SAMPLE CONTENT

SMART NOTES

Based on New Paper Pattern
and Latest Textbook

Std. XII

Mathematics and Statistics
Commerce (Part - 1)

The battle rope exercise in cross fit training is a representation of the mathematical concept of maxima and minima.

Target Publications® Pvt. Ltd.

Written as per the latest textbook prescribed by the Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune.

SMART NOTES

MATHEMATICS & STATISTICS Part - I

Std. XII Commerce

Salient Features

- Written as per the new textbook
- Exhaustive coverage of entire syllabus
- Topic-wise distribution of textual questions at the start of every chapter.
- Precise theory for every topic
- Covers answers to all exercises and miscellaneous exercises given in the textbook.
- Includes MCQs and additional problems for practice
- 'Smart Recap' at the end of the book
- Activity Based Questions covered in every chapter
- Topic Test at the end of each chapter for self-assessment
- Includes **Q.R. code** for students to access the 'Solutions' of the Topic tests.
- Includes Board Question Paper of March 2022 (Solution in pdf format through QR code)

Scan the adjacent QR Code to download Quill-The Padhai App.
Enter Product Key PK-HSCEMMATHS12022 in Quill App to get 30 days Free Access of the e-book version of this book & experience digital interactive learning.

Printed at: **Print to Print,** Mumbai

Balbharati Registration No.: 2018MH0022

TEID: 911

P.O. No. 5121

PREFACE

Mathematics & Statistics Part – I 'Smart Notes' is intended for every Maharashtra State Board aspirant of Std. XII, Commerce. The scope, sequence, and level of the book are designed to match the new textbook issued by the Maharashtra State Board.

At this crucial juncture in their lives, when the students are grappling with the pressures of cracking a career-defining board examination, we wanted to create a book that not only develops the necessary knowledge, tools and skills required to excel in the examination, but also enables students to appreciate the beauty of the subject and piques their curiosity.

We believe that students respond favourably to meaningful content, if it is presented in a way that is easy to read and understand, rather than being mired down with facts and information. Consequently, we have always placed the highest priority on writing clear and lucid explanations of fundamental concepts. Moreover, special care has been taken to ensure that the topics are presented in a logical order.

The primary purpose of this book is to assist the students in preparing for the board examination. However, this is closely linked to other goals: to exemplify how important and how incredibly interesting mathematics is, and to help the student become an expert thinker and problem solver.

Practice, practice & more practice is the key to score high in mathematics!

To help the students, this book amalgamates problems that are rich in both variety and number which provides the student with ample practice, ensuring mastery of each concept.

In addition, the chapter-test have been carefully crafted to focus on concepts, thus providing the students with a quick opportunity for self-assessment and giving them an increased appreciation of chapter-preparedness.

*Our **Mathematics & Statistics Part – I 'Smart Notes'** adheres to our vision and achieves several goals: **building concepts**, **developing competence to solve problems, recapitulation, self-study, self-assessment** and **student engagement** — all while encouraging students toward cognitive thinking.*

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.
Please write to us on: mail@targetpublications.org

A book affects eternity; one can never tell where its influence stops.

Best of luck to all the aspirants!

From,
Publisher

Edition: Third

KEY FEATURES

Topic-wise distribution of questions

In this section we have provided topic-wise distribution of the textual questions as well as practice questions which will help students to get an idea about the types types of the questions.

Illustrative Example

Illustrative Example provides a detailed approach towards solving a problem.

Multiple Choice Questions

In this section we have provided multiple choice questions for practice which will help students to test their concepts.

Additional Problems for Practice

In this section, we have provided additional problems which will help students to test their understanding of the chapter.

Activities for Practice

In this section we have provided multiple activities for practice which will help students understand the concepts.

Topic Test

Topic Test covers questions from the chapter for self-evaluation purpose.
This is our attempt to provide the students with revision and help them assess their knowledge of chapter.

Smart Recap

Smart Recap given at the end of the book includes important and relevant concepts and formulae in the chapters.
This is our attempt to offer students a handy tool to solve problems and ace the last minute revision.

