(1)(16)$
$= 100000000 + 8000000 + 240000 + 3200 + 16$
$= 108243216$

---

6. Using binomial theorem, find the value of **[2 Marks Each]**

i. $(9.9)^3$ **ii. $(0.9)^4$**

Solution:

ii. $(0.9)^4 = (1 - 0.1)^4$
$= {}^4C_0(1)^4(0.1)^0 - {}^4C_1(1)^3(0.1)^1 + {}^4C_2(1)^2(0.1)^2$
$- {}^4C_3(1)^1(0.1)^3 + {}^4C_4(1)^0(0.1)^4$

Now, ${}^4C_0 = {}^4C_4 = 1$, ${}^4C_1 = {}^4C_3 = 4$, ${}^4C_2 = 6$

$\therefore$ $(0.9)^4 = 1(1)(1) - 4(1)(0.1) + 6(1)(0.01)$
$- 4(1)(0.001) + 1(1)(0.0001)$
$= 1 - 0.4 + 0.06 - 0.004 + 0.0001$
$= 1.0601 - 0.404$
$= 0.6561$

---

7. Without expanding, find the value of **[2 Marks Each]**

i. $(x+1)^4 - 4(x+1)^3(x-1) + 6(x+1)^2(x-1)^2 - 4(x+1)(x-1)^3 + (x-1)^4$

ii. $(2x-1)^4 + 4(2x-1)^3(3-2x) + 6(2x-1)^2 .(3-2x)^2 + 4(2x-1)^1(3-2x)^3 + (3-2x)^4$

Solution:

i. Let $x + 1 = a$ and $x - 1 = b$

We notice that 1, 4, 6, 4, 1 are the values of ${}^4C_0$, ${}^4C_1$, ${}^4C_2$, ${}^4C_3$, ${}^4C_4$ respectively.

$\therefore$ The given expression becomes

${}^4C_0 a^4b^0 - {}^4C_1 a^3 b + {}^4C_2 a^2b^2 - {}^4C_3 a b^3 + {}^4C_4 a^0b^4$
$= (a - b)^4$
$= [x + 1 - (x - 1)]^4 = (x + 1 - x + 1)^4$
$= 2^4$
$= 16$

---

8. Find the value of $(1.02)^6$, correct upto four places of decimals. **[2 Marks]**

Solution:

$(1.02)^6 = (1 + 0.02)^6$
$= {}^6C_0(1)^6(0.02)^0 + {}^6C_1(1)^5(0.02)^1$
$+ {}^6C_2(1)^4(0.02)^2 + {}^6C_3(1)^3(0.02)^3$
$+ {}^6C_4(1)^2(0.02)^4 + {}^6C_5(1)^1(0.02)^5$
$+ {}^6C_6(1)^0(0.02)^6$

Now, ${}^6C_0 = {}^6C_6 = 1$, ${}^6C_1 = {}^6C_5 = 6$,

${}^6C_2 = {}^6C_4 = \dfrac{6\times5}{2\times1} = 15$, ${}^6C_3 = \dfrac{6\times5\times4}{3\times2\times1} = 20$

$\therefore$ $(1.02)^6 = 1(1)(1) + 6(1)(0.02) + 15(1)(0.0004)$
$+ 20(1)(0.000008) + \ldots$
$= 1 + 0.12 + 0.0060 + 0.000160 + \ldots$
$= 1.12616$
$= 1.1262$, correct upto 4 decimal places.

9. Find the value of $(1.01)^5$, correct upto three places of decimals. [2 Marks]

10. Find the value of $(0.9)^6$, correct upto four places of decimals. [2 Marks]

Practice Based On Exercise 4.2

+1. i. Expand $(x^2 + 3y)^5$ **[2 Marks]**

ii. Expand $\left(2x - \frac{y}{2}\right)^5$ **[3 Marks]**

2. Expand by using binomial theorem: **[2 Marks Each]**

i. $\left(x + \frac{1}{x}\right)^5$

ii. $(x^2 - 3y)^5$

iii. $\left(2 + \sqrt{3}\right)^4$

iv. $\left(\sqrt{5} - \sqrt{3}\right)^4$

+3. Expand

i. $\left(\sqrt{5} + \sqrt{3}\right)^4$ **[3 Marks]**

ii. $\left(\sqrt{2} + 1\right)^5 - \left(\sqrt{2} - 1\right)^5$ **[4 Marks]**

4. i. Find the value of: $\left(\sqrt{2} + 1\right)^5 + \left(\sqrt{2} - 1\right)^5$ **[4 Marks]**

ii. Find the value of: $\left(\sqrt{5} + \sqrt{2}\right)^4 + \left(\sqrt{5} - \sqrt{2}\right)^4$ **[4 Marks]**

iii. Prove that: $\left(\sqrt{2} + 1\right)^6 - \left(\sqrt{2} - 1\right)^6 = 140\sqrt{2}$ **[4 Marks]**

iv. Prove that: $\left(3 + \sqrt{2}\right)^5 + \left(3 - \sqrt{2}\right)^5 = 1686$. **[4 Marks]**

+5. Find the value of $(2.02)^5$ correct upto 4 decimal places. **[2 Marks]**

6. Using binomial theorem, find the value of: **[2 Marks Each]**

i. $(1.02)^4$ ii. $(98)^3$

+7. Without expanding, find the value of
$(2x - 1)^5 + 5(2x - 1)^4(1 - x) + 10(2x - 1)^3(1 - x)^2 + 10(2x - 1)^2(1 - x)^3 + 5(2x - 1)(1 - x)^4 + (1 - x)^5$ **[2 Marks]**

8. Without expanding find the value of
$(4x - 1)^5 + 5(4x - 1)^4(1 - 3x) + 10(4x - 1)^3(1 - 3x)^2 + 10(4x - 1)^2(1 - 3x)^3 + 5(4x - 1)(1 - 3x)^4 + (1 - 3x)^5$ **[2 Marks]**

Focus on Exercise 4.3

1. In the following expansions, find the indicated term. [2 Marks Each]

i. $\left(2x^2 + \frac{3}{2x}\right)^8$, **3rd term**

ii. $\left(x^2 - \frac{4}{x^3}\right)^{11}$, **5th term**

iii. $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$, **7th term**

iv. $\left(\frac{1}{3} + a^2\right)^{12}$, **9th term**

v. $\left(3a + \frac{4}{a}\right)^{13}$, **10th term**

Solution:

ii. Here, $a = x^2$, $b = \frac{-4}{x^3}$, $n = 11$.

For 5th term, $r = 4$

We have $t_{r+1} = {}^nC_r\ a^{n-r}\ b^r$

$\therefore\ t_5 = {}^{11}C_4 (x^2)^{11-4}\left(\frac{-4}{x^3}\right)^4 = \frac{11!}{4!\,7!} \times (x^2)^7 \times \left(\frac{-4}{x^3}\right)^4$

$= \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1} \times x^{14} \times \frac{256}{x^{12}}$

$= 330 \times 256 \times x^2$

$= 84480\,x^2$

$\therefore$ 5th term in the expansion of $\left(x^2 - \frac{4}{x^3}\right)^{11}$ is $84480\,x^2$.

---

iv. Here, $a = \frac{1}{3}$, $b = a^2$, $n = 12$.

For 9th term, $r = 8$

We have $t_{r+1} = {}^nC_r\ a^{n-r}\ b^r$

$\therefore\ t_9 = {}^{12}C_8\left(\frac{1}{3}\right)^{12-8}(a^2)^8 = \frac{12!}{8!\,4!} \times \left(\frac{1}{3}\right)^4 \times a^{16}$

$= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} \times \frac{1}{81} \times a^{16} = \frac{55}{9}a^{16}$

$\therefore$ 9th term in the expansion of $\left(\frac{1}{3} + a^2\right)^{12}$ is $\frac{55}{9}a^{16}$.

2. In the following expansions, find the indicated coefficients. [3 Marks Each]

i. x^3 **in** $\left(x^2 + \frac{3\sqrt{2}}{x}\right)^9$ **ii.** x^8 **in** $\left(2x^5 - \frac{5}{x^3}\right)^8$

iii. x^9 **in** $\left(\frac{1}{x} + x^2\right)^{18}$ **iv.** x^{-3} **in** $\left(x - \frac{1}{2x}\right)^5$

v. x^{-20} **in** $\left(x^3 - \frac{1}{2x^2}\right)^{15}$

Solution:

ii. Here, $a = 2x^5$, $b = \frac{-5}{x^3}$, $n = 8$.

We have $t_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

$= {}^8C_r\,(2x^5)^{8-r}\left(\frac{-5}{x^3}\right)^r$

$= {}^8C_r\,(2)^{8-r}\, x^{40-5r}\,(-5)^r\, x^{-3r}$

$= {}^8C_r\,(2)^{8-r}\,(-5)^r\, x^{40-8r}$

To get the coefficient of x^8, we must have

$x^{40-8r} = x^8$

$\therefore$ $40 - 8r = 8$

$\therefore$ $8r = 32$

$\therefore$ $r = 4$

$\therefore$ Coefficient of x^8 $= {}^8C_4\,(2)^4\,(-5)^4$

$= \frac{8!}{4!\,4!}\,(2)^4(-5)^4$

$= \frac{8\times7\times6\times5}{4\times3\times2\times1} \times 16 \times 625$

$= 700000$

---

iii. Here, $a = \frac{1}{x}$, $b = x^2$, $n = 18$.

We have $t_{r+1} = {}^nC_r\, a^{n-r}.b^r$

$= {}^{18}C_r\left(\frac{1}{x}\right)^{18-r}(x^2)^r$

$= {}^{18}C_r\,(x^{-1})^{18-r}.x^{2r}$

$= {}^{18}C_r\, x^{r-18}.\, x^{2r}$

$= {}^{18}C_r\, x^{3r-18}$

To get the coefficient of x^9, we must have

$x^{3r-18} = x^9$

$\therefore$ $3r - 18 = 9$

$\therefore$ $3r = 27$

$\therefore$ $r = 9$

$\therefore$ Coefficient of x^9

$= {}^{18}C_9$

$= \frac{18!}{9!9!}$

$= \frac{18\times17\times16\times15\times14\times13\times12\times11\times10}{9\times8\times7\times6\times5\times4\times3\times2\times1}$

$= 48620$

---

v. Here, $a = x^3$, $b = \frac{-1}{2x^2}$, $n = 15$.

