SAMPLE CONTENT

Precise MATHEMATICS

PART - 1

Std. XI
Science

Mrs. Swarada Kirloskar
M.Sc. (Applied Maths),
M.Phil (Computer Applications)

Mr. Shantanu R. Pal
B.E. (Electronics)

Ms. Suchitra Yadav
M.Sc. (Mathematics) B.Ed.

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PRECISE

MATHEMATICS - I

Std. XI Sci. & Arts

Salient Features

- Written as per the latest textbook
- Exhaustive coverage of entire syllabus
- Covers all derivations and theorems
- Tentative marks allocation for all the problems
- The chapters include:
  - 'Precise Theory' for every topic
  - Solutions to all Exercises and Miscellaneous exercises given in the textbook
  - 'One Mark Questions' and 'Multiple choice questions' (MCQs)
- Includes Important Features for holistic learning:
  - **Smart Check** - **Important Formulae** - **Remember This**

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PREFACE

"Everything should be made as simple as possible, but not simpler." - Albert Einstein.

Inspired by this vision, we present **'Precise Mathematics – I: Std. XI'**, tailored to the latest Maharashtra State Board textbook. This compact yet comprehensive book aims to boost students' confidence and prepare them for the crucial Std. XI final exam, laying a solid foundation for their concepts.

Inside, you'll find **answers to all textbook exercises, including miscellaneous problems**. To aid in understanding, we've provided **precise theory** where needed, along with essential **theorems and their derivations**. A recap of all **important formulae** is included at the end of the book for quick revision.

We recognize that many problems can be tackled using various methods. That's why we've included an **'Alternate Method'** section to introduce students to different problem-solving approaches. To ensure accuracy, **'Smart Check'** helps students verify their answers effectively. Additionally, each chapter concludes with **'One Mark Questions'** and **'Multiple Choice Questions'**, along with their answers.
'Precise Mathematics – I: Std. XI' embodies our vision and achieves multiple goals: building concepts, developing problem-solving competence, and promoting self-study, all while encouraging cognitive thinking.

Refer to the flow chart on the adjacent page for an overview of the book's key features and how they are designed to enhance student learning.

We hope the book benefits the learner as we have envisioned.

Publisher
Edition: First

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.
Please write to us on: mail@targetpublications.org

KEY FEATURES

Smart Check

Smart Check is a technique to verify the answers. This is our attempt to cross-check the accuracy of the answer. Smart check is indicated by ✓ symbol

One Mark Questions

These questions require very short solutions with direct application of mathematical concepts.

Multiple Choice Questions

Multiple Choice Questions include textual as well as additional MCQs.

Important Formulae

Important Formulae given at the end of the book include all of the key formulae in the chapter.
This is our attempt to offer students a handy tool to solve problems and ace the last minute revision.

CONTENTS

[Reference: Maharashtra State Board of Secondary and Higher Secondary Education, Pune - 04**]**

Smart check is indicated by ✓ symbol.

1 Angle and its Measurement

Contents and Concepts

- Directed Angles
- Angles of Different Measurements
- Units of Measure of an Angle
- Length of an Arc of a Circle
- Area of a Sector of a Circle

Let's Study

Directed angles

Suppose OX is the initial position of a ray. This ray rotates about O from initial position OX and takes a finite position along ray OP. In such a case we say that rotating ray OX describes a directed angle XOP.
It is also denoted by ∠XOP.

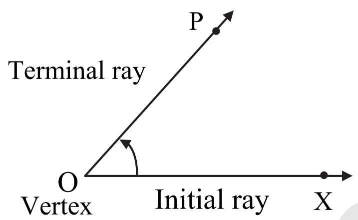


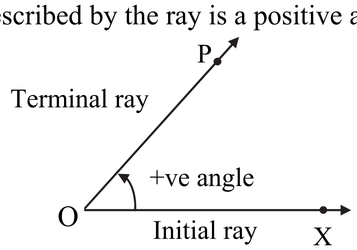
Fig. (i)

In figure (i), the point O is called the **vertex**. The ray OX is called the **initial ray** and ray OP is called the **terminal ray** of an angle XOP. The pair of rays are also called the **arms** of angle XOP.

In general, an angle can be defined as an ordered pair of initial and terminal rays or arms rotating from initial position to terminal position.

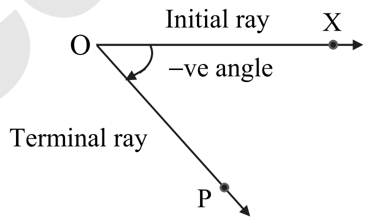
The directed angle includes two things:
i. Amount of rotation (magnitude of angle).
ii. Direction of rotation (sign of the angle).

Positive angle:
If a ray rotates about the vertex (the point) O from initial position OX in anticlockwise direction, then the angle described by the ray is a positive angle.



In the given figure, ∠XOP is obtained by the rotation of a ray in anticlockwise direction denoted by arrow. Hence, ∠XOP is positive, i.e., +∠XOP.

Negative angle:
If a ray rotates about the vertex (the point) O, from initial position OX in clockwise direction, then the angle described by the ray is a negative angle.



In the above figure, ∠XOP is obtained by the rotation of a ray in clockwise direction denoted by arrow. Hence, ∠XOP is negative, i.e., −∠XOP.

Angle of any magnitude:

i. Suppose a ray starts from the initial position OX in anticlockwise sense and makes complete rotation (revolution) about O and takes the final position along OX as shown in the figure (i), then the angle described by the ray is 360°.

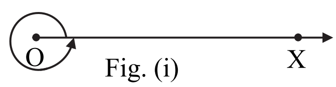


Fig. (i)

In figure (ii), initial ray rotates about O in anticlockwise sense and completes two rotations (revolutions). Hence, the angle described by the ray is $2 \times 360° = 720°$.

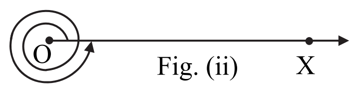
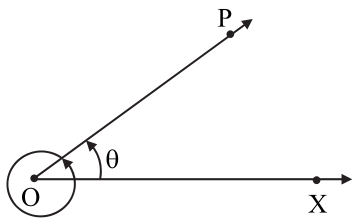


Fig. (ii)

In figure (iii), initial ray rotates about O in clockwise sense and completes two rotations (revolutions). Hence, the angle described by the ray is $-2 \times 360° = -720°$

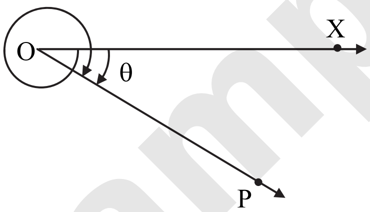
Fig. (iii)

ii. Suppose a ray starting from the initial position OX makes one complete rotation in anticlockwise sense and takes the position OP as shown in figure, then the angle described by the revolving ray is $360° + \angle XOP$.



If $\angle XOP = \theta$, then the traced angle is $360° + \theta$.
If the rotating ray completes two rotations, then the angle described is $2 \times 360° + \theta = 720° + \theta$ and so on.

iii. Suppose the initial ray makes one complete rotation about O in clockwise sense and attains its terminal position OP, then the described angle is $-(360° + \angle XOP)$.



If $\angle XOP = \theta$, then the traced angle is $-(360° + \theta)$.
If final position OP is obtained after 2,3,4, …. complete rotations in clockwise sense, then angle described are $-(2 \times 360° + \theta)$, $-(3 \times 360° + \theta)$, $-(4 \times 360° + \theta)$, …..

Types of angles:

Zero angle:

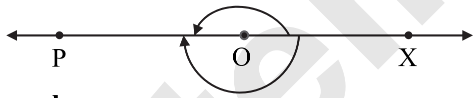
If the initial ray and the terminal ray lie along same line and same direction, i.e., they coincide, then the angle obtained is of measure zero and is called zero angle.

One rotation angle:

After one complete rotation, if the initial ray OA coincides with the terminal ray OB, then the angle obtained is called one rotation angle. $m\angle AOB = 360°$.

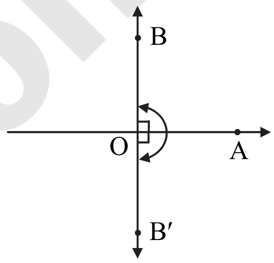
Straight angle:

In the figure, OX is the initial position and OP is the final position of rotating ray. The rays OX and OP lie along the same line but in opposite direction. In this case $\angle XOP$ is called a straight angle and $m\angle XOP = 180°$.



Right angle:

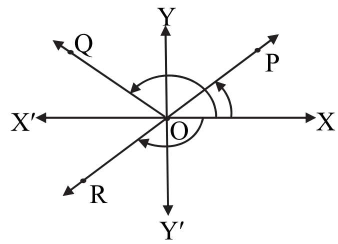
One fourth of one rotation angle is called as one right angle; it is also half of a straight angle. One complete rotation angle is four right angles.



In the figure, $m\angle AOB = 90°$ and $m\angle AOB' = -90°$.

Angles in standard position:

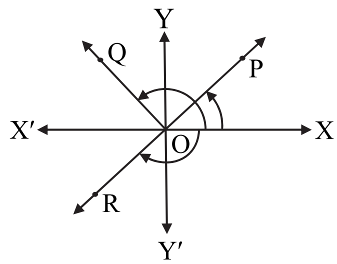
An angle which has vertex at origin and initial arm along positive X–axis is called standard angle or angle in standard position.



In the figure, $\angle XOP$, $\angle XOQ$, $\angle XOR$ with vertex O and initial ray along positive X–axis are called standard angles or angles in standard position.