QR Codes

QR codes provide:

i. The solutions of the Topic tests

ii. Solution to board question paper of March 2022

CONTENTS

7 Applications of Definite Integration

Type of Problems	Exercise	Q. Nos.
Area of region bounded by Curves, axes and the given lines	7.1	Q.1 to 4
	Practice Problems (Based on Exercise 7.1)	Q.1 to 5
	Miscellaneous Exercise 7	Q.IV (1, 3, 4, 5, 6, 7)
	Practice Problems (Based on Miscellaneous Exercise 7)	Q.1, 3, 4, 5, 6, 7
Area between two curves	Miscellaneous Exercise 7	Q.IV (2)
	Practice Problems (Based on Miscellaneous Exercise 7)	Q.2

Syllabus

- Area under the curve

Let's Study

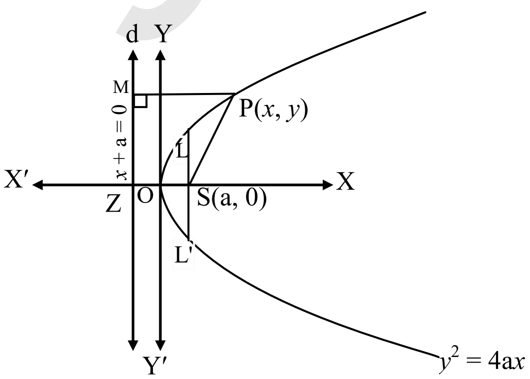
Introduction

Definite integration has a large number of applications in Science, Engineering, and various fields. It involves the geometrical applications of definite integrals, particularly in finding area under the curve.

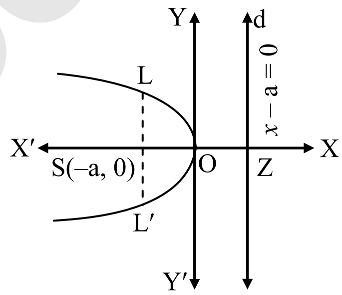
To find the area, we first draw the sketch (if possible) of the curve which encloses the region. For evaluation of area bounded by the given curves, we need to know the nature of the curves and their graphs. The shapes of different types of curves are as follows:

Standard forms of parabola and their shapes:

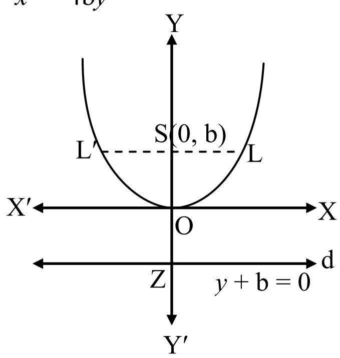
1. $y^2 = 4ax$



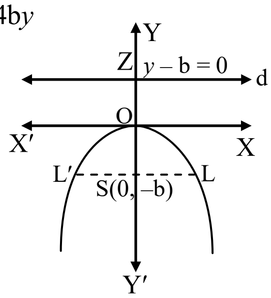
2. $y^2 = -4ax$



3. $x^2 = 4by$

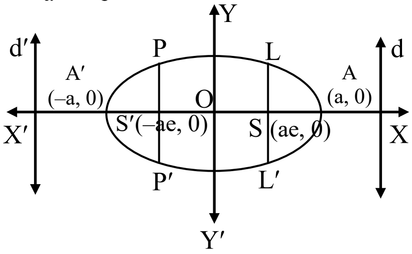


4. $x^2 = -4by$

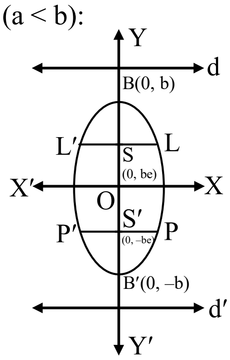


Standard forms of ellipse and their shapes:

1. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (a > b):



2. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (a < b):

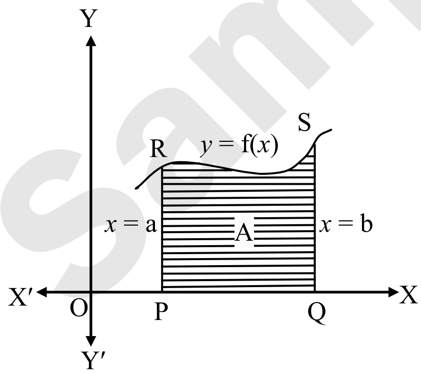


Area under the curve

i. The area 'A' bounded by the curve $y = f(x)$, X-axis and bounded between the lines $x = a$ and $x = b$ (see the figure below) is given by