We have $t_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

$= {}^{15}C_r\,(x^3)^{15-r}\left(\frac{-1}{2x^2}\right)^r$

$= {}^{15}C_r\, x^{45-3r}\left(\frac{-1}{2}\right)^r x^{-2r}$

$= {}^{15}C_r\left(\frac{-1}{2}\right)^r x^{45-5r}$

To get the coefficient of x^{-20}, we must have

$x^{45-5r} = x^{-20}$

$\therefore$ $45 - 5r = -20$

$\therefore$ $5r = 65$

$\therefore$ $r = 13$

$\therefore$ Coefficient of $x^{-20} = {}^{15}C_{13}\left(\frac{-1}{2}\right)^{13}$

$= \frac{15!}{13!\,2!}\left(\frac{-1}{2}\right)^{13}$

$= \frac{15\times14}{2\times1}\times\left(\frac{-1}{8192}\right)$

$= \frac{-105}{8192}$

---

3. Find the constant term (term independent of x) in the expansion of [3 Marks Each]

i. $\left(2x + \frac{1}{3x^2}\right)^9$ **ii.** $\left(x - \frac{2}{x^2}\right)^{15}$

iii. $\left(\sqrt{x} - \frac{3}{x^2}\right)^{10}$ **iv.** $\left(x^2 - \frac{1}{x}\right)^9$

v. $\left(2x^2 - \frac{5}{x}\right)^9$

Solution:

ii. Here, $a = x$, $b = \frac{-2}{x^2}$, $n = 15$.

We have $t_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

$= {}^{15}C_r\,(x)^{15-r}\left(\frac{-2}{x^2}\right)^r$

$= {}^{15}C_r\, x^{15-r}\,(-2)^r\, x^{-2r}$

$= {}^{15}C_r\,(-2)^r\, x^{15-3r}$

To get the term independent of x, we must have

$x^{15-3r} = x^0$

$\therefore$ $15 - 3r = 0$

$\therefore$ $r = 5$

$\therefore$ The term independent of x

$= {}^{15}C_5\,(-2)^5$

$= \frac{15!}{5!10!}(-2)^5$

$= \frac{15\times14\times13\times12\times11}{5\times4\times3\times2\times1} \times (-32)$

$= -96096$

---

iii. Here, $a = \sqrt{x}$, $b = \frac{-3}{x^2}$, $n = 10$.

We have $t_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

$= {}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{-3}{x^2}\right)^r$

$= {}^{10}C_r\,(x)^{\frac{10-r}{2}}\,(-3)^r\, x^{-2r}$

$= {}^{10}C_r\,(-3)^r\, x^{\frac{10-r}{2}}\, x^{-2r}$

$= {}^{10}C_r\,(-3)^r\, x^{\frac{10-5r}{2}}$

To get the term independent of x, we must have

$x^{\frac{10-5r}{2}} = x^0$

∴ $\frac{10-5r}{2} = 0$

∴ $10 - 5r = 0$

∴ $r = 2$

∴ The term independent of x $= {}^{10}C_2\,(-3)^2$

$= \frac{10!}{2!\,8!}(-3)^2$

$= \frac{10\times9}{2\times1}\times 9$

$= 405$

---

4. Find the middle terms in the expansion of

i. $\left(\frac{x}{y}+\frac{y}{x}\right)^{12}$ **[2 Marks]**

ii. $\left(x^2+\frac{1}{x}\right)^{7}$ **[3 Marks]**

iii. $\left(x^2-\frac{2}{x}\right)^{8}$ **[2 Marks]**

iv. $\left(\frac{x}{a}-\frac{a}{x}\right)^{10}$ **[2 Marks]**

v. $\left(x^4-\frac{1}{x^3}\right)^{11}$ **[3 Marks]**

Solution:

i. Here, $a = \frac{x}{y}$, $b = \frac{y}{x}$, $n = 12$.

Now, n is even.

∴ $\frac{n+2}{2} = \frac{12+2}{2} = 7$

∴ Middle term is t_7, for which $r = 6$.

We have $t_{r+1} = {}^{n}C_r\,a^{n-r}\,b^r$

∴ $t_7 = {}^{12}C_6\left(\frac{x}{y}\right)^6.\left(\frac{y}{x}\right)^6 = \frac{12!}{6!\,6!}$

$= \frac{12\times11\times10\times9\times8\times7}{6\times5\times4\times3\times2\times1} = 924$

∴ Middle term is 924.

---

iii. Here, $a = x^2$, $b = \frac{-2}{x}$, $n = 8$.

Now, n is even.

∴ $\frac{n+2}{2} = \frac{8+2}{2} = 5$

∴ Middle term is t_5, for which $r = 4$.

We have $t_{r+1} = {}^{n}C_r\,a^{n-r}\,b^r$

∴ $t_5 = {}^{8}C_4\,(x^2)^4\left(\frac{-2}{x}\right)^4$

$= \frac{8!}{4!\,4!}(x^8)\left(\frac{16}{x^4}\right)$

$= \frac{8\times7\times6\times5}{4\times3\times2\times1}\times\frac{x^8}{x^4}\times 16$

$= 1120x^4$

∴ Middle term is $1120x^4$.

iv. Here, $a = \frac{x}{a}$, $b = \frac{-a}{x}$, $n = 10$.

Now, n is even.

∴ $\frac{n+2}{2} = \frac{10+2}{2} = 6$

∴ Middle term is t_6, for which $r = 5$.

We have $t_{r+1} = {}^{n}C_r\,a^{n-r}\,b^r$

∴ $t_6 = {}^{10}C_5.\left(\frac{x}{a}\right)^5.\left(-\frac{a}{x}\right)^5$

$= {}^{10}C_5\times -1$

$= \frac{-10!}{5!5!}$

$= -\frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1} = -252$

∴ Middle term is -252.

---

5. In the expansion of $(k + x)^8$, the coefficient of x^5 is 10 times the coefficient of x^6. Find the value of k. **[2 Marks]**

Solution:

Coefficient of x^5 in $(k + x)^8 = {}^{8}C_5\;k^{8-5}$

$= {}^{8}C_5\,k^3$

Coefficient of x^6 in $(k + x)^8 = {}^{8}C_6\;k^2$

The coefficient of x^5 is 10 times the coefficient of x^6.

∴ ${}^{8}C_5\,k^3 = 10\left({}^{8}C_6\,k^2\right)$

∴ $\frac{8\times7\times6}{3\times2\times1}\,k = 10\left(\frac{8\times7}{2\times1}\right)$

∴ $k = 5$

SMART CHECK

In the expansion of $(5 + x)^8$, if the coefficient of x^5 is 10 times the coefficient of x^6, then our answer is correct.

Coefficient of x^5 in $(5 + x)^8 = {}^{8}C_5 5^3$

$= 7000 = 10(700)$

Coefficient of x^6 in $(5 + x)^8 = {}^{8}C_6\,5^2 = 700$

Thus, our answer is correct.

---

6. Find the term containing x^6 in the expansion of $(2 - x)\,(3x + 1)^9$. **[3 Marks]**

Solution:

$(2 - x)\,(3x + 1)^9 = 2(3x + 1)^9 - x(3x + 1)^9$

Consider $(3x + 1)^9$

Here, $a = 3x$, $b = 1$, $n = 9$

We have $t_{r+1} = {}^{n}C_r\,a^{n-r}.\,b^r$

$= {}^{9}C_r\,(3x)^{9-r}.(1)^r$

$= {}^{9}C_r\,3^{9-r}.x^{9-r}$

To get the coefficient of x^6 in $2(3x + 1)^9$, we must have

$x^{9-r} = x^6$

∴ $9 - r = 6$

∴ $r = 3$ …(i)

Also, to get the coefficient of x^6 in $x(3x + 1)^9$, we must have

$x.x^{9-r} = x^6$

∴ $x^{9-r} = x^5$

∴ $9 - r = 5$

∴ $r = 4$ …(ii)

∴ The term containing x^6 in the expansion of $2(3x + 1)^9 - x(3x + 1)^9$

$= 2\,{}^9C_r 3^{9-r} - {}^9C_r 3^{9-r}$

$= 2\,{}^9C_3 3^{9-3} - {}^9C_4 3^{9-4}$ …[From (i) and (ii)]

$= 2 \times 84 \times (3^6) - 126 \times 3^5$

$= 2 \times 3^5 (3 \times 84 - 63)$

$= 2 \times 243(252 - 63)$

$= 486 \times 189 = 91854$

7. The coefficient of x^2 in the expansion of $(1 + 2x)^m$ is 112. Find m. [3 Marks]

Solution:

The coefficient of x^2 in $(1 + 2x)^m = {}^mC_2\,(2^2)$

Given that the coefficient of $x^2 = 112$

∴ ${}^mC_2\,(4) = 112$

∴ ${}^mC_2 = 28$

∴ $\dfrac{m!}{2!(m-2)!} = 28$

∴ $\dfrac{m(m-1)(m-2)!}{2 \times (m-2)!} = 28$

∴ $m(m - 1) = 56$

∴ $m^2 - m - 56 = 0$

∴ $(m - 8)(m + 7) = 0$

As m cannot be negative.

∴ $m = 8$

SMART CHECK

If the coefficient of x^2 in $(1 + 2x)^8$ is 112, then our answer is correct.

Coefficient of x^2 in $(1 + 2x)^8 = {}^8C_2\,(2^2) = 28(4) = 112$

Thus, our answer is correct.

Practice Based On Exercise 4.3

1. i. Find the third term in the expansion of $\left(2x + \dfrac{1}{3x^2}\right)^9$. **[2 Marks]**

   ii. Find the seventh term in the expansion of $\left(2x^2 + \dfrac{1}{x}\right)^{11}$. **[2 Marks]**

   iii. Find the fifth term in the expansion of $\left(x^2 - \dfrac{1}{2x}\right)^8$. **[2 Marks]**

   iv. Find the sixth term in the expansion of $\left(2x^2 - \dfrac{1}{3x^2}\right)^{10}$. **[2 Marks]**

+2. Find the fifth term in the expansion of $\left(2x^2 + \dfrac{3}{2x}\right)^8$ **[2 Marks]**

+3. i. Find the coefficient of x^7 in the expansion of $\left(x^2 + \dfrac{1}{x}\right)^{11}$ **[3 Marks]**

   ii. Find the coefficient of x^{-2} in the expansion of $\left(2x - \dfrac{1}{\sqrt{3}x^2}\right)^{10}$ **[4 Marks]**

4. Find the coefficient of: **[3 Marks Each]**

   i. x^2 in the expansion of $\left(x^2 + \dfrac{1}{x}\right)^7$.

   ii. x^4 in the expansion of $\left(\dfrac{x}{2} - \dfrac{3}{x^2}\right)^{10}$.

   iii. x^{10} in the expansion of $\left(2x^2 - \dfrac{1}{x}\right)^{20}$.

   iv. x^{-9} in the expansion of $\left(\dfrac{x^2}{2} - \dfrac{2}{x}\right)^9$.

   v. x^{-15} in the expansion of $\left(x^3 - \dfrac{1}{x^2}\right)^{15}$.