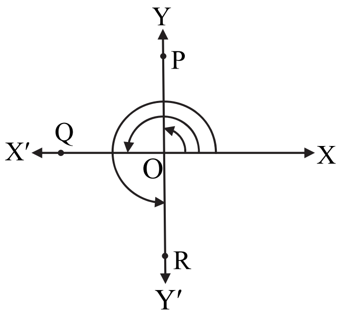
Angle in a Quadrant:

An angle is said to be in a particular quadrant, if the terminal ray of the angle in standard position lies in that quadrant.



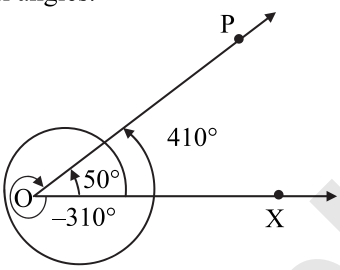
In the figure, $\angle XOP$, $\angle XOQ$ and $\angle XOR$ lie in first, second and third quadrants respectively.

Quadrantal Angles:
If the terminal arm of an angle in standard position lie along any one of the co-ordinate axes, then it is called as quadrantal angle.



In the figure, ∠XOP, ∠XOQ and ∠XOR are quadrantal angles.
Note:
The quadrantal angles are integral multiples of 90°, i.e., $\pm n\frac{\pi}{2}$, where $n \in N$.

Coterminal angles:
Two angles with different measures but having the same positions of initial ray and terminal ray are called as coterminal angles.

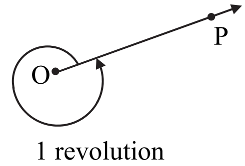


In the figure, the directed angles having measures 50°, 410°, –310° have the same initial arm, ray OX and the same terminal arm, ray OP. Hence, these angles are coterminal angles.
Note:
If two directed angles are co-terminal angles, then the difference between measures of these two directed angles is an integral multiple of 360°.

Measures of angles

There are two systems of measurement of an angle:
i. Sexagesimal system (Degree measure)
ii. Circular system (Radian measure)

i. Sexagesimal system (Degree Measure):
In this system, the unit of measurement of an angle is ‘degree’.
Suppose a ray OP starts rotating in the anticlockwise sense about O and attains the original position for the first time, then the amount of rotation caused is called 1 revolution.
Divide 1 revolution into 360 equal parts. Each part is called as one degree(1°).



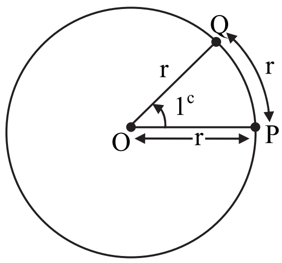
i.e., $\boxed{1 \text{ revolution} = 360^\circ}$

Divide 1° into 60 equal parts. Each part is called as one minute (1′).
i.e., $\boxed{1^\circ = 60'}$

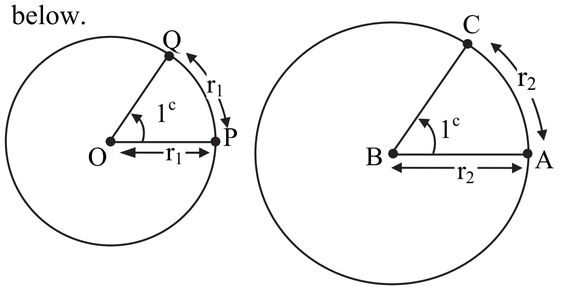
Divide 1′ into 60 equal parts. Each part is called as one second (1″).
i.e., $\boxed{1' = 60''}$

Note:
The sexagesimal system is extensively used in engineering, astronomy, navigation and surveying.

ii. Circular system (Radian measure):
In this system, the unit of measurement of an angle is ‘radian’.
Angle subtended at the centre of a circle by an arc whose length is equal to the radius is called as one radian denoted by 1^c.
Draw any circle with centre O and radius r. Take the points P and Q on the circle such that the length of arc PQ is equal to radius of the circle. Join OP and OQ.



Then by the definition, the measure of ∠POQ is 1 radian (1^c).
Note:
i. This system of measuring an angle is used in all the higher branches of mathematics.
ii. The radian is a constant angle, therefore radian does not depend on the circle, i.e., it does not depend on the radius of the circle as shown below.



In the figure, we draw two circles of different radii r_1 and r_2 and centres O and B respectively. Then the angle at the centre of both circles is equal to 1^c.
i.e., $\angle POQ = 1^c = \angle ABC$.

Theorem:

The radian is independent of the radius of the circle and $\pi^c = 180°$.

Proof:

Let O be the centre and r be the radius of the circle. Take points P, Q and R on the circle such that arc PQ = r and $\angle POR = 90°$.

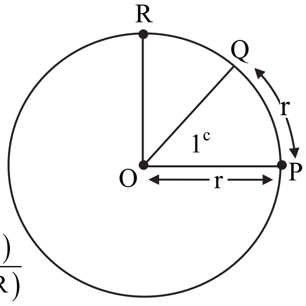
By definition of radian,

$\angle POQ = 1^c$

$l(\text{arc PQR}) = \frac{1}{4} \times$ circumference of the circle

$= \frac{1}{4} \times 2\pi r$

$= \frac{\pi r}{2}$



By proportionality theorem,

$$\frac{m\angle POQ}{m\angle POR} = \frac{l(\text{arc PQ})}{l(\text{arc PQR})}$$

$\therefore$ $m\angle POQ = \frac{l(\text{arc PQ})}{l(\text{arc PQR})} \times m\angle POR$

$\therefore$ $1^c = \frac{r}{\left(\frac{\pi r}{2}\right)} \times 1 \text{ right angle} = \frac{2}{\pi} \times (1 \text{ right angle})$

i.e., 1^c is a constant, which is independent of r.

Also, $\pi^c = 2$ right angles $= 180°$

$\therefore$ $\pi^c = 180°$

Remember This

i. To convert degree measure into radian measure, multiply degree measure by $\frac{\pi}{180}$.

ii. To convert radian measure into degree measure, multiply radian measure by $\frac{180}{\pi}$.

iii. $1^c = 57° 19' 29''$ (approx.)

iv. Certain degree measures in terms of radians:

Degree	Radian
15	$\frac{\pi}{12}$
30	$\frac{\pi}{6}$
45	$\frac{\pi}{4}$
60	$\frac{\pi}{3}$
90	$\frac{\pi}{2}$

Degree	Radian
120	$\frac{2\pi}{3}$
180	π
270	$\frac{3\pi}{2}$
360	2π

v. Relation between angle and time in a clock. (R is rotation.):

Minute Hand	**Hour Hand**
1R = 360°	1R = 360°
1R = 60 min.	1R = 12 Hrs.
60 min. = 360°	12Hrs. = 360°
1 min. = 6° rotation	1 Hr. = 30°
	1 Hr. = 60 min.
	60 min = 30°
	1 min = $\left(\frac{1}{2}\right)^\circ$

vi. 'Minute' in time and 'Minute' as a fraction of a degree angle are different.

Exercise 1.1

1. (A) Determine which of the following pairs of angles are co-terminal. [1 Mark Each]

i. 210°, − 150° **ii. 360°, − 30°**
iii. − 180°, 540° **iv. − 405°, 675°**
v. 860°, 580° **vi. 900°, − 900°**

Solution:

i. 210°, –150°

$210° - (-150°) = 210° + 150°$
$= 360°$
$= 1(360°)$,

which is a multiple of 360°.

$\therefore$ The given pair of angles is co-terminal.

ii. 360°, – 30°

$360° - (-30°) = 360° + 30°$
$= 390°$,

which is not a multiple of 360°.

$\therefore$ The given pair of angles is not co-terminal.

iii. –180°, 540°

$540° - (-180°) = 540° + 180°$
$= 720°$
$= 2(360°)$,

which is a multiple of 360°.

$\therefore$ The given pair of angles is co-terminal.

iv. – 405°, 675°

$675° - (-405°) = 675° + 405°$
$= 1080°$
$= 3(360°)$,

which is a multiple of 360°.

$\therefore$ The given pair of angles is co-terminal.

v. 860°, 580°

$860° - 580° = 280°$,

which is not a multiple of 360°.

$\therefore$ The given pair of angles is not co-terminal.

vi. $900°, -900°$

$900° - (-900°) = 900° + 900°$

$= 1800°$

$= 5(360°)$,

which is a multiple of 360°.

∴ The given pair of angles is co-terminal.

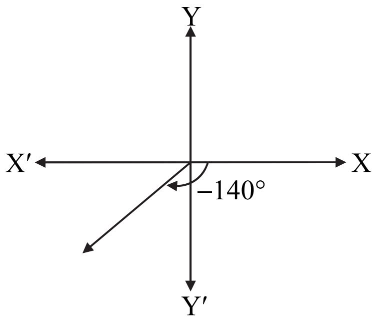
1. (B) Draw the angles of the following measures and determine their quadrants.

[2 Marks Each]

i.	**–140°**	**ii.**	**250°**	**iii.**	**420°**
iv.	**750°**	**v.**	**945°**	**vi.**	**1120°**
vii.	**– 80°**	**viii.**	**– 330°**	**ix.**	**– 500°**
x.	**– 820°**				

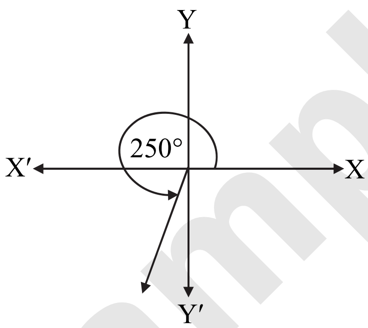
Solution:

i.