A = Area of the region PRSQ

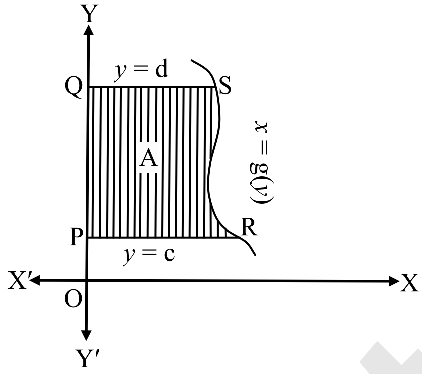
$\therefore$ $A = \int_a^b y\,dx = \int_{x=a}^{x=b} f(x)\,dx$



ii. The area 'A' bounded by the curve $x = g(y)$, Y-axis and bounded between the lines $y = c$ and $y = d$ (see the figure below) is given by

A = Area of the region PRSQ

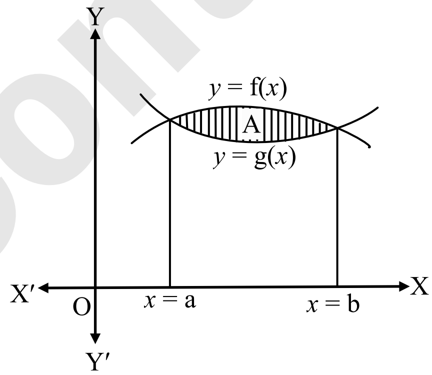
$\therefore$ $A = \int_c^d x\,dy = \int_{y=c}^{y=d} g(y)\,dy$



iii. The area of the shaded region bounded by two curves $y = f(x)$ and $y = g(x)$ as shown in the figure is given by

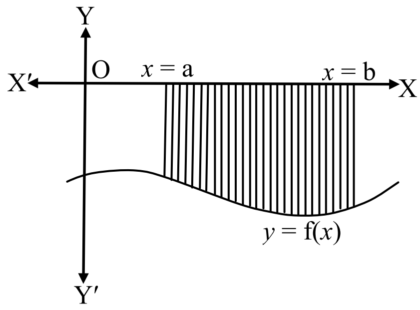
$$A = \left|\int_{x=a}^{x=b} f(x)\,dx - \int_{x=a}^{x=b} g(x)\,dx\right|$$

where the curves $y = f(x)$ and $y = g(x)$ intersect at the points [a, f(a)] and [b, f(b)].



Remarks:

i. If the area of the curve which lies below the X-axis and bounded by the lines $x = a$, $x = b$ is negative, then in such a case, we consider the absolute value.

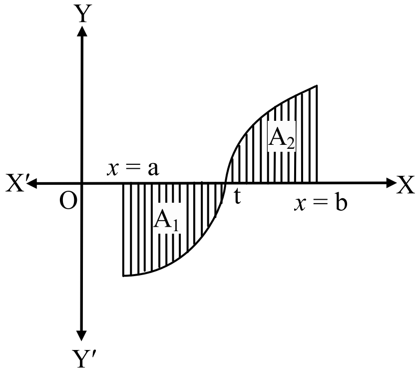


$\therefore$ required area = $\left|\int_{x=a}^{x=b} f(x)\,dx\right|$

ii. The area of the portion lying above the X-axis is positive.

iii. The curve which lies above as well as below the X-axis is as shown in the figure. Area $A_1 < 0$ and $A_2 > 0$, then total area is given by

$$A = |A_1| + A_2 = \left|\int_a^t f(x)\,dx\right| + \int_t^b f(x)\,dx$$



Exercise 7.1

1. Find the area of the region bounded by the following curves, the X-axis and the given lines:

i. $y = x^4, x = 1, x = 5$ **[Mar 19]**

ii. $y = \sqrt{6x+4}, x = 0, x = 2$

iii. $y = \sqrt{16-x^2}, x = 0, x = 4$

iv. $2y = 5x + 7, x = 2, x = 8$

v. $2y + x = 8, x = 2, x = 4$ **[Mar 16]**

vi. $y = x^2 + 1, x = 0, x = 3$

vii. $y = 2 - x^2, x = -1, x = 1$

Solution:

i. Let A be the required area.

Consider the equation $y = x^4$.

$\therefore$ $A = \int_1^5 y\,dx = \int_1^5 x^4 dx = \left[\frac{x^5}{5}\right]_1^5 = \frac{1}{5}\left[x^5\right]_1^5$

$= \frac{1}{5}[(5)^5 - (1)^5]$

$= \frac{1}{5}(3125 - 1)$

$\therefore$ $A = \frac{3124}{5}$ sq. units.