+5. Find the term independent of x, in the expansion of $\left(\sqrt{x} - \dfrac{2}{x^2}\right)^{10}$ **[4 Marks]**

6. Find the constant term (term independent of x) in the expansion of: **[3 Marks Each]**

   i. $\left(x - \dfrac{1}{x}\right)^{18}$ ii. $\left(x^2 - \dfrac{1}{3x}\right)^9$

   iii. $\left(2x^2 - \dfrac{1}{4x}\right)^{12}$ iv. $\left(2x + \dfrac{1}{\sqrt{x}}\right)^{15}$

+7. i. Find the middle term(s) in the expansion of $\left(x^2 + \dfrac{2}{x}\right)^8$ **[3 Marks]**

   ii. Find the middle terms in the expansion of $\left(2x + \dfrac{1}{4x}\right)^9$ **[4 Marks]**

8. Find the middle term(s) in the expansion of:

   i. $\left(x + \dfrac{1}{x}\right)^{10}$ **[2 Marks]**

ii. $\left(\frac{x}{y}+\frac{y}{x}\right)^8$ **[2 Marks]**

iii. $\left(\frac{x}{2}-\frac{1}{x^2}\right)^{10}$ **[2 Marks]**

iv. $\left(3x-\frac{x^3}{6}\right)^9$ **[3 Marks]**

Focus on Exercise 4.4

1. State, by writing first four terms, the expansion of the following, where | x | < 1.
[2 Marks Each]

i. $(1+x)^{-4}$ **ii.** $(1-x)^{\frac{1}{3}}$

iii. $(1-x^2)^{-3}$ **iv.** $(1+x)^{-\frac{1}{5}}$

v. $(1+x^2)^{-1}$

Solution:

ii. $(1-x)^{\frac{1}{3}}$

$$= 1 - \frac{1}{3}x + \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)}{2!}x^2 - \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}x^3 + \ldots$$

$$= 1 - \frac{x}{3} + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2}x^2 - \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{6} + x^3 + \ldots$$

$$= 1 - \frac{x}{3} - \frac{1}{9}x^2 - \frac{5}{81}x^3 - \ldots$$

iv. $(1+x)^{-1/5}$

$$= 1 + \left(\frac{-1}{5}\right)x + \frac{\left(\frac{-1}{5}\right)\left(\frac{-1}{5}-1\right)}{2!}x^2 + \frac{\left(\frac{-1}{5}\right)\left(\frac{-1}{5}-1\right)\left(\frac{-1}{5}-2\right)}{3!}x^3 + \ldots$$

$$= 1 - \frac{x}{5} + \frac{\left(\frac{-1}{5}\right)\left(\frac{-6}{5}\right)}{2}x^2 + \frac{\left(\frac{-1}{5}\right)\left(\frac{-6}{5}\right)\left(\frac{-11}{5}\right)}{6}x^3 + \ldots$$

$$= 1 - \frac{x}{5} + \frac{3x^2}{25} - \frac{11x^3}{125} + \ldots$$

2. State by writing first four terms, the expansion of the following, where | b | < | a |.
[2 Marks Each]

i. $(a-b)^{-3}$ **ii.** $(a+b)^{-4}$

iii. $(a+b)^{1/4}$ **iv.** $(a-b)^{-1/4}$

v. $(a+b)^{-1/3}$

Solution:

ii. $(a+b)^{-4}$

$$= \left[a\left(1+\frac{b}{a}\right)\right]^{-4}$$

$$= a^{-4}\left(1+\frac{b}{a}\right)^{-4}$$

$$= a^{-4}\left[1+(-4)\frac{b}{a}+\frac{(-4)(-4-1)}{2!}\left(\frac{b}{a}\right)^2 + \frac{(-4)(-4-1)(-4-2)}{3!}\left(\frac{b}{a}\right)^3+\ldots\right]$$

$$= a^{-4}\left[1-\frac{4b}{a}+\frac{(-4)(-5)}{2}\cdot\frac{b^2}{a^2}+\frac{(-4)(-5)(-6)}{6}\cdot\frac{b^3}{a^3}+\ldots\right]$$

$$= a^{-4}\left[1-\frac{4b}{a}+\frac{10b^2}{a^2}-\frac{20b^3}{a^3}+\ldots\right]$$

iii. $(a+b)^{1/4}$

$$= \left[a\left(1+\frac{b}{a}\right)\right]^{\frac{1}{4}}$$

$$= a^{\frac{1}{4}}\left(1+\frac{b}{a}\right)^{\frac{1}{4}}$$

$$= a^{\frac{1}{4}}\left[1+\left(\frac{1}{4}\right)\frac{b}{a}+\frac{\frac{1}{4}\left(\frac{1}{4}-1\right)}{2!}\left(\frac{b}{a}\right)^2 + \frac{\frac{1}{4}\left(\frac{1}{4}-1\right)\left(\frac{1}{4}-2\right)}{3!}\left(\frac{b}{a}\right)^3+\ldots\right]$$

$$= a^{\frac{1}{4}}\left[1+\frac{b}{4a}+\frac{\frac{1}{4}\left(\frac{-3}{4}\right)}{2}\cdot\frac{b^2}{a^2}+\frac{\frac{1}{4}\left(\frac{-3}{4}\right)\left(\frac{-7}{4}\right)}{6}\cdot\frac{b^3}{a^3}+\ldots\right]$$

$$= a^{\frac{1}{4}}\left[1+\frac{b}{4a}-\frac{3b^2}{32a^2}+\frac{7b^3}{128a^3}-\ldots\right]$$

v. $(a+b)^{-1/3} = \left[a\left(1+\frac{b}{a}\right)\right]^{\frac{-1}{3}}$

$$= a^{\frac{-1}{3}}\left(1+\frac{b}{a}\right)^{\frac{-1}{3}}$$

$$= a^{\frac{-1}{3}}\left[1+\left(\frac{-1}{3}\right)\frac{b}{a}+\frac{\left(\frac{-1}{3}\right)\left(-\frac{1}{3}-1\right)}{2!}\left(\frac{b}{a}\right)^2 + \frac{\left(\frac{-1}{3}\right)\left(\frac{-1}{3}-1\right)\left(\frac{-1}{3}-2\right)}{3!}\left(\frac{b}{a}\right)^3+\ldots\right]$$

$$= a^{\frac{-1}{3}}\left[1-\frac{b}{3a}+\frac{\left(\frac{-1}{3}\right)\left(\frac{-4}{3}\right)}{2}\cdot\frac{b^2}{a^2}+\frac{\left(\frac{-1}{3}\right)\left(\frac{-4}{3}\right)\left(\frac{-7}{3}\right)}{6}\cdot\frac{b^3}{a^3}+\ldots\right]$$

$$= a^{\frac{-1}{3}}\left[1-\frac{b}{3a}+\frac{2b^2}{9a^2}-\frac{14b^3}{81a^3}+\ldots\right]$$

---

3. Simplify first three terms in the expansion of the following: [2 Marks Each]

i. $(1 + 2x)^{-4}$ **ii.** $(1 + 3x)^{-1/2}$

iii. $(2 - 3x)^{1/3}$ **iv.** $(5 + 4x)^{-1/2}$

v. $(5 - 3x)^{-1/3}$

Solution:

ii. $(1+3x)^{\frac{-1}{2}}$

$$= 1 + \left(\frac{-1}{2}\right)(3x) + \frac{\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)}{2!}(3x)^2 + \ldots$$

$$= 1 - \frac{3x}{2} + \frac{\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{2}(9x^2) + \ldots$$

$$= 1 - \frac{3x}{2} + \frac{27}{8}x^2 + \ldots$$

iii. $(2-3x)^{\frac{1}{3}} = \left[2\left(1-\frac{3x}{2}\right)\right]^{\frac{1}{3}}$

$$= 2^{\frac{1}{3}}\left(1-\frac{3x}{2}\right)^{\frac{1}{3}}$$

$$= 2^{\frac{1}{3}}\left[1-\left(\frac{1}{3}\right)\left(\frac{3x}{2}\right)+\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)}{2!}\left(\frac{3x}{2}\right)^2-\ldots\right]$$

$$= 2^{\frac{1}{3}}\left[1-\frac{x}{2}+\frac{\frac{1}{3}\left(\frac{-2}{3}\right)}{2}\cdot\left(\frac{9x^2}{4}\right)-\ldots\right]$$

$$= 2^{\frac{1}{3}}\left[1-\frac{x}{2}-\frac{x^2}{4}-\ldots\right]$$

---

4. Use binomial theorem to evaluate the following upto four places of decimals. [3 Marks Each]

i. $\sqrt{99}$ **ii.** $\sqrt[3]{126}$

iii. $\sqrt[4]{16.08}$ **iv.** $(1.02)^{-5}$

v. $(0.98)^{-3}$

Solution:

ii. $\sqrt[3]{126} = (125+1)^{\frac{1}{3}}$

$$= \left[125\left(1+\frac{1}{125}\right)\right]^{\frac{1}{3}}$$

$$= (125)^{\frac{1}{3}}\left(1+\frac{1}{125}\right)^{\frac{1}{3}}$$

$$= 5(1+0.008)^{\frac{1}{3}}$$

$$= 5\left[1+\frac{1}{3}(0.008)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(0.008)^2+\ldots\right]$$

$$= 5\left[1+\frac{1}{3}(0.008)+\frac{\frac{1}{3}\left(\frac{-2}{3}\right)}{2}(0.008)^2+\ldots\right]$$

$$= 5\left[1+0.00266-\frac{0.000064}{9}\right]$$

= 5 (1.00266 – 0.000007)

= 5 (1.002653)

= 5.013265

= 5.0133

iii. $\sqrt[4]{16.08} = (16+0.08)^{1/4}$

$$= [16(1+0.005)]^{1/4}$$

$$= 16^{\frac{1}{4}}(1+0.005)^{1/4}$$

$$= 2\left[1+\frac{1}{4}(0.005)+\frac{\frac{1}{4}\left(\frac{1}{4}-1\right)}{2!}(0.005)^2+\ldots\right]$$

$$= 2\left[1+\frac{1}{4}(0.005)+\frac{\frac{1}{4}\left(\frac{-3}{4}\right)}{2}(0.005)^2+\ldots\right]$$

$$= 2\left[1+0.00125-\frac{0.000075}{32}+\ldots\right]$$

= 2 [1 + 0.00125 – 0.0000023 + …]

= 2 (1.0012477)

= 2.0024954

= 2.0025

iv. $(1.02)^{-5}$

$= (1 + 0.02)^{-5}$

$$= 1 + (-5)(0.02) + \frac{(-5)(-5-1)}{2!}(0.02)^2 + \frac{(-5)(-5-1)(-5-2)}{3!}(0.02)^3 + \ldots$$

$= 1 + (-5)(0.02) + \frac{(-5)(-6)}{2}(0.02)^2 + \frac{(-5)(-6)(-7)}{6}(0.02)^3 + \ldots$

$= 1 - 0.1 + 0.006 - 0.00028 + \ldots$

$= 0.90572$

$= 0.9057$

---

v. $(0.98)^{-3}$

$= (1 - 0.02)^{-3}$

$= 1 + (-3)(-0.02) + \frac{(-3)(-3-1)}{2!}(-0.02)^2 + \frac{(-3)(-3-1)(-3-2)}{3!}(-0.02)^3 + \ldots$