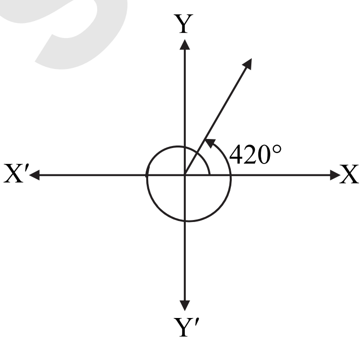
From the figure, the given angle terminates in quadrant III.

ii.



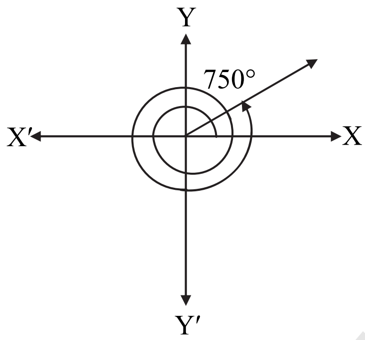
From the figure, the given angle terminates in quadrant III.

iii.



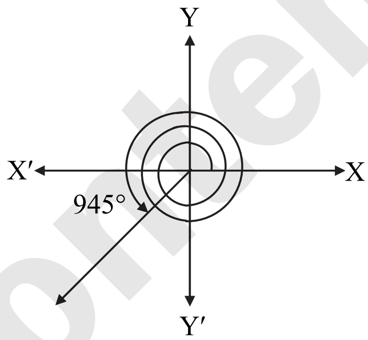
From the figure, the given angle terminates in quadrant I.

iv.



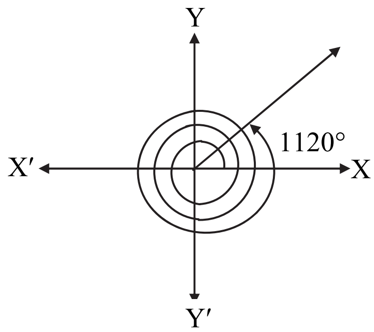
From the figure, the given angle terminates in quadrant I.

v.



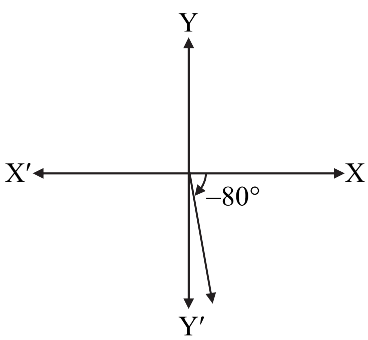
From the figure, the given angle terminates in quadrant III.

vi.



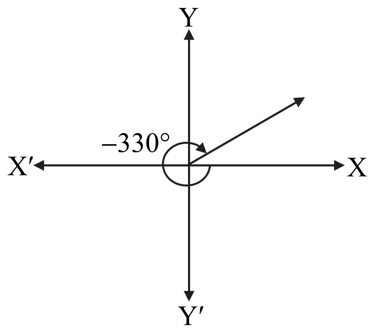
From the figure, the given angle terminates in quadrant I.

vii.



From the figure, the given angle terminates in quadrant IV.

viii.



From the figure, the given angle terminates in quadrant I.

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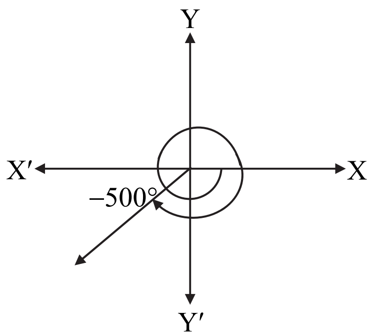
ix.



From the figure, the given angle terminates in quadrant III.

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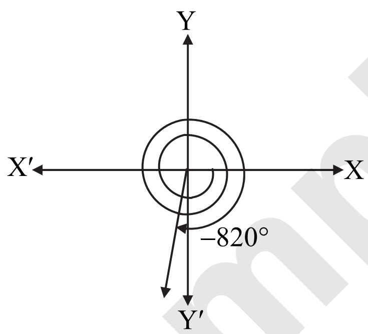
x.



From the figure, the given angle terminates in quadrant III.

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2. Convert the following angles into radians.

i. 85° ii. 250° iii. −132° [1 Mark Each]

iv. 65°30′ v. 75°30′ vi. 40°48′ [2 Marks Each]

Solution:

We know that $\theta^\circ = \left(\theta \times \frac{\pi}{180}\right)^c$

i. $85^\circ = \left(85 \times \frac{\pi}{180}\right)^c$

$= \left(\frac{17\pi}{36}\right)^c$

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ii. $250^\circ = \left(250 \times \frac{\pi}{180}\right)^c$

$= \left(\frac{25\pi}{18}\right)^c$

---

iii. $-132^\circ = \left(-132 \times \frac{\pi}{180}\right)^c$

$= \left(-\frac{11\pi}{15}\right)^c$

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iv. $65^\circ 30' = 65^\circ + 30'$

$= 65^\circ + \left(\frac{30}{60}\right)^\circ \quad \ldots\left[\because 1' = \left(\frac{1}{60}\right)^\circ\right]$

$= 65^\circ + \left(\frac{1}{2}\right)^\circ$

$= \left(65 + \frac{1}{2}\right)^\circ$

$= \left(\frac{131}{2}\right)^\circ$

$= \left(\frac{131}{2} \times \frac{\pi}{180}\right)^c$

$= \left(\frac{131\pi}{360}\right)^c$

---

v. $75^\circ 30' = 75^\circ + 30'$

$= 75^\circ + \left(\frac{30}{60}\right)^\circ \quad \ldots\left[\because 1' = \left(\frac{1}{60}\right)^\circ\right]$

$= 75^\circ + \left(\frac{1}{2}\right)^\circ$

$= \left(75 + \frac{1}{2}\right)^\circ$

$= \left(\frac{151}{2}\right)^\circ$

$= \left(\frac{151}{2} \times \frac{\pi}{180}\right)^c = \left(\frac{151\pi}{360}\right)^c$

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vi. $40^\circ 48' = 40^\circ + 48'$

$= 40^\circ + \left(\frac{48}{60}\right)^\circ \quad \ldots\left[\because 1' = \left(\frac{1}{60}\right)^\circ\right]$

$= 40^\circ + \left(\frac{4}{5}\right)^\circ$

$= \left(40 + \frac{4}{5}\right)^\circ$

$= \left(\frac{204}{5}\right)^\circ$

$= \left(\frac{204}{5} \times \frac{\pi}{180}\right)^c = \left(\frac{17\pi}{75}\right)^c$

3. Convert the following angles in degrees. **[1 Mark Each]**

i. $\frac{7\pi^c}{12}$ **ii.** $\frac{-5\pi^c}{3}$ **iii.** 5^c

iv. $\frac{11\pi^c}{18}$ **v.** $\left(\frac{-1}{4}\right)^c$

Solution:

We know that $\theta^c = \left(\theta \times \frac{180}{\pi}\right)^o$

i. $\frac{7\pi^c}{12} = \left(\frac{7\pi}{12} \times \frac{180}{\pi}\right)^o = 105°$

ii. $-\frac{5\pi^c}{3} = \left(-\frac{5\pi}{3} \times \frac{180}{\pi}\right)^o = -300°$

iii. $5^c = \left(5 \times \frac{180}{\pi}\right)^o = \left(\frac{900}{\pi}\right)^o$

iv. $\frac{11\pi^c}{18} = \left(\frac{11\pi}{18} \times \frac{180}{\pi}\right)^o = 110°$

v. $\left(-\frac{1}{4}\right)^c = \left(-\frac{1}{4} \times \frac{180}{\pi}\right)^o = \left(-\frac{45}{\pi}\right)^o$

4. Express the following angles in degrees, minutes and seconds.

i. (183.7)° **[1 Mark]**

ii. (245.33)° **[1 Mark]**

iii. $\left(\frac{1}{5}\right)^c$ **[2 Marks]**

Solution:

We know that 1° = 60′ and 1′ = 60″

i. (183.7)° = 183° + (0.7)°
= 183° + (0.7 × 60)′
= 183° + 42′
= 183° 42′

ii. (245.33)° = 245° + (0.33)°
= 245° + (0.33 × 60)′
= 245° + (19.8)′
= 245° + 19′ + (0.8)′
= 245° 19′ + (0.8 × 60)″
= 245° 19′ + 48″
= 245° 19′ 48″

iii. We know that $\theta^c = \left(\theta \times \frac{180}{\pi}\right)^o$

∴ $\left(\frac{1}{5}\right)^c = \left(\frac{1}{5} \times \frac{180}{\pi}\right)^o$

$= \left(\frac{36}{\pi}\right)^o$

$= \left(\frac{36}{3.14}\right)^o$ …[∵ π = 3.14]

= (11.46)°
= 11° + (0.46)°
= 11° + (0.46 × 60)′
= 11° + (27.6)′
= 11° + 27′ + (0.6)′
= 11° + 27′ + (0.6 × 60)″
= 11°27′ + 36″
= 11°27′36″ (approx.)

5. In Δ ABC, if m∠A = $\frac{7\pi^c}{36}$, m∠B = 120°, find m∠C in degree and radian. **[2 Marks]**

Solution:

We know that $\theta^c = \left(\theta \times \frac{180}{\pi}\right)^o$

In ΔABC,

m∠A = $\frac{7\pi^c}{36} = \left(\frac{7\pi}{36} \times \frac{180}{\pi}\right)^o = 35°$,

m∠B = 120°

∴ m∠A + m∠B + m∠C = 180°
…[Sum of the angles of a triangle is 180°]

∴ 35° + 120° + m∠C = 180°

∴ m∠C = 180° – 35° – 120°

∴ m∠C = 25°

$= \left(25 \times \frac{\pi}{180}\right)^c$ …$\left[\because \theta^o = \left(\theta \times \frac{\pi}{180}\right)^c\right]$

$= \left(\frac{5\pi}{36}\right)^c$

∴ m∠C = 25° = $\left(\frac{5\pi}{36}\right)^c$

Smart Check

If the sum of the angles of ΔABC is 180°, then our answer is correct.