---

ii. Let A be the required area.

Consider the equation $y = \sqrt{6x+4}$.

$\therefore$ $A = \int_0^2 y\,dx = \int_0^2 \sqrt{6x+4}\,dx$

$= \int_0^2 (6x+4)^{\frac{1}{2}}\,dx$

$= \left[\frac{(6x+4)^{\frac{3}{2}}}{\frac{3}{2}\times 6}\right]_0^2 = \frac{1}{9}\left[(6x+4)^{\frac{3}{2}}\right]_0^2$

$= \frac{1}{9}\left[(6\times 2+4)^{\frac{3}{2}} - (6\times 0+4)^{\frac{3}{2}}\right]$

$= \frac{1}{9}\left[(16)^{\frac{3}{2}} - (4)^{\frac{3}{2}}\right]$

$= \frac{1}{9}\left[(4^2)^{\frac{3}{2}} - (2^2)^{\frac{3}{2}}\right]$

$= \frac{1}{9}[(4)^3 - (2)^3]$

$= \frac{1}{9}(64 - 8)$

$\therefore$ $A = \frac{56}{9}$ sq. units.

[**Note:** Answer given in the textbook is $\frac{56}{3}$ sq. units. However, as per our calculation it is $\frac{56}{9}$ sq .units.]

---

iii. Let A be the required area.

Consider the equation $y = \sqrt{16-x^2}$.

$\therefore$ $A = \int_0^4 y\,dx = \int_0^4 \sqrt{16-x^2}\,dx$

$= \int_0^4 \sqrt{(4)^2-(x)^2}\,dx$

$= \left[\frac{x}{2}\sqrt{(4)^2-x^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$

$= \left[\frac{4}{2}\sqrt{16-(4)^2} + \frac{16}{2}\sin^{-1}\left(\frac{4}{4}\right)\right] - \left[\frac{0}{2}\sqrt{16-(0)^2} + \frac{16}{2}\sin^{-1}\left(\frac{0}{4}\right)\right]$

$= [2(0) + 8\sin^{-1}(1)] - [0 + 0]$

$= 8 \times \frac{\pi}{2}$

$\therefore$ $A = 4\pi$ sq. units.

---

iv. Let A be the required area.

Consider the equation $2y = 5x + 7$.

i.e., $y = \frac{5x+7}{2}$

$\therefore$ $A = \int_2^8 y\,dx = \int_2^8 \frac{5x+7}{2}\,dx = \frac{1}{2}\int_2^8 (5x+7)\,dx$

$= \frac{1}{2}\left[\frac{5x^2}{2} + 7x\right]_2^8$

$= \frac{1}{2}\left[\left(\frac{5\times 8^2}{2} + 7\times 8\right) - \left(\frac{5\times 2^2}{2} + 7\times 2\right)\right]$

$= \frac{1}{2}\left[(160+56) - (10+14)\right]$

$= \frac{1}{2}(216 - 24)$

$= \frac{1}{2}\times 192$

$\therefore$ $A = 96$ sq. units.

v. Let A be the required area.

Consider the equation $2y + x = 8$.

i.e., $y = \dfrac{8-x}{2}$

$\therefore$ $A = \int_2^4 y\,dx = \int_2^4 \dfrac{8-x}{2}\,dx = \dfrac{1}{2}\int_2^4 (8-x)\,dx$

$= \dfrac{1}{2}\left[8x - \dfrac{x^2}{2}\right]_2^4$

$= \dfrac{1}{2}\left[\left(8\times4 - \dfrac{4^2}{2}\right) - \left(8\times2 - \dfrac{2^2}{2}\right)\right]$

$= \dfrac{1}{2}\left[(32-8)-(16-2)\right]$

$= \dfrac{1}{2}(24-14)$

$= \dfrac{1}{2}\times10$

$\therefore$ A = 5 sq. units.

---

vi. Let A be the required area.

Consider the equation $y = x^2 + 1$.

$\therefore$ $A = \int_0^3 y\,dx = \int_0^3 (x^2+1)\,dx$

$= \left[\dfrac{x^3}{3} + x\right]_0^3$

$= \left(\dfrac{3^3}{3} + 3\right) - (0)$

$= (9 + 3)$

$\therefore$ A = 12 sq. units.