$= 1 + 0.06 + \frac{(-3)(-4)}{2}(0.02)^2 - \frac{(-3)(-4)(-5)}{6}(0.02)^3 + \ldots$

$= 1 + 0.06 + 0.0024 + 0.00008 + \ldots$

$= 1.06248$

$= 1.0625$

Practice Based On Exercise 4.4

+1. i. State first four terms in the expansion of $\frac{1}{(a-b)^4}$ where $|b| > |a|$ **[3 Marks]**

ii. State first four terms in the expansion of $\frac{1}{(a+b)}$, $|b| < |a|$ **[3 Marks]**

2. Find first four terms in the expansion of the following, where $|x| < 1$: **[2 Marks Each]**

i. $(1 + x)^{-3}$ ii. $(1 - x)^{-2}$

iii. $(1+x)^{\frac{1}{3}}$ iv. $(1-x^2)^{-\frac{1}{2}}$

3. Find the first four terms in the expansion of the following, where $|b| < |a|$: **[2 Marks Each]**

i. $(a - b)^{\frac{1}{3}}$ ii. $(a + b)^{-\frac{2}{3}}$

+4. State first four terms in the expansion of $(2-3x)^{-1/2}$ if $|x| < \frac{2}{3}$ **[3 Marks]**

5. Simplify first three terms in the expansion of the following: **[2 Marks Each]**

i. $(1 + 6x)^{-2}$ ii. $(1-2x)^{\frac{3}{2}}$

iii. $(4-3x)^{-\frac{1}{2}}$ iv. $(4+5x)^{-\frac{1}{2}}$

+6. Find the value of $\sqrt{30}$ upto 4 decimal places. **[3 Marks]**

7. Use binomial theorem to evaluate the following upto four places of decimals: **[3 Marks Each]**

i. $\sqrt{90}$ ii. $\sqrt[3]{217}$

iii. $(0.98)^{-\frac{1}{3}}$

Focus on Exercise 4.5

Show that

1. $C_0 + C_1 + C_2 + \ldots + C_8 = 256$ **[1 Mark]**

Solution:

Since $C_0 + C_1 + C_2 + C_3 + \ldots + C_n = 2^n$

Putting n = 8, we get

$C_0 + C_1 + C_2 + \ldots + C_8 = 2^8$

$\therefore$ $C_0 + C_1 + C_2 + \ldots + C_8 = 256$

---

2. $C_0 + C_1 + C_2 + \ldots + C_9 = 512$ **[1 Mark]**

---

3. $C_1 + C_2 + C_3 + \ldots + C_7 = 127$ **[2 Marks]**

Solution:

Since $C_0 + C_1 + C_2 + C_3 + \ldots + C_n = 2^n$

Putting n = 7, we get

$C_0 + C_1 + C_2 + \ldots + C_7 = 2^7$

$\therefore$ $C_0 + C_1 + C_2 + \ldots + C_7 = 128$

But, $C_0 = 1$

$\therefore$ $1 + C_1 + C_2 + \ldots + C_7 = 128$

$\therefore$ $C_1 + C_2 + \ldots + C_7 = 128 - 1 = 127$

---

4. $C_1 + C_2 + C_3 + \ldots + C_6 = 63$ **[2 Marks]**

---

5. $C_0+C_2+C_4+C_6 + C_8 = C_1 + C_3 + C_5 + C_7 = 128$ **[3 Marks]**

Solution:

Since $C_0 + C_1 + C_2 + C_3 + \ldots + C_n = 2^n$

Putting n = 8, we get

$C_0 + C_1 + C_2 + C_3 + \ldots + C_8 = 2^8$

But, sum of even coefficients = sum of odd coefficients

$\therefore$ $C_0 + C_2 + C_4 + C_6 + C_8 = C_1 + C_3 + C_5 + C_7$

Let $C_0 + C_2 + C_4 + C_6 + C_8 = C_1 + C_3 + C_5 + C_7 = k$

Now, $C_0 + C_1 + C_2 + C_3 + C_4 + C_5 + C_6 + C_7 + C_8 = 256$

$\therefore$ $(C_0 + C_2 + C_4 + C_6 + C_8) + (C_1 + C_3 + C_5 + C_7) = 256$

$\therefore$ $k + k = 256$

$\therefore$ $2k = 256$

$\therefore$ $k = 128$

$\therefore$ $C_0 + C_2 + C_4 + C_6 + C_8 = C_1 + C_3 + C_5 + C_7 = 128$

---

6. $C_1 + C_2 + C_3 + \ldots + C_n = 2^n - 1$ **[1 Mark]**

7. $\mathbf{C_0 + 2C_1 + 3C_2 + 4C_3 + \ldots + (n+1)C_n = (n+2)2^{n-1}}$ **[3 Marks]**

Solution:

Since $C_0 + C_1 + C_2 + C_3 + \ldots + C_n = 2^n$

$C_0 + 2C_1 + 3C_2 + 4C_3 + \ldots + (n+1)C_n$

$= (C_0 + C_1 + C_2 + C_3 + \ldots + C_n) + (C_1 + 2C_2 + 3C_3 + \ldots + nC_n)$

$= 2^n + (C_1 + 2C_2 + 3C_3 + \ldots + nC_n)$

$= 2^n + \left[n + \frac{2n(n-1)}{2!} + \frac{3n(n-1)(n-2)}{3!} + \ldots + n(1)\right]$

$= 2^n + n\left[1 + \frac{(n-1)}{1!} + \frac{(n-1)(n-2)}{2!} + \ldots + 1\right]$

$= 2^n + n\left[{}^{n-1}C_0 + {}^{n-1}C_1 + {}^{n-1}C_2 + \ldots + {}^{n-1}C_{n-1}\right]$

$= 2^n + n.\ 2^{n-1}$

$= 2^{n-1}.2 + n.2^{n-1}$

$= (2+n).2^{n-1}$

$= (n+2).2^{n-1}$

Practice Based On Exercise 4.5

+1. Show that

i. $C_0 + C_1 + C_2 + \ldots + C_{10} = 1024$ **[1 Mark]**

ii. $C_0 + C_2 + C_4 + \ldots + C_{12} = C_1 + C_3 + C_5 + \ldots + C_{11} = 2048$ **[3 Marks]**

Show that

2. $C_0 + C_1 + C_2 + \ldots + C_7 = 128$ **[1 Mark]**

3. $C_1 + C_2 + C_3 + \ldots + C_8 = 255$ **[2 Marks]**

4. $C_0 + C_2 + C_4 + \ldots + C_{10} = C_1 + C_3 + C_5 + \ldots + C_9 = 512$ **[3 Marks]**

5. ${}^{30}C_2 + {}^{30}C_3 + {}^{30}C_4 + \ldots + {}^{30}C_{30} = 2^{30} - 31$ **[2 Marks]**

6. ${}^{n}C_0 - {}^{n}C_1 + {}^{n}C_2 - {}^{n}C_3 + \ldots + (-1)^n\ .\ {}^{n}C_n = 0$ **[3 Marks]**

+7. Prove that

i. $C_1 + 2C_2 + 3C_3 + 4C_4 + \ldots + nC_n = n.2^{n-1}$ **[3 Marks]**

ii. $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$ **[4 Marks]**

8. Show that:

$C_0 + 3C_1 + 5C_2 + \ldots + (2n+1)C_n = (n+1)\ 2^n$ **[3 Marks]**

Focus on Miscellaneous Exercise – 4

I. Select the correct answers from the given alternatives. **[2 Marks Each]**

1. The total number of terms in the expression of $(x+y)^{100} + (x-y)^{100}$ after simplification is:
(A) 50 (B) 51
(C) 100 (D) 202

2. The middle term in the expansion of $(1+x)^{2n}$ will be:
(A) $(n-1)^{th}$ (B) n^{th}
(C) $(n+1)^{th}$ (D) $(n+2)^{th}$

3. In the expansion of $(x^2 - 2x)^{10}$, the coefficient of x^{16} is
(A) –1680 (B) 1680
(C) 3360 (D) 6720

4. The term not containing x in expansion of $(1-x)^2\left(x + \frac{1}{x}\right)^{10}$ is
(A) ${}^{11}C_5$ (B) ${}^{10}C_5$ (C) ${}^{10}C_4$ (D) ${}^{10}C_7$

5. The number of terms in expansion of $(4y+x)^8 - (4y-x)^8$ is
(A) 4 (B) 5 (C) 8 (D) 9

6. The value of ${}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots. + {}^{14}C_{11}$ is
(A) $2^{14} - 1$ (B) $2^{14} - 14$
(C) 2^{12} (D) $2^{13} - 14$

7. The value of ${}^{11}C_2 + {}^{11}C_4 + {}^{11}C_6 + {}^{11}C_8$ is equal to
(A) $2^{10} - 1$ (B) $2^{10} - 11$
(C) $2^{10} + 12$ (D) $2^{10} - 12$

8. In the expansion of $(3x+2)^4$, the coefficient of middle term is
(A) 36 (B) 54 (C) 81 (D) 216

9. The coefficient of the 8^{th} term in the expansion of $(1+x)^{10}$ is:
(A) 7 (B) 120 (C) ${}^{10}C_8$ (D) 210

10. If the coefficients of x^2 and x^3 in the expansion of $(3+ax)^9$ are the same, then the value of a is
(A) $-\frac{7}{9}$ (B) $-\frac{9}{7}$ (C) $\frac{7}{9}$ (D) $\frac{9}{7}$

Answers:

1.	(B)	2.	(C)	3.	(C)	4.	(A)
5.	(A)	6.	(D)	7.	(D)	8.	(D)
9.	(B)	10.	(D)				

Hints:

1. $(x+y)^{100}$
$= x^{100} + {}^{100}C_1\, x^{99}\, y + {}^{100}C_2\, x^{98}\, y^2 + \ldots + y^{100}$
$(x-y)^{100}$
$= x^{100} - {}^{100}C_1\, x^{99}\, y + {}^{100}C_2\, x^{98}\, y^2 - \ldots + y^{100}$
$(x+y)^{100} + (x-y)^{100}$
$= 2\underbrace{\left[x^{100} + {}^{100}C_2\, x^{98}\, y^2 + \ldots + y^{100}\right]}_{51\text{ terms}}$

3. $(x^2 - 2x)^{10} = x^{10}(x-2)^{10}$

To get the coefficient of x^{16} in $(x^2 - 2x)^{10}$, we need to check coefficient of x^6 in $(x-2)^{10}$

$\therefore$ Required coefficient = ${}^{10}C_6(-2)^4 = 210 \times 16$

$= 3360$

---

4. $(1-x)^2\left(x+\frac{1}{x}\right)^{10}$

$= (1-2x+x^2)\underbrace{\left(x+\frac{1}{x}\right)^{10}}_{\text{This expansion has even indices of } x}$