∠A + ∠B + ∠C = 35° + 120° + 25° = 180°

Thus, our answer is correct.

6. Two angles of a triangle are $\frac{5\pi^c}{9}$ and $\frac{5\pi^c}{18}$. Find the degree and radian measures of third angle. **[2 Marks]**

Solution:

We know that $\theta^c = \left(\theta \times \frac{180}{\pi}\right)^o$

The measures of two angles of a triangle are $\frac{5\pi^c}{9}$, $\frac{5\pi^c}{18}$, i.e., $\left(\frac{5\pi}{9}\times\frac{180}{\pi}\right)^{o}$, $\left(\frac{5\pi}{18}\times\frac{180}{\pi}\right)^{o}$,

i.e., 100°, 50°

Let the measure of third angle of the triangle be x°.

∴ 100° + 50° + x° = 180°

...[Sum of the angles of a triangle is 180°]

∴ x° = 180° – 100° – 50°

∴ x° = 30°

$= \left(30\times\frac{\pi}{180}\right)^{c}$ $\quad\ldots\left[\because \theta^{o} = \left(\theta\times\frac{\pi}{180}\right)^{c}\right]$

$= \left(\frac{\pi}{6}\right)^{c}$

∴ The degree and radian measures of the third angle are 30° and $\left(\frac{\pi}{6}\right)^{c}$ respectively.

7. In a right angled triangle, the acute angles are in the ratio 4:5. Find the angles of the triangle in degrees and radians. [3 Marks]

Solution:

Since the triangle is a right angled triangle, one of the angles is 90°.

In the right angled triangle, the acute angles are in the ratio 4:5.

Let the measures of the acute angles of the triangle in degrees be 4k and 5k, where k is a constant.

∴ 4k + 5k + 90° = 180°

…[Sum of the angles of a triangle is 180°]

∴ 9k = 180° – 90°

∴ 9k = 90°

∴ k = 10°

∴ The measures of the angles in degrees are

4k = 4 × 10° = 40°,

5k = 5 × 10° = 50°

and 90°.

We know that $\theta° = \left(\theta\times\frac{\pi}{180}\right)^{c}$

∴ The measures of the angles in radians are

$40° = \left(40\times\frac{\pi}{180}\right)^{c} = \left(\frac{2\pi}{9}\right)^{c}$

$50° = \left(50\times\frac{\pi}{180}\right)^{c} = \left(\frac{5\pi}{18}\right)^{c}$

$90° = \left(90\times\frac{\pi}{180}\right)^{c} = \left(\frac{\pi}{2}\right)^{c}$

8. The sum of two angles is $5\pi^{c}$ and their difference is 60°. Find their measures in degrees. [3 Marks]

Solution:

Let the measures of the two angles in degrees be x and y.

Sum of two angles is $5\pi^{c}$.

∴ $x + y = 5\pi^{c}$

∴ $x + y = \left(5\pi\times\frac{180}{\pi}\right)^{o}$ $\quad\ldots\left[\because \theta^{c} = \left(\theta\times\frac{180}{\pi}\right)^{o}\right]$

∴ $x + y = 900°$ …(i)

Difference of two angles is 60°.

$x - y = 60°$ …(ii)

Adding (i) and (ii), we get

$2x = 960°$

∴ $x = 480°$

Substituting the value of x in (i), we get

$480° + y = 900°$

∴ $y = 900° - 480° = 420°$

∴ The measures of the two angles in degrees are 480° and 420°.

Smart Check

If the difference of the two angles is 60°, then our answer is correct.

Difference = 480° – 420° = 60°

Thus, our answer is correct.

9. The measures of the angles of a triangle are in the ratio 3:7:8. Find their measures in degrees and radians. [4 Marks]

Solution:

The measures of the angles of the triangle are in the ratio 3:7:8.

Let the measures of the angles of the triangle in degrees be 3k, 7k and 8k, where k is a constant.

∴ 3k + 7k + 8k = 180°

…[Sum of the angles of a triangle is 180°]

∴ 18k = 180°

∴ k = 10°

∴ The measures of the angles in degrees are

3k = 3 × 10° = 30°,

7k = 7 × 10° = 70° and

8k = 8 × 10° = 80°.

We know that $\theta° = \left(\theta\times\frac{\pi}{180}\right)^{c}$

∴ The measures of the angles in radians are

$30° = \left(30\times\frac{\pi}{180}\right)^{c} = \left(\frac{\pi}{6}\right)^{c}$

$70° = \left(70\times\frac{\pi}{180}\right)^{c} = \left(\frac{7\pi}{18}\right)^{c}$

$80° = \left(80\times\frac{\pi}{180}\right)^{c} = \left(\frac{4\pi}{9}\right)^{c}$

10. The measures of the angles of a triangle are in A.P. and the greatest is 5 times the smallest (least). Find the angles in degrees and radians. [4 Marks]

Solution:

Let the measures of the angles of the triangle in degrees be $a - d$, a, $a + d$, where $a > d > 0$.

$\therefore$ $a - d + a + a + d = 180°$

…[Sum of the angles of a triangle is 180°]

$\therefore$ $3a = 180°$

$\therefore$ $a = 60°$ …(i)

According to the given condition, greatest angle is 5 times the smallest angle.

$\therefore$ $a + d = 5(a - d)$

$\therefore$ $a + d = 5a - 5d$

$\therefore$ $6d = 4a$

$\therefore$ $3d = 2a$

$\therefore$ $3d = 2(60°)$ …[From (i)]

$\therefore$ $d = \frac{120°}{3} = 40°$

$\therefore$ The measures of the angles in degrees are

$a - d = 60° - 40° = 20°$,

$a = 60°$ and

$a + d = 60° + 40° = 100°$.

We know that $\theta° = \left(\theta \times \frac{\pi}{180}\right)^c$

$\therefore$ The measures of the angles in radians are

$$20° = \left(20 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{9}\right)^c$$

$$60° = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$$

$$100° = \left(100 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{9}\right)^c$$

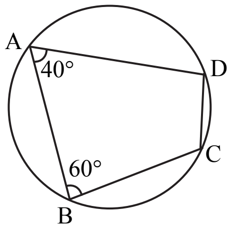
11. In a cyclic quadrilateral two adjacent angles are 40° and $\frac{\pi^c}{3}$. Find the angles of the quadrilateral in degrees. [3 Marks]

Solution:

Let ABCD be the cyclic quadrilateral such that $\angle A = 40°$ and

$\angle B = \frac{\pi^c}{3} = \left(\frac{\pi}{3} \times \frac{180}{\pi}\right)^o$ …$\left[\because \theta^c = \left(\theta \times \frac{180}{\pi}\right)^o\right]$

$= 60°$



$\angle A + \angle C = 180°$

…[Opposite angles of a cyclic quadrilateral are supplementary]

$\therefore$ $40° + \angle C = 180°$

$\therefore$ $\angle C = 180° - 40° = 140°$

Also, $\angle B + \angle D = 180°$

…[Opposite angles of a cyclic quadrilateral are supplementary]

$\therefore$ $60° + \angle D = 180°$

$\therefore$ $\angle D = 180° - 60° = 120°$

$\therefore$ The angles of the quadrilateral in degrees are 40°, 60°, 140° and 120°.

12. One angle of a quadrilateral has measure $\frac{2\pi^c}{5}$ and the measures of other three angles are in the ratio 2 : 3 : 4. Find their measures in degrees and radians. [4 Marks]

Solution:

We know that $\theta^c = \left(\theta \times \frac{180}{\pi}\right)^o$

One angle of the quadrilateral has measure

$\frac{2\pi^c}{5} = \left(\frac{2\pi}{5} \times \frac{180}{\pi}\right)^o = 72°$

Measures of other three angles are in the ratio 2 : 3 : 4.

Let the measures of the other three angles of the quadrilateral in degrees be $2k$, $3k$, $4k$, where k is a constant.