---

vii. Let A be the required area.

Consider the equation $y = 2 - x^2$.

$\therefore$ $A = \int_{-1}^1 y\,dx = \int_{-1}^1 (2-x^2)\,dx$

$= \left[2x - \dfrac{x^3}{3}\right]_{-1}^1$

$= \left[2\times1 - \dfrac{1^3}{3}\right] - \left[2\times(-1) - \dfrac{(-1)^3}{3}\right]$

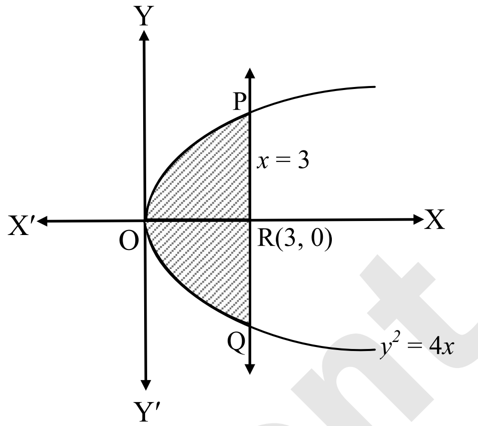
$= \left(2 - \dfrac{1}{3}\right) - \left(-2 + \dfrac{1}{3}\right)$

$= \dfrac{5}{3} - \left(\dfrac{-5}{3}\right)$

$\therefore$ $A = \dfrac{10}{3}$ sq. units.

2. Find the area of the region bounded by the parabola $y^2 = 4x$ and the line $x = 3$. [July 17]

***Solution*:**



Given equation of the parabola is $y^2 = 4x$

$\therefore$ $y = 2\sqrt{x}$ …[$\because$ In first quadrant, $y > 0$]

and equation of the line is $x = 3$

$\therefore$ Required area = area of the region OQRPO

= 2 (area of the region ORPO)

$= 2\int_0^3 y\,dx$

$= 2\int_0^3 2\sqrt{x}\,dx$

$= 4\int_0^3 \sqrt{x}\,dx$

$= 4\int_0^3 x^{1/2}\,dx$

$= 4\left[\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^3$

$= 4 \times \dfrac{2}{3}\left[(3)^{\frac{3}{2}} - 0\right]$

$= \dfrac{8}{3}\left(3\sqrt{3}\right)$

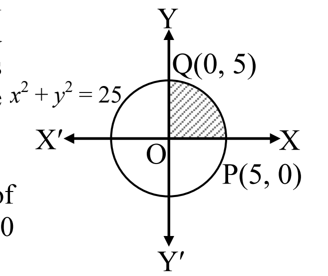
$\therefore$ Required area = $8\sqrt{3}$ sq. units.

---

3. Find the area of circle $x^2 + y^2 = 25$.

***Solution*:**

By the symmetry of the circle, required area of the circle is 4 times the area of the region OPQO.



For the region OPQO, the limits of integration are $x = 0$ and $x = 5$.

Given equation of the circle is $x^2 + y^2 = 25$.

$\therefore \quad y^2 = 25 - x^2$

$\therefore \quad y = \pm\sqrt{25 - x^2}$

$\therefore \quad y = \sqrt{25 - x^2}$ …[$\because$ In first quadrant, $y > 0$]

$\therefore$ Required area = 4 (area of the region OPQO)

$= 4 \times \int_0^5 y.\mathrm{d}x$

$= 4 \times \int_0^5 \sqrt{25 - x^2}\,\mathrm{d}x$

$= 4\int_0^5 \sqrt{(5)^2 - x^2}\,\mathrm{d}x$

$= 4\left[\frac{x}{2}\sqrt{(5)^2 - x^2} + \frac{(5)^2}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5$

$= 4\left\{\left[\frac{5}{2}\sqrt{25 - (5)^2} + \frac{25}{2}\sin^{-1}\left(\frac{5}{5}\right)\right] - \left[\frac{0}{2}\sqrt{25 - (0)^2} + \frac{25}{2}\sin^{-1}\left(\frac{0}{5}\right)\right]\right\}$

$= 4\left\{\left[\frac{5}{2}(0) + \frac{25}{2}\sin^{-1}(1)\right] - [0 + 0]\right\}$

$= 4\left(\frac{25}{2} \times \frac{\pi}{2}\right)$

$= 25\pi$ sq. units.