Term not containing x = A + B,

where A = ${}^{10}C_r x^{10-r}\left(\frac{1}{x}\right)^r$ for r such that $10 - r - r = 0$, i.e., r = 5

$\therefore$ A = ${}^{10}C_5$

and B = ${}^{10}C_r\, x^{10-r}\left(\frac{1}{x}\right)^r$

for r such that $10 - r - r = -2$

i.e., r = 6

$\therefore$ B = ${}^{10}C_6$

Required coefficient = A + B = ${}^{10}C_5 + {}^{10}C_6$

$= {}^{10}C_5 + {}^{10}C_4 = {}^{11}C_5$

---

7. ${}^{11}C_0 + {}^{11}C_2 + ... + {}^{11}C_8 + {}^{11}C_{10} = 2^{11-1} = 2^{10}$

$\therefore$ ${}^{11}C_2 + {}^{11}C_4 + {}^{11}C_6 + {}^{11}C_8$

$= 2^{10} - ({}^{11}C_0 + {}^{11}C_{10})$

$= 2^{10} - (1 + 11)$

$= 2^{10} - 12$

---

9. r = 7,

$t_8 = {}^{10}C_7\, x^7 = {}^{10}C_3 x^7$

$\therefore$ coefficient of 8th term = ${}^{10}C_3 = 120$

II. Answer the following.

1. Prove by method of induction, for all n $\in$ N.

[4 Marks Each]

i. $8 + 17 + 26 + \ldots + (9n - 1) = \frac{n}{2}(9n + 7)$

ii. $1^2 + 4^2 + 7^2 + \ldots + (3n-2)^2 = \frac{n}{2}(6n^2 - 3n - 1)$

iii. $2 + 3.2 + 4.2^2 + \ldots + (n+1)2^{n-1} = n.2^n$

iv. $\frac{1}{3.4.5} + \frac{2}{4.5.6} + \frac{3}{5.6.7} + \ldots + \frac{n}{(n+2)(n+3)(n+4)} = \frac{n(n+1)}{6(n+3)(n+4)}$

***Solution*:**

ii. Let P(n) $\equiv 1^2 + 4^2 + 7^2 + \ldots + (3n-2)^2$

$= \frac{n}{2}(6n^2 - 3n - 1)$, for all n $\in$ N.

Step I:

Put n = 1

L.H.S. = $1^2 = 1$

R.H.S. = $\frac{1}{2}[6(1)^2 - 3(1) - 1] = 1$

$\therefore$ L.H.S. = R.H.S.

$\therefore$ P(n) is true for n = 1.

Step II:

Let us assume that P(n) is true for n = k.

$\therefore$ $1^2 + 4^2 + 7^2 + ... + (3k-2)^2 = \frac{k}{2}(6k^2 - 3k - 1)$ …(i)

Step III:

We have to prove that P(n) is true for n = k + 1,
i.e., to prove that

$1^2 + 4^2 + 7^2 + ... + [3(k+1) - 2]^2$

$= \frac{(k+1)}{2}[6(k+1)^2 - 3(k+1) - 1]$

$= \frac{(k+1)}{2}(6k^2 + 12k + 6 - 3k - 3 - 1)$

$= \frac{(k+1)}{2}(6k^2 + 9k + 2)$

L.H.S. = $1^2 + 4^2 + 7^2 + \ldots + [3(k+1) - 2]^2$

$= 1^2 + 4^2 + 7^2 + \ldots + (3k-2)^2 + (3k+1)^2$

$= \frac{k}{2}(6k^2 - 3k - 1) + (3k+1)^2$ …[From (i)]

$= \frac{(6k^3 - 3k^2 - k) + 2(9k^2 + 6k + 1)}{2}$

$= \frac{6k^3 + 15k^2 + 11k + 2}{2}$

$= \frac{(k+1)(6k^2 + 9k + 2)}{2}$

= R.H.S.

$\therefore$ P(n) is true for n = k + 1.

Step IV:

From all the steps above, by the principle of mathematical induction, P(n) is true for all n $\in$ N.

$\therefore$ $1^2 + 4^2 + 7^2 + \ldots + (3n-2)^2 = \frac{n}{2}(6n^2 - 3n - 1)$

for all n $\in$ N.

---

iv. Let P(n) $\equiv \frac{1}{3.4.5} + \frac{2}{4.5.6} + \frac{3}{5.6.7} + \ldots + \frac{n}{(n+2)(n+3)(n+4)}$

$= \frac{n(n+1)}{6(n+3)(n+4)}$, for all n $\in$ N.

Step I:

Put n = 1

L.H.S. = $\frac{1}{3.4.5} = \frac{1}{60}$

R.H.S. = $\frac{1(1+1)}{6(1+3)(1+4)} = \frac{2}{120} = \frac{1}{60}$

∴ L.H.S. = R.H.S.

∴ P(n) is true for n = 1.

Step II:

Let us assume that P(n) is true for n = k.

$$\therefore \quad \frac{1}{3.4.5} + \frac{2}{4.5.6} + \frac{3}{5.6.7} + \ldots + \frac{k}{(k+2)(k+3)(k+4)} = \frac{k(k+1)}{6(k+3)(k+4)} \quad \ldots(i)$$

Step III:

We have to prove that P(n) is true for n = k+1, i.e., to prove that

$$\frac{1}{3.4.5} + \frac{2}{4.5.6} + \frac{3}{5.6.7} + \ldots + \frac{k+1}{(k+3)(k+4)(k+5)} = \frac{(k+1)(k+2)}{6(k+4)(k+5)}$$

$$\text{L.H.S.} = \frac{1}{3.4.5} + \frac{2}{4.5.6} + \frac{3}{5.6.7} + \ldots + \frac{k+1}{(k+3)(k+4)(k+5)}$$

$$= \frac{1}{3.4.5} + \frac{2}{4.5.6} + \frac{3}{5.6.7} + \ldots + \frac{k}{(k+2)(k+3)(k+4)} + \frac{k+1}{(k+3)(k+4)(k+5)}$$

$$= \frac{k(k+1)}{6(k+3)(k+4)} + \frac{k+1}{(k+3)(k+4)(k+5)} \quad \ldots[\text{From (i)}]$$

$$= \frac{k+1}{(k+3)(k+4)}\left[\frac{k}{6} + \frac{1}{k+5}\right]$$

$$= \frac{(k+1)}{(k+3)(k+4)}\left[\frac{k^2+5k+6}{6(k+5)}\right]$$

$$= \frac{(k+1)(k+2)(k+3)}{6(k+3)(k+4)(k+5)}$$

$$= \frac{(k+1)(k+2)}{6(k+4)(k+5)}$$

= R.H.S.

∴ P(n) is true for n = k + 1.

Step IV:

From all the steps above, by the principle of mathematical induction, P(n) is true for all n ∈ N.

$$\therefore \quad \frac{1}{3.4.5} + \frac{2}{4.5.6} + \frac{3}{5.6.7} + \ldots + \frac{n}{(n+2)(n+3)(n+4)} = \frac{n(n+1)}{6(n+3)(n+4)}$$

for all n ∈ N.

---

2. Given that $t_{n+1} = 5t_n - 8$, $t_1 = 3$, prove by method of induction that $t_n = 5^{n-1} + 2$.

[4 Marks]

---

3. Prove by method of induction

$$\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^n = \begin{bmatrix} 2n+1 & -4n \\ n & -2n+1 \end{bmatrix}, \forall\, n \in N$$

[4 Marks]

***Solution*:**

Let $P(n) \equiv \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^n = \begin{bmatrix} 2n+1 & -4n \\ n & -2n+1 \end{bmatrix}$, for all n ∈ N.

Step I:

Put n = 1

L.H.S. = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^1 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

R.H.S. = $\begin{bmatrix} 2(1)+1 & -4(1) \\ 1 & -2(1)+1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

∴ P(n) is true for n = 1.

Step II:

Let us assume that P(n) is true for n = k.

$$\therefore \quad \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^k = \begin{bmatrix} 2k+1 & -4k \\ k & -2k+1 \end{bmatrix} \quad \ldots(i)$$

Step III:

We have to prove that P(n) is true for n = k +1, i.e., to prove that

$$\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^{k+1} = \begin{bmatrix} 2(k+1)+1 & -4(k+1) \\ k+1 & -2(k+1)+1 \end{bmatrix}$$

$$\text{L.H.S.} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^k \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2k+1 & -4k \\ k & -2k+1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \ldots[\text{From (i)}]$$

$$= \begin{bmatrix} 6k+3-4k & -8k-4+4k \\ 3k-2k+1 & -4k+2k-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2(k+1)+1 & -4(k+1) \\ k+1 & -2(k+1)+1 \end{bmatrix}$$

= R.H.S.

∴ P(n) is true for n = k + 1.

Step IV:

From all the steps above, by the principle of mathematical induction, P(n) is true for all n∈N.

$\therefore \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^n = \begin{bmatrix} 2n+1 & -4n \\ n & -2n+1 \end{bmatrix}$, for all n∈N.

---

4. Expand $(3x^2 + 2y)^5$ [2 Marks]

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5. Expand $\left(\frac{2x}{3} - \frac{3}{2x}\right)^4$ [2 Marks]

Solution:

Here, a = $\frac{2x}{3}$, b = $\frac{3}{2x}$, n = 4.

Using binomial theorem, $\left(\frac{2x}{3} - \frac{3}{2x}\right)^4$

$$= {}^4C_0\left(\frac{2x}{3}\right)^4\left(\frac{3}{2x}\right)^0 - {}^4C_1\left(\frac{2x}{3}\right)^3\left(\frac{3}{2x}\right)^1 + {}^4C_2\left(\frac{2x}{3}\right)^2\left(\frac{3}{2x}\right)^2 - {}^4C_3\left(\frac{2x}{3}\right)^1\left(\frac{3}{2x}\right)^3 + {}^4C_4\left(\frac{2x}{3}\right)^0\left(\frac{3}{2x}\right)^4$$

Now, ${}^4C_0 = {}^4C_4 = 1$, ${}^4C_1 = {}^4C_3 = 4$,

${}^4C_2 = \frac{4 \times 3}{2 \times 1} = 6$,

$$\therefore \left(\frac{2x}{3} - \frac{3}{2x}\right)^4$$

$$= 1\left(\frac{16x^4}{81}\right)(1) - 4\left(\frac{8x^3}{27}\right)\left(\frac{3}{2x}\right) + 6\left(\frac{4x^2}{9}\right)\left(\frac{9}{4x^2}\right) - 4\left(\frac{2x}{3}\right)\left(\frac{27}{8x^3}\right) + 1(1)\left(\frac{81}{16x^4}\right)$$

$$= \frac{16x^4}{81} - \frac{16x^2}{9} + 6 - \frac{9}{x^2} + \frac{81}{16x^4}$$

---

6. Find third term in the expansion of $\left(9x^2 - \frac{y^3}{6}\right)^4$. [2 Marks]

---

7. Find tenth term in the expansion of $\left(2x^2 + \frac{1}{x}\right)^{12}$. [2 Marks]

Solution:

Here, a = $2x^2$, b = $\frac{1}{x}$, n = 12.