$\therefore$ $72° + 2k + 3k + 4k = 360°$

…[Sum of the angles of a quadrilateral is 360°]

$\therefore$ $9k = 288°$

$\therefore$ $k = 32°$

$\therefore$ The measures of the angles in degrees are

$2k = 2 \times 32° = 64°$

$3k = 3 \times 32° = 96°$

$4k = 4 \times 32° = 128°$

We know that $\theta° = \left(\theta \times \frac{\pi}{180}\right)^c$

$\therefore$ The measures of the angles in radians are

$$64° = \left(64 \times \frac{\pi}{180}\right)^c = \left(\frac{16\pi}{45}\right)^c$$

$$96° = \left(96 \times \frac{\pi}{180}\right)^c = \left(\frac{8\pi}{15}\right)^c$$

$$128° = \left(128 \times \frac{\pi}{180}\right)^c = \left(\frac{32\pi}{45}\right)^c$$

13. Find the degree and radian measures of exterior and interior angles of a regular [3 Marks Each]

i. pentagon **ii. hexagon**

iii. heptagon **iv. octagon**

Solution:

i. Pentagon:

Number of sides = 5

Number of exterior angles = 5

Sum of exterior angles = 360°

$\therefore$ Each exterior angle $= \dfrac{360^\circ}{\text{no. of sides}} = \dfrac{360^\circ}{5} = 72^\circ$

$= \left(72 \times \dfrac{\pi}{180}\right)^c = \left(\dfrac{2\pi}{5}\right)^c$

Interior angle + Exterior angle = 180°

$\therefore$ Each interior angle $= 180^\circ - 72^\circ = 108^\circ$

$= \left(108 \times \dfrac{\pi}{180}\right)^c = \left(\dfrac{3\pi}{5}\right)^c$

---

ii. Hexagon:

Number of sides = 6

Number of exterior angles = 6

Sum of exterior angles = 360°

$\therefore$ Each exterior angle $= \dfrac{360^\circ}{\text{no. of sides}} = \dfrac{360^\circ}{6} = 60^\circ$

$= \left(60 \times \dfrac{\pi}{180}\right)^c = \left(\dfrac{\pi}{3}\right)^c$

Interior angle + Exterior angle = 180°

$\therefore$ Each interior angle $= 180^\circ - 60^\circ = 120^\circ$

$= \left(120 \times \dfrac{\pi}{180}\right)^c$

$= \left(\dfrac{2\pi}{3}\right)^c$

---

iii. Heptagon:

Number of sides = 7

Number of exterior angles = 7

Sum of exterior angles = 360°

$\therefore$ Each exterior angle $= \dfrac{360^\circ}{\text{no. of sides}} = \dfrac{360^\circ}{7}$

$= (51.43)^\circ$

$= \left(\dfrac{360}{7} \times \dfrac{\pi}{180}\right)^c = \left(\dfrac{2\pi}{7}\right)^c$

Interior angle + Exterior angle = 180°

$\therefore$ Each interior angle $= 180^\circ - \left(\dfrac{360}{7}\right)^\circ$

$= \left(\dfrac{1260-360}{7}\right)^\circ$

$= \left(\dfrac{900}{7}\right)^\circ = (128.57)^\circ$

$= \left(\dfrac{900}{7} \times \dfrac{\pi}{180}\right)^c$

$= \left(\dfrac{5\pi}{7}\right)^c$

---

iv. Octagon:

Number of sides = 8

Number of exterior angles = 8

Sum of exterior angles = 360°

$\therefore$ Each exterior angle $= \dfrac{360^\circ}{\text{no. of sides}}$

$= \dfrac{360^\circ}{8}$

$= 45^\circ$

$= \left(45 \times \dfrac{\pi}{180}\right)^c = \left(\dfrac{\pi}{4}\right)^c$

Interior angle + Exterior angle = 180°

$\therefore$ Each interior angle $= 180^\circ - 45^\circ = 135^\circ$

$= \left(135 \times \dfrac{\pi}{180}\right)^c = \left(\dfrac{3\pi}{4}\right)^c$

---

14. Find the angle between hour-hand and minute-hand in a clock at [2 Marks Each]

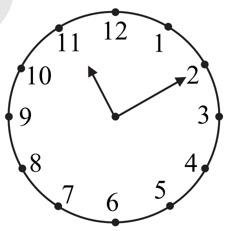
i. ten past eleven **ii. twenty past seven**

iii. thirty five past one **iv. quarter to six**

v. 2:20 **vi. 10:10**

Solution:

i. At 11:10, the minute-hand is at mark 2 and hour-hand has crossed $\left(\dfrac{1}{6}\right)^{\text{th}}$ of the angle between 11 and 12.



Angle between two consecutive marks $= \dfrac{360^\circ}{12}$

$= 30^\circ$

Angle traced by hour-hand in 10 minutes $= \dfrac{1}{6}(30^\circ)$

$= 5^\circ$

Angle between marks 11 and 2 = 3 × 30° = 90°

$\therefore$ Angle between two hands of the clock at ten past eleven = 90° – 5° = 85°

Smart Check

The angle between marks 11 and 2 is 90°.

But hour-hand has crossed 11.

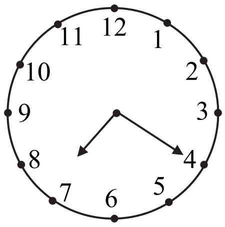
$\therefore$ Required angle will be less than 90°.

Angle made by hour-hand in one minute is $\left(\dfrac{1}{2}\right)^\circ$.

$\therefore$ In 10 minutes it makes $\left(10 \times \dfrac{1}{2}\right)^\circ = 5^\circ$

$\therefore$ Required angle = 90° – 5° = 85°

ii. At 7 : 20, the minute-hand is at mark 4 and hour-hand has crossed $\left(\frac{1}{3}\right)^{rd}$ of angle between 7 and 8.



Angle between two consecutive marks = $\frac{360°}{12}$

= 30°

Angle traced by hour-hand in 20 minutes

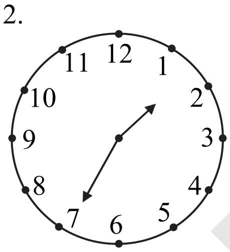
= $\frac{1}{3}$ (30°) = 10°

Angle between marks 4 and 7 = 3 × 30° = 90°

∴ Angle between two hands of the clock at twenty past seven = 90° + 10° = 100°

---

iii. At 1:35, the minute-hand is at mark 7 and hour-hand has crossed $\left(\frac{7}{12}\right)^{th}$ of the angle between 1 and 2.



Angle between two consecutive marks

= $\frac{360°}{12}$ = 30°

Angle traced by hour-hand in 35 minutes

$= \frac{7}{12}(30°) = \left(\frac{35}{2}\right)^{\circ} = \left(17\frac{1}{2}\right)^{\circ}$

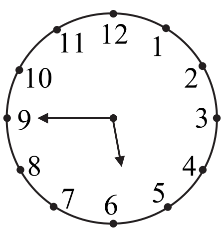
Angle between marks 1 and 7 = 6 × 30° = 180°

∴ Angle between two hands of the clock at thirty five past one = $180° - \left(17\frac{1}{2}\right)^{\circ} = \left(162\frac{1}{2}\right)^{\circ}$

$= 162° + \frac{1}{2}^{\circ} = 162°30'$

---

iv. At 5:45, the minute-hand is at mark 9 and hour-hand has crossed $\left(\frac{3}{4}\right)^{th}$ of the angle between 5 and 6.



Angle between two consecutive marks

= $\frac{360°}{12}$ = 30°

Angle traced by hour-hand in 45 minutes

$= \frac{3}{4}(30°) = (22.5)° = \left(22\frac{1}{2}\right)^{\circ}$

Angle between marks 5 and 9

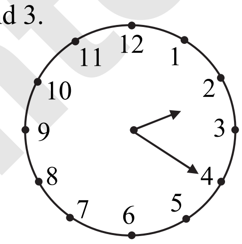
= 4 × 30° = 120°

∴ Angle between two hands of the clock at quarter to six = $120° - \left(22\frac{1}{2}\right)^{\circ}$

$= \left(97\frac{1}{2}\right)^{\circ} = 97° + \frac{1}{2}^{\circ} = 97°30'$

---

v. At 2 : 20, the minute-hand is at mark 4 and hour-hand has crossed $\left(\frac{1}{3}\right)^{rd}$ of the angle between 2 and 3.



Angle between two consecutive marks

= $\frac{360°}{12}$ = 30°

Angle traced by hour-hand in 20 minutes

= $\frac{1}{3}$ (30°) = 10°

Angle between marks 2 and 4 = 2 × 30° = 60°

∴ Angle between two hands of the clock at 2:20 = 60° – 10° = 50°

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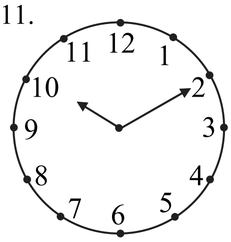
vi. At 10:10, the minute-hand is at mark 2 and hour-hand has crossed $\left(\frac{1}{6}\right)^{th}$ of the angle between 10 and 11.



Angle between two consecutive marks

= $\frac{360°}{12}$ = 30°

Angle traced by hour-hand in 10 minutes

$= \frac{1}{6}(30°) = 5°$

Angle between marks 10 and 2 = 4 × 30° = 120°

∴ Angle between two hands of the clock at 10:10 = 120° – 5° = 115°

Let's Study

Arc length and Area of a sector

Theorem:

If S is the length of an arc of a circle of radius r which subtends an angle θ^c at the centre of the circle, then S = rθ.

Proof:

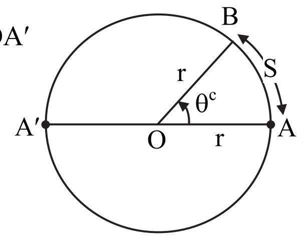
Let O be the centre and r be the radius of the circle.

Let AB be an arc of the circle with length 'S' units and m$\angle$AOB = θ^c.

Let AA′ be the diameter of the circle.

Now, l(arc AB) $\propto$ m $\angle$AOB

and l (arc ABA′) $\propto$ m $\angle$AOA′



$\therefore \quad \dfrac{l(\text{arc AB})}{l(\text{arc ABA}')} = \dfrac{\theta^c}{\pi}$

$\therefore \quad \dfrac{S}{\frac{1}{2}(\text{circumference})} = \dfrac{\theta}{\pi}$

$\therefore \quad \dfrac{S}{\pi r} = \dfrac{\theta}{\pi}$

$\therefore \quad S = r\theta$

$\therefore$ Length of an arc, S = rθ.

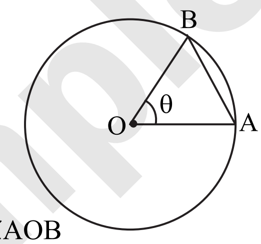
Theorem:

If θ^c is an angle between two radii of the circle of radius r, then the area of the corresponding sector is $\frac{1}{2}r^2\theta$.

Proof:

Let O be the centre and r be the radius of the circle and m $\angle$AOB = θ^c.

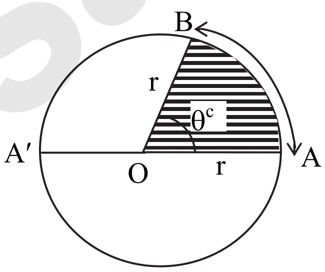
Let AA′ be the diameter of the circle.



Now, Area of sector AOB $\propto$ m $\angle$AOB

and area of sector ABA′ $\propto$ m $\angle$AOA′

$\therefore \quad \dfrac{\text{Area of sector AOB}}{\text{Area of sector ABA}'} = \dfrac{m\angle AOB}{m\angle AOA'} = \dfrac{\theta}{\pi}$



$\therefore$ Area of sector AOB = Area of sector ABA′ $\times \dfrac{\theta}{\pi}$

$= \dfrac{1}{2}(\pi r^2) \times \dfrac{\theta}{\pi}$

$\therefore$ Area of sector AOB = $\dfrac{1}{2}r^2\,\theta$.

Note:

The above theorems are not asked in examination but are provided just for reference.

Remember This

$$A(\text{sector}) = \frac{1}{2} \times r^2 \times \theta = \frac{1}{2} \times r \times r\theta$$
$$= \frac{1}{2} \times r \times S$$

Exercise 1.2

1. Find the length of an arc of a circle which subtends an angle of 108° at the centre, if the radius of the circle is 15 cm. [1 Mark]

Solution:

Here, r = 15cm and

$\theta = 108° = \left(108 \times \dfrac{\pi}{180}\right)^c = \left(\dfrac{3\pi}{5}\right)^c$

Since S = r.θ,

S = 15 $\times \dfrac{3\pi}{5}$ = 9π cm.

---

2. The radius of a circle is 9 cm. Find the length of an arc of this circle which cuts off a chord of length equal to length of radius. [2 Marks]

Solution:

Here, r = 9cm

Let the arc AB cut off a chord equal to the radius of the circle.

Since OA = OB = AB,

Δ OAB is an equilateral triangle.

$\therefore$ m$\angle$AOB = 60°

$\therefore$ θ = 60°

$= \left(60 \times \dfrac{\pi}{180}\right)^c = \left(\dfrac{\pi}{3}\right)^c$

Since S = r.θ,

S = 9 $\times \dfrac{\pi}{3}$ = 3π cm.

---

3. Find the angle in degree subtended at the centre of a circle by an arc whose length is 15 cm, if the radius of the circle is 25 cm. [1 Mark]

Solution:

Here, r = 25 cm and S = 15 cm

Since S = r.θ,

15 = 25 $\times$ θ

$\therefore \quad \theta = \left(\dfrac{15}{25}\right)^c$

$\therefore$ $\theta = \left(\frac{3}{5}\right)^c = \left(\frac{3}{5} \times \frac{180}{\pi}\right)^o$

$= \left(\frac{108}{\pi}\right)^o = \left(\frac{108}{3.14}\right)^o$ ...[$\because \pi = 3.14$]

$= (34.40)°$ (approx.)

$\therefore$ The required angle in degree is $\left(\frac{108}{\pi}\right)^o$ or $(34.40)°$(approx.).

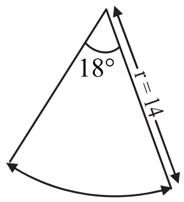
---

4. A pendulum of length 14 cm oscillates through an angle of 18°. Find the length of its path. [1 Mark]

Solution:

Here, r = 14 cm and

$\theta = 18° = \left(18 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{10}\right)^c$



Since $S = r.\theta$,

$S = 14 \times \frac{\pi}{10}$

$\therefore$ $S = \frac{7\pi}{5} = \frac{7(3.14)}{5}$ ...[$\because \pi = 3.14$]

$= \frac{21.98}{5}$

$= 4.4$ cm. (approx.)

---

5. Two arcs of the same length subtend angles of 60° and 75° at the centres of the two circles. What is the ratio of radii of two circles? [3 Marks]

Solution:

Let r_1 and r_2 be the radii of the two circles and let their arcs of same length S subtend angles of 60° and 75° at their centres.

Angle subtended at the centre of the first circle,

$\theta_1 = 60° = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$

$\therefore$ $S = r_1\theta_1 = r_1\left(\frac{\pi}{3}\right)$ …(i)

Angle subtended at the centre of the second circle,

$\theta_2 = 75° = \left(75 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{12}\right)^c$

$\therefore$ $S = r_2\theta_2 = r_2\left(\frac{5\pi}{12}\right)$ …(ii)

From (i) and (ii), we get

$r_1\left(\frac{\pi}{3}\right) = r_2\left(\frac{5\pi}{12}\right)$

$\therefore$ $\frac{r_1}{r_2} = \frac{15}{12}$

$\therefore$ $\frac{r_1}{r_2} = \frac{5}{4}$

$\therefore$ $r_1 : r_2 = 5 : 4$.

---

6. The area of the circle is 25π sq.cm. Find the length of its arc subtending an angle of 144° at the centre. Also find the area of the corresponding sector. [3 Marks]

Solution:

Area of circle $= \pi r^2$

But area is given to be 25 π sq.cm

$\therefore$ $25\pi = \pi r^2$

$\therefore$ $r^2 = 25$

$\therefore$ $r = 5$ cm

$\theta = 144° = \left(144 \times \frac{\pi}{180}\right)^c = \left(\frac{4\pi}{5}\right)^c$

Since $S = r\theta$,

$S = 5\left(\frac{4\pi}{5}\right) = 4\pi$ cm.

Also, A(sector) $= \frac{1}{2} \times r \times S$

$= \frac{1}{2} \times 5 \times 4\pi$

$= 10\pi$ sq.cm.

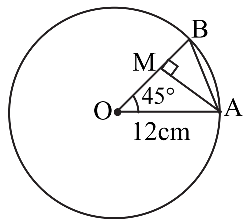
---

7. OAB is a sector of the circle having centre at O and radius 12 cm. If m∠AOB = 45°, find the difference between the area of sector OAB and ΔAOB. [3 Marks]

Solution:

Here, r = 12 cm

$\theta = 45° = \left(45 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{4}\right)^c$



Draw AM ⊥ OB

In ΔOAM,

$\sin 45° = \frac{AM}{12}$

$\therefore$ $\frac{1}{\sqrt{2}} = \frac{AM}{12}$

$\therefore$ $AM = \frac{12}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 6\sqrt{2}$ cm

$\therefore$ A (sector OAB) − A(ΔAOB)

$= \frac{1}{2}r^2\theta - \frac{1}{2} \times OB \times AM$

$= \frac{1}{2} \times (12)^2 \times \frac{\pi}{4} - \frac{1}{2} \times 12 \times 6\sqrt{2}$

$= \frac{1}{2} \times 144 \times \frac{\pi}{4} - 36\sqrt{2}$

$= 18\pi - 36\sqrt{2}$

$= 18(\pi - 2\sqrt{2})$ sq.cm.

8. OPQ is the sector of a circle having centre at O and radius 15 cm. If m∠POQ = 30°, find the area enclosed by arc PQ and chord PQ. **[4 Marks]**

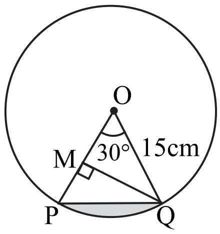
Solution:

Here, r = 15 cm

m∠POQ = 30°

$= \left(30 \times \frac{\pi}{180}\right)^c$

∴ $\theta = \left(\frac{\pi}{6}\right)^c$



Draw QM ⊥ OP

In ΔOQM,

$\sin 30° = \frac{QM}{15}$

∴ $\frac{1}{2} = \frac{QM}{15}$

∴ $QM = 15 \times \frac{1}{2} = \frac{15}{2}$

Shaded portion indicates the area enclosed by arc PQ and chord PQ.

∴ A(shaded portion)

= A(sector OPQ) – A(ΔOPQ)

$= \frac{1}{2}r^2\theta - \frac{1}{2} \times OP \times QM$

$= \frac{1}{2} \times (15)^2 \times \frac{\pi}{6} - \frac{1}{2} \times 15 \times \frac{15}{2}$

$= \frac{225\pi}{12} - \frac{225}{4} = \frac{225}{4}\left(\frac{\pi}{3} - 1\right)$ sq.cm.

9. The perimeter of a sector of the circle of area 25π sq.cm is 20 cm. Find the area of sector. **[3 Marks]**

Solution:

Area of circle = πr^2

But area is given to be 25π sq.cm.

∴ $25\pi = \pi r^2$

∴ $r^2 = 25$

∴ r = 5 cm

Perimeter of sector = 2r + S

But perimeter is given to be 20 cm.

∴ 20 = 2(5) + S

∴ 20 = 10 + S

∴ S = 10 cm

Area of sector $= \frac{1}{2} \times r \times S$

$= \frac{1}{2} \times 5 \times 10 = 25$ sq.cm.

10. The perimeter of a sector of the circle of area 64 π sq.cm is 56 cm. Find the area of the sector. **[3 Marks]**

Solution:

Area of circle = πr^2

But area is given to be 64 π sq.cm.

∴ $64\pi = \pi r^2$

∴ $r^2 = 64$

∴ r = 8 cm

Perimeter of sector = 2r + S

But perimeter is given to be 56 cm.