For 10^{th} term, r = 9

We have $t_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

$$\therefore t_{10} = {}^{12}C_9\,(2x^2)^{12-9}\left(\frac{1}{x}\right)^9$$

$$= \frac{12!}{9!3!}\left(8x^6\right)\left(\frac{1}{x^9}\right)$$

$$= \frac{12 \times 11 \times 10}{3 \times 2 \times 1} \times \frac{8x^6}{x^9}$$

$$= \frac{1760}{x^3}$$

$\therefore$ 10^{th} term in the expansion of $\left(2x^2 + \frac{1}{x}\right)^{12}$ is $\frac{1760}{x^3}$.

---

8. Find the middle term(s) in the expansion of

i. $\left(\frac{2a}{3} - \frac{3}{2a}\right)^6$ [2 Marks]

ii. $\left(x - \frac{1}{2y}\right)^{10}$ [2 Marks]

iii. $(x^2 + 2y^2)^7$ [3 Marks]

iv. $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ [3 Marks]

Solution:

ii. Here, a = x, b = $-\frac{1}{2y}$, n = 10.

Now, n is even.

$\therefore \frac{n+2}{2} = \frac{10+2}{2} = 6$

$\therefore$ Middle term is t_6, for which r = 5

We have $t_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

$$\therefore t_6 = {}^{10}C_5\,(x)^5\left(\frac{-1}{2y}\right)^5 = \frac{10!}{5!\,5!}(x)^5\left(\frac{-1}{32y^5}\right)$$

$$= -\frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5 \times 4 \times 3 \times 2 \times 1 \times 5!} \times \frac{x^5}{32y^5}$$

$$= -\frac{63x^5}{8y^5}$$

$\therefore$ Middle term is $-\frac{63x^5}{8y^5}$.

---

iii. Here, a = x^2, b = $2y^2$, n = 7.

Now, n is odd.

$\therefore \frac{n+1}{2} = \frac{7+1}{2} = 4$, $\frac{n+3}{2} = \frac{7+3}{2} = 5$

$\therefore$ Middle terms are t_4 and t_5, for which r = 3 and r = 4 respectively.

We have $t_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

$$\therefore t_4 = {}^7C_3\,(x^2)^4\,(2y^2)^3$$

$$= \frac{7!}{3!4!}\,(x^8)\,(8y^6)$$

$$= \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} \times 8x^8y^6$$

$$= 280x^8y^6$$

and $t_5 = {}^7C_4\,(x^2)^3.\,(2y^2)^4$

$$= \frac{7!}{4!3!}\,(x^6).(16y^8)$$

$= \dfrac{7\times6\times5\times4!}{4!\times3\times2\times1}\times16x^6y^8$

$= 560x^6y^8$

$\therefore$ Middle terms are $280x^8y^6$ and $560x^6y^8$.

---

9. Find the coefficients of [3 Marks Each]

i. x^6 in the expantion of $\left(3x^2 - \dfrac{1}{3x}\right)^9$

ii. x^{60} in the expansion of $\left(\dfrac{1}{x^2} + x^4\right)^{18}$

***Solution*:**

ii. Here, $a = \dfrac{1}{x^2}$, $b = x^4$, $n = 18$.

We have, $t_{r+1} = {}^nC_r\, a^{n-r}.b^r$

$= {}^{18}C_r\left(\dfrac{1}{x^2}\right)^{18-r}(x^4)^r$

$= {}^{18}C_r\, x^{2r-36}.x^{4r}$

$= {}^{18}C_r\, x^{6r-36}$

To get the coefficient of x^{60}, we must have $x^{6r-36} = x^{60}$

$\therefore$ $6r - 36 = 60$

$\therefore$ $6r = 96$

$\therefore$ $r = 16$

$\therefore$ Coefficient of x^{60}

$= {}^{18}C_{16} = \dfrac{18!}{16!\,2!} = \dfrac{18\times17\times16!}{16!\times2\times1} = 153$

---

10. Find the constant term in the expansion of [3 Marks Each]

i. $\left(\dfrac{4x^2}{3} + \dfrac{3}{2x}\right)^9$ ii. $\left(2x^2 - \dfrac{1}{x}\right)^{12}$

Solution:

i. Here, $a = \dfrac{4x^2}{3}$, $b = \dfrac{3}{2x}$, $n = 9$.

We have $t_{r+1} = {}^nC_r\, a^{n-r}.b^r$

$= {}^9C_r\left(\dfrac{4x^2}{3}\right)^{9-r}\left(\dfrac{3}{2x}\right)^r$

$= {}^9C_r\left(\dfrac{4}{3}\right)^{9-r}\left(\dfrac{3}{2}\right)^r.x^{18-2r}.x^{-r}$

$= {}^9C_r\left(\dfrac{4}{3}\right)^{9-r}.\left(\dfrac{3}{2}\right)^r.x^{18-3r}$

To get the constant term, we must have $x^{18-3r} = x^0$

$\therefore$ $18 - 3r = 0$

$\therefore$ $r = 6$

$\therefore$ Constant term $= {}^9C_6\left(\dfrac{4}{3}\right)^3\left(\dfrac{3}{2}\right)^6$

$= \dfrac{9!}{6!\,3!}\,\dfrac{64}{27}\times\dfrac{729}{64}$

$= \dfrac{9\times8\times7\times6!}{6!\times3\times2\times1}\times27$

$= 2268$

11. Prove the following by using by method of Induction [4 Marks Each]

i. $\log_a x^n = n\log_a x$, $x > 0$, $n \in N$

ii. $15^{2n-1} + 1$ is divisible by 16, for all $n \in N$.

iii. $5^{2n} - 2^{2n}$ is divisible by 3, for all $n \in N$.

Solution:

i. Let $P(n) \equiv \log_a x^n = n\log_a x$, for all $n \in N$.

Step I:

Put n = 1

L.H.S. $= \log_a(x^1) = \log_a x$

R.H.S. $= 1.(\log_a x) = \log_a x$

$\therefore$ L.H.S. = R.H.S.

$\therefore$ P(n) is true for n = 1.

Step II:

Let us assume that P(n) is true for n = k.

$\therefore$ $\log_a x^k = k.\log_a x$ …(i)

Step III:

We have to prove that P(n) is true for n = k + 1, i.e., to prove that

$\log_a x^{k+1} = (k+1).\log_a x$

L.H.S. $= \log_a(x^{k+1})$

$= \log_a(x^k.x) = \log_a x^k + \log_a x$

$= k.\log_a x + \log_a x$ …[From (i)]

$= (k+1).\log_a x$

= R.H.S.

$\therefore$ P(n) is true for n = k + 1.

Step IV:

From all the steps above, by the principle of mathematical induction, P(n) is true for all $n \in N$.

$\therefore$ $\log_a x^n = n\log_a x$, $x > 0$, $n \in N$.

---

12. If the coefficient of x^{16} in the expansion of $(x^2 + ax)^{10}$ is 3360, find a. [3 Marks]

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13. If the middle term in the expansion of $\left(x + \dfrac{b}{x}\right)^6$ is 160, find b. [3 Marks]

Solution:

Here, $a = x$, $b = \dfrac{b}{x}$, $n = 6$.

Now, n is even.

$\therefore$ $\dfrac{n+2}{2} = \dfrac{6+2}{2} = 4$

$\therefore$ Middle term is t_4, for which r = 3

$\therefore$ $t_4 = 160$

We have $t_{r+1} = {}^nC_r\, a^{n-r}.b^r$

$\therefore$ $t_4 = {}^6C_3\,(x)^3\left(\dfrac{b}{x}\right)^3$

$\therefore$ $160 = \dfrac{6!}{3!3!}\,(x)^3\left(\dfrac{b^3}{x^3}\right)$

$\therefore \quad 160 = \frac{6\times5\times4\times3!}{3\times2\times1\times3!}\times b^3$

$\therefore \quad 160 = 20b^3$

$\therefore \quad 8 = b^3$

$\therefore \quad b = 2$

---

14. If the coefficients of x^2 and x^3 in the expansion of $(3 + kx)^9$ are equal, find k. **[4 Marks]**

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15. If the constant term in the expansion of $\left(x^3 + \frac{k}{x^8}\right)^{11}$ is 1320, find k. **[4 Marks]**

Solution:

Here, $a = x^3$, $b = \frac{k}{x^8}$, n = 11 and

constant term = 1320.

We have $t_{r+1} = {}^nC_r\, a^{n-r}.\, b^r$

$= {}^{11}C_r\,(x^3)^{11-r}.\left(\frac{k}{x^8}\right)^r$

$= {}^{11}C_r\, k^r. x^{33-3r}. x^{-8r}$

$= {}^{11}C_r\, k^r\, x^{33-11r}$

To get the constant term, we must have

$x^{33-11r} = x^0$

$\therefore \quad 33 - 11r = 0$

$\therefore \quad r = 3$

$\therefore \quad$ Constant term $= {}^{11}C_3.k^3$

$\therefore \quad 1320 = \frac{11!}{3!\,8!}.k^3$

$\therefore \quad 1320 = \frac{11\times10\times9\times8!}{3\times2\times1\times8!}.k^3$

$\therefore \quad 1320 = 165\,k^3$

$\therefore \quad k^3 = 8$

$\therefore \quad k = 2$

---

16. Show that there is no term containing x^6 in the expansion of $\left(x^2 - \frac{3}{x}\right)^{11}$. **[3 Marks]**

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17. Show that there is no constant term in the expansion of $\left(2x - \frac{x^2}{4}\right)^9$. **[3 Marks]**

Solution:

Here, $a = 2x$, $b = \frac{-x^2}{4}$, n = 9.

We have $t_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

$= {}^9C_r\,(2x)^{9-r}\left(-\frac{x^2}{4}\right)^r$

$= {}^9C_r\,(2)^{9-r}\left(-\frac{1}{4}\right)^r x^{9-r}\, x^{2r}$

$= {}^9C_r\, 2^{9-r}\left(-\frac{1}{4}\right)^r x^{9+r}$

To get the constant term, we must have

$x^{9+r} = x^0$

$\therefore \quad 9 + r = 0$

$\therefore \quad r = -9$, which is not possible

$\therefore \quad$ There is no constant term in the expansion of $\left(2x - \frac{x^2}{4}\right)^9$.

---

18. State, first four terms in the expansion of $\left(1 - \frac{2x}{3}\right)^{-\frac{1}{2}}$. **[2 Marks]**

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19. State, first four terms in the expansion of $(1-x)^{-\frac{1}{4}}$. **[2 Marks]**

Solution:

$(1-x)^{-1/4}$

$$= 1 + \left(\frac{-1}{4}\right)(-x) + \frac{\left(\frac{-1}{4}\right)\left(\frac{-1}{4}-1\right)}{2!}(-x)^2 + \frac{\left(\frac{-1}{4}\right)\left(\frac{-1}{4}-1\right)\left(\frac{-1}{4}-2\right)}{3!}(-x)^3 + \ldots$$

$$= 1 + \left(\frac{-1}{4}\right)(-x) + \frac{\left(\frac{-1}{4}\right)\left(\frac{-5}{4}\right)}{2}x^2 - \frac{\left(\frac{-1}{4}\right)\left(\frac{-5}{4}\right)\left(\frac{-9}{4}\right)}{6}x^3 + \ldots$$

$$= 1 + \frac{x}{4} + \frac{5x^2}{32} + \frac{15x^3}{128} + \ldots$$

---

20. State, first three terms in the expansion of $(5+4x)^{-\frac{1}{2}}$ **[2 Marks]**

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21. Using binomial theorem, find the value of $\sqrt[3]{995}$ upto four places of decimals. **[3 Marks]**

Solution:

$$\sqrt[3]{995} = (1000-5)^{\frac{1}{3}} = \left[1000\left(1 - \frac{5}{1000}\right)\right]^{\frac{1}{3}}$$

$$= (1000)^{\frac{1}{3}}\,(1-0.005)^{\frac{1}{3}}$$

$$= 10\left[1 - \frac{1}{3}(0.005) + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(0.005)^2 - \ldots\right]$$

$$= 10\left[1-\frac{1}{3}(0.005)+\frac{\frac{1}{3}\left(\frac{-2}{3}\right)}{2}(0.005)^2-\ldots\right]$$

$$= 10\left[1-0.00166-\frac{0.000025}{9}-\ldots\right]$$

$= 10(1 - 0.00166 - 0.0000028 - \ldots)$
$= 10(0.9983372)$
$= 9.9833$ upto 4 decimal places.

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22. Find approximate value of $\frac{1}{4.08}$ upto four places of decimals. [3 Marks]

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23. Find the term independent of x in the expansion of $(1 - x^2)\left(x+\frac{2}{x}\right)^6$. [4 Marks]

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24. $(a + bx)(1 - x)^6 = 3 - 20x + cx^2 + \ldots$, then find a, b, c. [3 Marks]

***Solution*:**

Consider $(a + bx)(1 - x)^6$

$= a(1 - x)^6 + bx(1 - x)^6$

$= a(1 - {}^6C_1 x + {}^6C_2 x^2 - {}^6C_3 x^3 + \ldots)$
$\quad + bx(1 - {}^6C_1 x + {}^6C_2 x^2 - {}^6C_3 x^3 + \ldots)$

$= a(1 - 6x + 15x^2 - 20x^3 + \ldots)$
$\quad + bx(1 - 6x + 15x^2 - 20x^3 + \ldots)$

$= a + (b - 6a)x + (15a - 6b)x^2 + \ldots$ …(i)

Since $(a + bx)(1 - x)^6 = 3 - 20x + cx^2 + \ldots$

$\therefore$ $a + (b - 6a)x + (15a - 6b)x^2 + \ldots$
$= 3 - 20x + cx^2 + \ldots$ …[From (i)]

Equating both sides, we get

$a = 3$, $b - 6a = -20$, $15a - 6b = c$

$\therefore$ $a = 3$, $b = -2$, $c = 15(3) - 6(-2) = 57$

---

25. The 3rd term of $(1 + x)^n$ is $36x^2$. Find 5th term. [3 Marks]

***Solution*:**

Here, $a = 1$, $b = x$.

For 3rd term, $r = 2$

We have $t_{r+1} = {}^nC_r\, a^{n-r}.b^r$

$\therefore$ $t_3 = {}^nC_2 (1)^{n-2}.\, x^2$

$\therefore$ $36x^2 = {}^nC_2.\, x^2$

$\therefore$ ${}^nC_2 = 36$

$\therefore$ $\frac{n(n-1)}{2} = 36$

$\therefore$ $n^2 - n - 72 = 0$

$\therefore$ $(n - 9)(n + 8) = 0$

$\therefore$ $n = 9$

Now, $t_{r+1} = {}^nC_r\, a^{n-r}.b^r$

For 5th term $r = 4$

$\therefore$ $t_5 = {}^9C_4\, 1^{9-4}.x^4$

$\therefore$ $t_5 = {}^9C_4\, x^4 = 126x^4$

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26. Suppose $(1 + kx)^n = 1 - 12x + 60x^2 - \ldots$, find k and n. [3 Marks]

Practice Based On Miscellaneous Exercise – 4

1. Prove the following by the method of induction, for all $n \in N$:
$5^2 + 8^2 + 11^2 + \ldots + (3n + 2)^2$
$= \frac{n}{2}(6n^2 + 21n + 23)$ **[4 Marks]**

2. Expand by using binomial theorem: **[2 Marks Each]**
i. $\left(\frac{x}{2}+\frac{1}{x}\right)^4$ ii. $\left(x-\frac{1}{2x}\right)^5$

3. Find the tenth term in the expansion of $\left(x^2+\frac{1}{2x}\right)^{12}$. **[2 Marks]**

4. Find the middle term(s) in the expansion of:
i. $\left(x^2-\frac{1}{x}\right)^{12}$ **[2 Marks]**
ii. $\left(2x-\frac{1}{4x}\right)^9$ **[3 Marks]**

5. Find the coefficient of x^{32} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$. **[3 Marks]**

6. Find the constant term in the expansion of $\left(\sqrt{x}-\frac{2}{x^2}\right)^{10}$. **[3 Marks]**

7. If the coefficient of x in the expansion of $\left(x^2+\frac{a}{x}\right)^5$ is 270, find a. **[4 Marks]**

8. If the middle term in the expansion of $\left(x+\frac{b}{x}\right)^8$ is 1120, find b. **[3 Marks]**

9. If the coefficients of x^7 and x^8 in the expansion of $(2 + kx)^{55}$ are equal, find k. **[4 Marks]**

10. Show that there is no constant term in the expansion of $\left(x^3+\frac{a}{x}\right)^{10}$. **[4 Marks]**

11. Find the first four terms in the expansion of $(1 + x)^{-5}$. **[2 Marks]**

One Mark Questions

1. Find 2nd term in the the expansion of $(1 + x)^9$.
2. Find the value of $C_1 + C_2 + C_3 + C_4$.
3. If 4th term is the middle term in the expansion of $(1.7x + 3\sqrt{5}\,y)^n$, then find the value of n.
4. Find the general term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{11}$.
5. If there are 17 terms in the expansion of $(a + b)^n$, then find the value of n.

Multiple Choice Questions

[2 Marks Each]

1. $(\sqrt{2} + 1)^6 - (\sqrt{2} - 1)^6 =$
(A) 101 (B) $70\sqrt{2}$
(C) $140\sqrt{2}$ (D) $120\sqrt{2}$

2. 6^{th} term in expansion of $\left(2x^2 - \frac{1}{3x^2}\right)^{10}$ is
(A) $\frac{4580}{17}$ (B) $-\frac{896}{27}$
(C) $\frac{5580}{17}$ (D) None of these

3. If the ratio of the coefficient of third and fourth term in the expansion of $\left(x - \frac{1}{2x}\right)^n$ is 1 : 2, then the value of n will be
(A) 18 (B) 16
(C) 12 (D) –10

4. 16^{th} term in the expansion of $(\sqrt{x} - \sqrt{y})^{17}$ is
(A) $136\,xy^7$ (B) $136\,xy$
(C) $-136xy^{15/2}$ (D) $-136xy^2$

5. If the third term in the binomial expansion of $(1 + x)^m$ is $-\frac{1}{8}x^2$, then the rational value of m is
(A) 2 (B) 1/2
(C) 3 (D) 4

6. Coefficient of x in the expansion of $\left(x^2 + \frac{a}{x}\right)^5$ is
(A) $9a^2$ (B) $10a^3$
(C) $10a^2$ (D) $10a$

7. In the expansion of $\left(x - \frac{1}{x}\right)^6$, the constant term is
(A) – 20 (B) 20
(C) 30 (D) – 30

8. In the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$, the coefficient of x^4 is
(A) $\frac{405}{256}$ (B) $\frac{504}{259}$
(C) $\frac{450}{263}$ (D) None of these

9. The coefficient of x^3 in the expansion of $\left(x - \frac{1}{x}\right)^7$ is
(A) 14 (B) 21
(C) 28 (D) 35

10. If the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ are the same, then the value of a is
(A) $-\frac{7}{9}$ (B) $-\frac{9}{7}$
(C) $\frac{7}{9}$ (D) $\frac{9}{7}$

11. In the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$, the term independent of x is
(A) ${}^9C_3 . \frac{1}{6^3}$ (B) ${}^9C_3\left(\frac{3}{2}\right)^3$
(C) ${}^9C_3$ (D) None of these

12. In the expansion of $\left(x - \frac{3}{x^2}\right)^9$, the term independent of x is
(A) ${}^9C_3$ (B) ${}^9C_2$
(C) 2268 (D) – 2268

13. The coefficient of middle term in the expansion of $(1 + x)^{10}$ is
(A) $\frac{10!}{5!6!}$ (B) $\frac{10!}{(5!)^2}$
(C) $\frac{10!}{5!7!}$ (D) None of these

14. The middle term in the expression of $\left(x - \frac{1}{x}\right)^{18}$ is
(A) ${}^{18}C_9$ (B) $-{}^{18}C_9$
(C) ${}^{18}C_0$ (D) $-{}^{18}C_{10}$

15. ${}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9 =$
(A) 2^9 (B) 2^{10}
(C) $2^{10} - 1$ (D) None of these

16. The value of ${}^{13}C_2 + {}^{13}C_3 + {}^{13}C_4 \ldots + {}^{13}C_{13}$ is
(A) $2^{13} - 13$
(B) $2^{13} - 14$
(C) an odd number $\neq 2^{13} - 12$
(D) an even number $\neq 2^{13} - 14$

17. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, then the value of $C_0 + C_2 + C_4 + C_6 + \ldots$ is
(A) 2^{n-1} (B) $2^n - 1$
(C) 2^n (D) $2^{n-1} - 1$

18. The fourth term in the expansion of $(1 - 2x)^{3/2}$ will be
(A) $-\frac{3}{4}x^4$ (B) $\frac{x^3}{2}$
(C) $-\frac{x^3}{2}$ (D) $\frac{3}{4}x^4$

19. $\frac{1}{(2+x)^4} =$
(A) $\frac{1}{2}\left(1-2x+\frac{5}{2}x^2-...\right)$
(B) $\frac{1}{16}\left(1-2x+\frac{5}{2}x^2-...\right)$
(C) $\frac{1}{16}\left(1+2x+\frac{5}{2}x^2+...\right)$
(D) $\frac{1}{2}\left(1+2x+\frac{5}{2}x^2+...\right)$