∴ 56 = 2(8) + S

∴ 56 = 16 + S

∴ S = 40 cm

Area of sector $= \frac{1}{2} \times r \times S$

$= \frac{1}{2} \times 8 \times 40 = 160$ sq.cm.

Miscellaneous Exercise – 1

I. Select the correct option from the given alternatives. **[2 Marks Each]**

1. $\left(\frac{22\pi}{15}\right)^c$ is equal to

(A) 246° (B) 264°
(C) 224° (D) 426°

2. 156° is equal to

(A) $\left(\frac{17\pi}{15}\right)^c$ (B) $\left(\frac{13\pi}{15}\right)^c$
(C) $\left(\frac{11\pi}{15}\right)^c$ (D) $\left(\frac{7\pi}{15}\right)^c$

3. A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 metres when it traces the angle of 72° at the centre, then the length of the rope is

(A) 70 m (B) 55 m
(C) 40 m (D) 35 m

4. If a 14 cm long pendulum oscillates through an angle of 12°, then find the length of its path

(A) $\frac{13\pi}{14}$ (B) $\frac{14\pi}{13}$ (C) $\frac{15\pi}{14}$ (D) $\frac{14\pi}{15}$

5. Angle between hands of a clock when it shows the time 9 : 45 is

(A) (7.5)° (B) (12.5)°
(C) (17.5)° (D) (22.5)°

6. 20 metres of wire is available for fencing off a flower-bed in the form of a circular sector of radius 5 metres, then the maximum area (in sq. m.) of the flower-bed is

(A) 15 (B) 20 (C) 25 (D) 30

7. If the angles of a triangle are in the ratio 1:2:3, then the smallest angle in radian is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{9}$

8. A semicircle is divided into two sectors whose angles are in the ratio 4:5. Find the ratio of their areas?
(A) 5:1 (B) 4:5
(C) 5:4 (D) 3:4

9. Find the measure of the angle between hour-hand and the minute hand of a clock at twenty minutes past two.
(A) 50° (B) 60°
(C) 54° (D) 65°

10. The central angle of a sector of circle of area 9π sq.cm is 60°, the perimeter of the sector is
(A) π (B) 3 + π
(C) 6 + π (D) 6

Answers:

1.	(B)	2.	(B)	3.	(A)	4.	(D)
5.	(D)	6.	(C)	7.	(B)	8.	(B)
9.	(A)	10.	(C)				

Hints:

3. $\theta = 72° = \left(72 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{5}\right)^c$

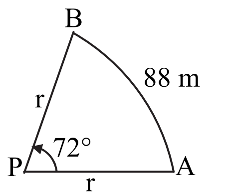
$S = 88$ m

$S = r\theta$

$\therefore$ $88 = r\left(\frac{2\pi}{5}\right)$

$\therefore$ $r = 88 \times \frac{5}{2\pi}$

$= 88 \times \frac{5}{2\left(\frac{22}{7}\right)} = 70$ m

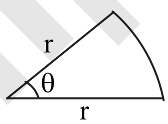


6. $r + r + r\theta = 20$m

$\therefore$ $2r + r\theta = 20$

$\therefore$ $\theta = \frac{20 - 2r}{r}$

$r = 5$m



Area $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}r^2\left(\frac{20 - 2r}{r}\right)$

$= \frac{1}{2}(5)^2\left(\frac{20 - 10}{5}\right) = 25$ sq. m

II. Answer the following.

1. Find the number of sides of a regular polygon, if each of its interior angles is $\frac{3\pi^c}{4}$. **[2 Marks]**

Solution:

Each interior angle of a regular polygon

$= \frac{3\pi^c}{4} = \left(\frac{3\pi}{4} \times \frac{180}{\pi}\right)^\circ = 135°$

Interior angle + Exterior angle = 180°

$\therefore$ Exterior angle = 180° − 135° = 45°

Let the number of sides of the regular polygon be n.

But in a regular polygon,

exterior angle $= \frac{360°}{\text{no.of sides}}$

$\therefore$ $45° = \frac{360°}{n}$

$\therefore$ $n = \frac{360°}{45°} = 8$

$\therefore$ Number of sides of a regular polygon = 8.

2. Two circles each of radius 7 cm, intersect each other. The distance between their centres is $7\sqrt{2}$ cm. Find the area of the portion common to both the circles. **[4 Marks]**

Solution:

Let O and O_1 be the centres of two circles intersecting each other at A and B.

Then $OA = OB = O_1A = O_1B = 7$ cm

and $OO_1 = 7\sqrt{2}$ cm

$\therefore$ $OO_1^2 = 98$ …(i)

Since $OA^2 + O_1A^2 = 7^2 + 7^2 = 98$

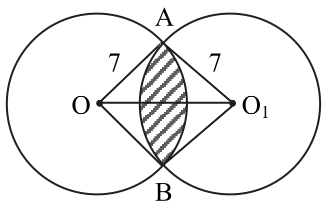
$= OO_1^2$ …[From (i)]

$\therefore$ $m\angle OAO_1 = 90°$

$\therefore$ □ OAO_1B is a square.

$m\angle AOB = m\angle AO_1B = 90°$

$= \left(90 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{2}\right)^c$



Now, A(sector OAB) $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 7^2 \times \frac{\pi}{2} = \frac{49\pi}{4}$ sq.cm

and A(sector O_1AB) $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 7^2 \times \frac{\pi}{2} = \frac{49\pi}{4}$ sq.cm

A(□ OAO_1B) = $(\text{side})^2 = (7)^2 = 49$ sq.cm

$\therefore$ Required area = area of shaded portion

= A(sector OAB) + A(sector O_1AB) − A(□ OAO_1B)

$= \frac{49\pi}{4} + \frac{49\pi}{4} - 49$

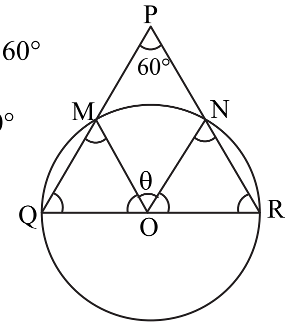
$= \frac{49\pi}{2} - 49 = 49\left(\frac{\pi}{2} - 1\right)$ sq.cm

3. ΔPQR is an equilateral triangle with side 18 cm. A circle is drawn on segment QR as diameter. Find the length of the arc of this circle within the triangle. [3 Marks]

Solution:

Let 'O' be the centre of the circle drawn on QR as a diameter.

Let the circle intersect seg PQ and seg PR at points M and N respectively.



Since $l(OQ) = l(OM)$,

$m\angle OMQ = m\angle OQM = 60°$

$\therefore$ $m\angle MOQ = 60°$

Similarly, $m\angle NOR = 60°$

Given, QR = 18 cm.

$\therefore$ r = 9 cm

$\therefore$ $\theta = 60° = \left(60 \times \frac{\pi}{180}\right)^c$

$= \left(\frac{\pi}{3}\right)^c$

$\therefore$ $l(\text{arc MN}) = S = r\theta = 9 \times \frac{\pi}{3} = 3\pi$ cm.

---

4. Find the radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm. [2 Marks]

Solution:

Let S be the length of the arc and r be the radius of the circle.

$\theta = 60° = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$

S = 37.4 cm

Since $S = r\theta$,

$37.4 = r \times \frac{\pi}{3}$

$\therefore$ $3 \times 37.4 = r \times \frac{22}{7}$ ... $\left[\because \pi = \frac{22}{7}\right]$

$\therefore$ $r = \frac{3 \times 37.4 \times 7}{22}$

$\therefore$ r = 35.7 cm

---

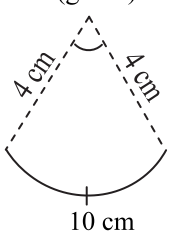
5. A wire of length 10 cm is bent so as to form an arc of a circle of radius 4 cm. What is the angle subtended at the centre in degrees? [2 Marks]

Solution:

S = 10 cm and r = 4 cm ...(given)

Since $S = r\theta$,

$10 = 4 \times \theta$



$\therefore$ $\theta = \left(\frac{5}{2}\right)^c = \left(\frac{5}{2} \times \frac{180}{\pi}\right)^\circ$

$= \left(\frac{450}{\pi}\right)^\circ$

6. If two arcs of the same length in two circles subtend angles 65° and 110° at the centre. Find the ratio of their radii. [3 Marks]

Solution:

Let r_1 and r_2 be the radii of the two circles and let their arcs of same length S subtend angles of 65° and 110° at their centres.

Angle subtended at the centre of the first circle,

$\theta_1 = 65° = \left(65 \times \frac{\pi}{180}\right)^c = \left(\frac{13\pi}{36}\right)^c$

$\therefore$ $S = r_1\theta_1 = r_1\left(\frac{13\pi}{36}\right)$ ...(i)

Angle subtended at the centre of the second circle,

$\theta_2 = 110° = \left(110 \times \frac{\pi}{180}\right)^c = \left(\frac{11\pi}{18}\right)^c$

$\therefore$ $S = r_2\theta_2 = r_2\left(\frac{11\pi}{18}\right)$ ...(ii)

From (i) and (ii), we get

$r_1\left(\frac{13\pi}{36}\right) = r_2\left(\frac{11\pi}{18}\right)$

$\therefore$ $\frac{r_1}{r_2} = \frac{22}{13}$

$\therefore$ $r_1 : r_2 = 22 : 13$

---

7. The area of a circle is 81π sq.cm. Find the length of the arc subtending an angle of 300° at the centre and also the area of corresponding sector. [3 Marks]

Solution:

Area of circle = πr^2

But area is given to be 81π sq.cm

$\therefore$ $\pi r^2 = 81\pi$

$\therefore$ $r^2 = 81$

$\therefore$ r = 9 cm

$\theta = 300° = \left(300 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{3}\right)^c$

Since $S = r\theta$,

$S = 9 \times \frac{5\pi}{3} = 15\pi$ cm

Area of sector $= \frac{1}{2} \times r \times S$

$= \frac{1}{2} \times 9 \times 15\pi = \frac{135\pi}{2}$ sq.cm

---

8. Show that minute-hand of a clock gains 5° 30′ on the hour-hand in one minute. [2 Marks]

Solution:

Angle made by hour-hand in one minute

$= \frac{360°}{12 \times 60} = \left(\frac{1}{2}\right)^\circ$

Angle made by minute-hand in one minute

$= \frac{360°}{60} = 6°$

$\therefore$ Gain by minute-hand on the hour-hand in one minute

$= 6° - \left(\frac{1}{2}\right)^{o} = \left(5\frac{1}{2}\right)^{o} = 5° 30'$

9. A train is running on a circular track of radius 1 km at the rate of 36 km per hour. Find the angle to the nearest minute, through which it will turn in 30 seconds. [3 Marks]

Solution:

r = 1km = 1000m

l(Arc covered by train in 30 seconds)

$= 30 \times \frac{36000}{60 \times 60}$ m

$\therefore$ S = 300 m

Since S = rθ,

300 = 1000 × θ

$\therefore \theta = \left(\frac{3}{10}\right)^{c} = \left(\frac{3}{10} \times \frac{180}{\pi}\right)^{\circ}$

$= \left(\frac{54}{\pi}\right)^{\circ}$

$= \left(\frac{54 \times 7}{22}\right)^{\circ}$ ...$\left[\because \pi = \frac{22}{7}\right]$

= (17.18)°

= 17° + (0.18)°

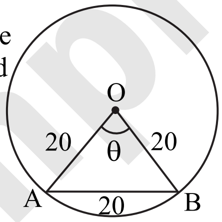
= 17° + (0.18 × 60)′ = 17° + (10.8)′

$\therefore$ θ = 17°11′ (approx.)

10. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord. [2 Marks]

Solution:

Let 'O' be the centre of the circle and AB be the chord of the circle.



Here, d = 40 cm

$\therefore r = \frac{40}{2} = 20$ cm

Since OA = OB = AB,

ΔOAB is an equilateral triangle.

$\therefore$ The angle subtended at the centre by the minor arc AOB is $\theta = 60° = \left(60 \times \frac{\pi}{180}\right)^{c} = \left(\frac{\pi}{3}\right)^{c}$

$\therefore$ l (minor arc of chord AB) $= r\theta = 20 \times \frac{\pi}{3}$

$= \frac{20\pi}{3}$ cm.

11. The angles of a quadrilateral are in A.P. and the greatest angle is double the least. Find angles of the quadrilateral in radians. [4 Marks]

Solution:

Let the measures of the angles of the quadrilateral in degrees be

a – 3d, a – d, a + d, a + 3d, where a > d > 0

$\therefore$ (a – 3d) + (a – d) + (a + d) + (a + 3d) = 360°

…[Sum of the angles of a quadrilateral is 360°]

$\therefore$ 4a = 360°

$\therefore$ a = 90°

According to the given condition, the greatest angle is double the least.

$\therefore$ a + 3d = 2.(a – 3d)

$\therefore$ 90° + 3d = 2.(90° – 3d)

$\therefore$ 90° + 3d = 180° – 6d

$\therefore$ 9d = 90°

$\therefore$ d = 10°

$\therefore$ The measures of the angles in degrees are

a – 3d = 90° – 3(10°) = 90° – 30° = 60°,

a – d = 90° – 10° = 80°,

a + d = 90° + 10° = 100°,

a + 3d = 90° + 3(10°) = 90° + 30° = 120°

We know that $\theta° = \left(\theta \times \frac{\pi}{180}\right)^{c}$

$\therefore$ The measures of the angles in radians are

$60° = \left(60 \times \frac{\pi}{180}\right)^{c} = \left(\frac{\pi}{3}\right)^{c}$

$80° = \left(80 \times \frac{\pi}{180}\right)^{c} = \left(\frac{4\pi}{9}\right)^{c}$

$100° = \left(100 \times \frac{\pi}{180}\right)^{c} = \left(\frac{5\pi}{9}\right)^{c}$

$120° = \left(120 \times \frac{\pi}{180}\right)^{c} = \left(\frac{2\pi}{3}\right)^{c}$

One Mark Questions

1. Find the degree measure of an angle traced by the hour-hand in 15 minutes.
2. Check whether the given pair of angles is co-terminal or not.
   45° and – 315°
3. Find the length of an arc of a circle of radius r cm which subtends an angle θ^{c} at the centre of the circle.
4. Determine the quadrant of angle 1105°.
5. Express the angle (0.13)° in seconds.

Multiple Choice Questions

1. The angle subtended at the centre of a circle of radius 3 metres by an arc of length 1 metre is equal to
   (A) 20° (B) 60°
   (C) $\frac{1}{3}$ radian (D) 3 radians

2. A wire that can cover a circle of radius 7 cm is bent again into an arc of a circle of radius 12 cm. The angle subtended by the arc at the centre is
(A) 50° (B) 210° (C) 100° (D) 60°

3. The radius of the circle whose arc of length 15 cm makes an angle of $\frac{3}{4}$ radian at the centre is
(A) 10 cm (B) 20 cm
(C) $11\frac{1}{4}$cm (D) $22\frac{1}{2}$cm

4. $\frac{4\pi^c}{5} =$
(A) 144° (B) 60°
(C) 120° (D) 135°

5. $\frac{8\pi^c}{3} =$
(A) 144° (B) 80°
(C) 480° (D) 180°

6. 36° =
(A) $\frac{\pi^c}{6}$ (B) $\frac{\pi^c}{5}$ (C) $\frac{\pi^c}{3}$ (D) $\frac{\pi^c}{2}$

7. −520° =
(A) $\frac{24}{9}\pi^c$ (B) $\frac{25}{9}\pi^c$
(C) $\frac{23}{9}\pi^c$ (D) $\frac{-26}{9}\pi^c$

8. The angles of a triangle are in A. P. such that greatest is 5 times the least. The angles in degrees are
(A) 30°, 60°, 100° (B) 30°, 45°, 90°
(C) 20°, 45°, 180° (D) 20°, 60°, 100°

9. The angles of a quadrilateral are in the ratio 2 : 3 : 3 : 4. Then the least angle in degrees is
(A) 90° (B) 45° (C) 30° (D) 60°

10. The angles of a triangle are in the ratio 3 : 7 : 8. Then the greatest angle in radians is
(A) $\frac{4\pi^c}{9}$ (B) $\frac{5\pi^c}{9}$ (C) $\frac{7\pi^c}{18}$ (D) $\frac{\pi^c}{6}$

11. The difference between two acute angles of a right angled triangle is $\frac{\pi}{9}$. Then the angles in degrees are
(A) 30°, 35° (B) 45°, 55°
(C) 55°, 35° (D) 60°, 75°

12. Angle between the hour hand and minute hand of a clock at quarter past eleven in degrees is
(A) $\left(\frac{15\pi}{24}\right)^c$ (B) 112°30′
(C) 107°73″ (D) $\left(\frac{2\pi}{3}\right)^c$

13. The interior angle of a regular polygon of 15 sides in radians is
(A) $\frac{13\pi^c}{15}$ (B) $\frac{9\pi^c}{20}$
(C) 156° (D) 135°

14. The length of arc of a circle of radius 9 cm subtending an angle of 40° at the centre is
(A) 2π cm (B) 12π cm
(C) $\frac{2\pi}{9}$ cm (D) $\frac{4\pi}{5}$ cm

15. OA and OB are two radii of a circle of radius 10 such that m∠AOB = 144°. Then area of the sector AOB is
(A) 8π sq.cm. (B) 20π sq.cm.
(C) 30π sq.cm. (D) 40π sq.cm.

16. The perimeter of a sector of a circle of area 36π sq. cm is 24 cm. Then the area of sector is
(A) 40 sq.cm. (B) 36 sq.cm.
(C) 46 sq.cm. (D) 26 sq.cm.

17. A semicircle is divided into two sectors, whose angles are in the ratio 1 : 2. Then the ratio of their areas is
(A) 1:3 (B) 1:4
(C) 2:3 (D) 1:2

18. If θ^c is the angle between two radii of a circle of radius r, then the area of corresponding sector is
(A) $r^2\theta$ (B) $\frac{1}{2}r^2\theta$ (C) $r\theta$ (D) $2\pi r$

19. A wire 121 cm. long is bent so as to lie along the arc of a circle of 180 cm radius. The angle subtended at the centre of the arc in degrees is
(A) 35° 37′ (B) 36° 30′
(C) 37° 30′ (D) 38° 30′

Answers

One Mark Questions

1. (7.5)°
2. Co-terminal
3. rθ cm
4. I quadrant
5. 468″

Multiple Choice Questions

1.	(C)	2.	(B)	3.	(B)	4.	(A)
5.	(C)	6.	(B)	7.	(D)	8.	(D)
9.	(D)	10.	(A)	11.	(C)	12.	(B)
13.	(A)	14.	(A)	15.	(D)	16.	(B)
17.	(D)	18.	(B)	19.	(D)		

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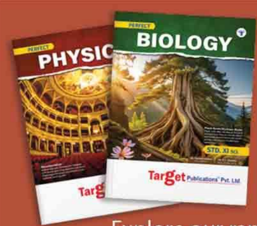

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