20. The approximate value of $(7.995)^{1/3}$ correct to four decimal places is
(A) 1.9995 (B) 1.9996
(C) 1.9990 (D) 1.9991

21. The first four terms in the expansion of $(1 - x)^{3/2}$ are
(A) $1 - \frac{3}{2}x + \frac{3}{8}x^2 - \frac{1}{16}x^3$
(B) $1 - \frac{3}{2}x - \frac{3}{8}x^2 - \frac{x^3}{16}$
(C) $1 - \frac{3}{2}x + \frac{3}{8}x^2 + \frac{x^3}{16}$
(D) None of these

22. Two middle terms in the expansion of $\left(x-\frac{1}{x}\right)^{11}$ are
(A) $231x$ and $\frac{231}{x}$ (B) $462\,x$ and $\frac{462}{x}$
(C) $-462\,x$ and $\frac{462}{x}$ (D) None of these

23. In the expansion of $(1 + x)^n$, the sum of coefficients of odd powers of x is
(A) 2^{n+1} (B) $2^n - 1$
(C) 2^n (D) 2^{n-1}

Answers

Focus on Exercise 4.2

1. i. $49 + 20\sqrt{6}$
ii. $145\sqrt{5} - 229\sqrt{2}$
2. i. $16x^8 + 96x^6 + 216x^4 + 216x^2 + 81$
ii. $64x^6 - 192x^4 + 240x^2 - 160 + \frac{60}{x^2} - \frac{12}{x^4} + \frac{1}{x^6}$
3. i. $32\sqrt{3}$ ii. 1364
5. i. 108243216 ii. 1.61051
6. i. 970.299 ii. 0.6561
7. i. 16 ii. 16
8. 1.1262
9. 1.051
10. 0.5314

Practice Based On Exercise 4.2

1. i. $x^{10} + 15x^8 y + 90x^6 y^2 + 270x^4 y^3 + 405x^2y^4 + 243y^5$
ii. $32x^5 - 40x^4 y + 20x^3 y^2 - 5x^2 y^3 + \frac{5}{8}xy^4 - \frac{y^5}{32}$
2. i. $x^5 + 5x^3 + 10x + \frac{10}{x} + \frac{5}{x^3} + \frac{1}{x^5}$
ii. $x^{10} - 15x^8y + 90x^6y^2 - 270x^4y^3 + 405x^2y^4 - 243y^5$
iii. $97 + 56\sqrt{3}$
iv. $124 - 32\sqrt{15}$
3. i. $\left(\sqrt{5}+\sqrt{3}\right)^4 = 124 + \left(32\sqrt{15}\right)$
ii. $\left(\sqrt{2}+1\right)^5 - \left(\sqrt{2}-1\right)^5 = 82$
4. i. $58\sqrt{2}$ ii. 178
5. 33.6323
6. i. 1.0824 ii. 941192
7. x^5 8. x^5

Focus on Exercise 4.3

1. i. $4032\,x^{10}$ ii. $84480\,x^2$
iii. $\frac{10500}{x^3}$ iv. $\frac{55}{9}a^{16}$
v. $\frac{15182069760}{a^5}$
2. i. $122472\sqrt{2}$ ii. 700000
iii. 48620 iv. $\frac{5}{16}$
v. $\frac{-105}{8192}$

3. i. $\frac{1792}{9}$ ii. -96096
   iii. 405 iv. 84
   v. 10500000

4. i. 924 ii. $35x^5$ and $35x^2$
   iii. $1120x^4$ iv. -252
   v. $-462x^9$ and $462x^2$

5. 5

6. 91854

7. 8

Practice Based On Exercise 4.3

1. i. $512\,x^3$ ii. $14784x^4$
   iii. $\frac{35x^4}{8}$ iv. $\frac{-896}{27}$

2. $5670x^4$

3. i. 462 ii. $\frac{4480}{3}$

4. i. 35 ii. $\frac{405}{256}$
   iii. ${}^{20}C_{10}(2^{10})$ iv. -512
   v. 455

5. 180

6. i. -48620 ii. $\frac{28}{243}$
   iii. $\frac{495}{4096}$ iv. 96096

7. i. $1120x^4$
   ii. $\left(\frac{63x}{4}\right)$ and $\frac{-63}{32x}$

8. i. 252 ii. 70
   iii. $\frac{-63}{8x^5}$ iv. $\frac{189}{8}x^{17}, \frac{-21}{16}x^{19}$

Focus on Exercise 4.4

1. i. $1 - 4x + 10x^2 - 20x^3 + \ldots$
   ii. $1 - \frac{x}{3} - \frac{1}{9}x^2 - \frac{5}{81}x^3 - \ldots$
   iii. $1 + 3x^2 + 6x^4 + 10x^6 + \ldots$
   iv. $1 - \frac{x}{5} + \frac{3x^2}{25} - \frac{11x^3}{125} + \ldots$
   v. $1 - x^2 + x^4 - x^6 + \ldots$

2. i. $a^{-3}\left[1 + \frac{3b}{a} + \frac{6b^2}{a^2} + \frac{10b^3}{a^3} + \ldots\right]$
   ii. $a^{-4}\left[1 - \frac{4b}{a} + \frac{10b^2}{a^2} - \frac{20b^3}{a^3} + \ldots\right]$
   iii. $a^{\frac{1}{4}}\left[1 + \frac{b}{4a} - \frac{3b^2}{32a^2} + \frac{7b^3}{128a^3} - \ldots\right]$
   iv. $a^{\frac{-1}{4}}\left[1 + \frac{b}{4a} + \frac{5b^2}{32a^2} + \frac{15b^3}{128a^3} + \ldots\right]$
   v. $a^{\frac{-1}{3}}\left[1 - \frac{b}{3a} + \frac{2b^2}{9a^2} - \frac{14b^3}{81a^3} + \ldots\right]$

3. i. $1 - 8x + 40x^2 + \ldots$
   ii. $1 - \frac{3x}{2} + \frac{27}{8}x^2 + \ldots$
   iii. $2^{\frac{1}{3}}\left[1 - \frac{x}{2} - \frac{x^2}{4} - \ldots\right]$
   iv. $5^{\frac{-1}{2}}\left[1 - \frac{2x}{5} + \frac{6x^2}{25} + \ldots\right]$
   v. $5^{\frac{-1}{3}}\left[1 + \frac{x}{5} + \frac{2x^2}{25} + \ldots\right]$

4. i. 9.9499 ii. 5.0133
   iii. 2.0025 iv. 0.9057
   v. 1.0625

Practice Based On Exercise 4.4

1. i. $a^{-4}\left[1 + \frac{4b}{a} + \frac{10b^2}{a^2} + \frac{20b^3}{a^3} + \ldots\right]$
   ii. $a^{-1}\left[1 - \frac{b}{a} + \frac{b^2}{a^2} - \frac{b^3}{a^3} + \ldots\right]$

2. i. $1 - 3x + 6x^2 - 10x^3 + \ldots$
   ii. $1 + 2x + 3x^2 + 4x^3 + \ldots$
   iii. $1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5x^3}{81} - \ldots$
   iv. $1 + \frac{x^2}{2} + \frac{3x^4}{8} + \frac{5x^6}{16} + \ldots$

3. i. $a^{\frac{1}{3}}\left(1 - \frac{b}{3a} - \frac{b^2}{9a^2} - \frac{5b^3}{81a^3} - \ldots\right)$
   ii. $a^{-\frac{2}{3}}\left(1 - \frac{2b}{3a} + \frac{5b^2}{9a^2} - \frac{40b^3}{81a^3} + \ldots\right)$

4. $2^{\frac{-1}{2}}\left[1 + \frac{3x}{4} + \frac{27x^2}{32} + \frac{135x^2}{128} + \ldots\right]$

5. i. $1 - 12x + 108x^2 - \ldots$
   ii. $1 - 3x + \frac{3x^2}{2} + \ldots$
   iii. $4^{-\frac{1}{2}}\left(1 + \frac{3x}{8} + \frac{27x^2}{128} + \ldots\right)$
   iv. $4^{-\frac{1}{2}}\left(1 - \frac{5x}{8} + \frac{75x^2}{128} - \ldots\right)$

6. 5.4775

7. i. 9.4869 ii. 6.0092
   iii. 1.0068

Focus on Miscellaneous Exercise – 4

II.

4. $243x^{10} + 810x^8y + 1080x^6y^2 + 720x^4y^3 + 240x^2y^4 + 32y^5$

5. $\frac{16x^4}{81} - \frac{16x^2}{9} + 6 - \frac{9}{x^2} + \frac{81}{16x^4}$

6. $\frac{27}{2}x^4y^6$

7. $\frac{1760}{x^3}$

8. i. -20
   ii. $-\frac{63x^5}{8y^5}$
   iii. $280x^8y^6$ and $560x^6y^8$
   iv. $\frac{189x^6}{16}$ and $\frac{-21x^3}{8}$

9. i. 378 ii. 153

10. i. 2268 ii. 7920

12. ± 2

13. 2

14. $\frac{9}{7}$

15. 2

18. $1 + \frac{x}{3} + \frac{x^2}{6} + \frac{5x^3}{54} + \ldots$

19. $1 + \frac{x}{4} + \frac{5x^2}{32} + \frac{15x^3}{128} + \ldots$

20. $5^{\frac{-1}{2}}\left[1 - \frac{2x}{5} + \frac{6x^2}{25} - \ldots\right]$

21. 9.9833

22. 0.2451

23. -80

24. a = 3, b = –2 , c = 57

25. $126x^4$

26. k = –2, n = 6

Practice Based On Miscellaneous Exercise – 4

2. i. $\frac{x^4}{16} + \frac{x^2}{2} + \frac{3}{2} + \frac{2}{x^2} + \frac{1}{x^4}$
   ii. $x^5 - \frac{5x^3}{2} + \frac{5x}{2} - \frac{5}{4x} + \frac{5}{16x^3} - \frac{1}{32x^5}$

3. $\frac{55}{128x^3}$

4. i. $924x^6$ ii. $\frac{63x}{4}$ and $\frac{-63}{32x}$

5. 1365

6. 180

7. 3

8. 2

9. $\frac{1}{3}$

11. $1 - 5x + 15x^2 - 35x^3 + \ldots$

Answers to One Mark Questions

1. $9x$
2. 15
3. 6
4. ${}^{11}C_r x^{22-3r}$
5. 16

Answers to Multiple Choice Questions

1.	(C)	2.	(B)	3.	(D)	4.	(C)
5.	(B)	6.	(B)	7.	(A)	8.	(A)
9.	(B)	10.	(D)	11.	(A)	12.	(D)
13.	(B)	14.	(B)	15.	(A)	16.	(B)
17.	(A)	18.	(B)	19.	(B)	20.	(A)
21.	(C)	22.	(C)	23.	(D)		