Smart Check

Area of the circle $x^2 + y^2 = r^2$ is πr^2 sq. units.

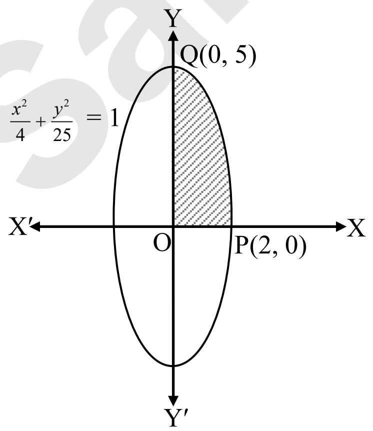
Here, $r^2 = 25$

$\therefore$ Required area $= 25\pi$ sq. units

4. Find the area of ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$.

[Oct 14; July 19]

***Solution*:**



By the symmetry of the ellipse, required area of the ellipse is 4 times the area of the region OPQO.

For the region OPQO, the limits of integration are $x = 0$ and $x = 2$.

Given equation of the ellipse is $\frac{x^2}{4} + \frac{y^2}{25} = 1$

$\therefore \quad \frac{y^2}{25} = 1 - \frac{x^2}{4}$

$\therefore \quad y^2 = 25\left(1 - \frac{x^2}{4}\right) = \frac{25}{4}(4 - x^2)$

$\therefore \quad y = \pm\frac{5}{2}\sqrt{4 - x^2}$

$\therefore \quad y = \frac{5}{2}\sqrt{4 - x^2}$ …[$\because$ In first quadrant, $y > 0$]

$\therefore$ Required area = 4(area of the region OPQO)

$= 4\int_0^2 y\,\mathrm{d}x = 4\int_0^2 \frac{5}{2}\sqrt{4 - x^2}\,\mathrm{d}x$

$= \frac{4 \times 5}{2}\int_0^2 \sqrt{(2)^2 - x^2}\,\mathrm{d}x$

$= 10\left[\frac{x}{2}\sqrt{(2)^2 - x^2} + \frac{(2)^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$= 10\left\{\left[\frac{2}{2}\sqrt{(2)^2 - (2)^2} + \frac{(2)^2}{2}\sin^{-1}\left(\frac{2}{2}\right)\right] - \left[\frac{0}{2}\sqrt{(2)^2 - (0)^2} + \frac{(2)^2}{2}\sin^{-1}\left(\frac{0}{2}\right)\right]\right\}$

$= 10\{[0 + 2\sin^{-1}(1)] - [0 + 0]\}$

$= 10\left(2 \times \frac{\pi}{2}\right)$

$= 10\pi$ sq. units.

Smart Check

Area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is π ab sq. units.

Here, a = 2, b = 5

$\therefore$ Required area $= \pi(2)(5) = 10\pi$ sq. units

Miscellaneous Exercise – 7

I. Choose the correct alternative.

1. Area of the region bounded by the curve $x^2 = y$, the X-axis and the lines $x = 1$ and $x = 3$ is ________.

(A) $\frac{26}{3}$ sq. units (B) $\frac{3}{26}$ sq. units

(C) 26 sq. units (D) 3 sq. units

2. The area of the region bounded by $y^2 = 4x$, the X-axis and the lines $x = 1$ and $x = 4$ is _______.
(A) 28 sq. units (B) 3 sq. units
(C) $\frac{56}{3}$ sq. units (D) $\frac{3}{28}$ sq. units

[**Note:** *Option (C) has been modified.*]

3. Area of the region bounded by $x^2 = 16\,y$, $y = 1$ and $y = 4$ and the Y-axis, lying in the first quadrant is _______.
(A) 63 sq. units (B) $\frac{3}{56}$ sq. units
(C) $\frac{56}{3}$ sq. units (D) $\frac{63}{7}$ sq. units

4. Area of the region bounded by $y = x^4$, $x = 1$, $x = 5$ and the X-axis is _____.
(A) $\frac{3142}{5}$ sq. units (B) $\frac{3124}{5}$ sq. units
(C) $\frac{3142}{3}$ sq. units (D) $\frac{3124}{3}$ sq. units

5. Using definite integration, area of circle $x^2 + y^2 = 25$ is _______.
(A) 5π sq. units (B) 4π sq. units
(C) 25π sq. units (D) 25 sq. units

Answers:
1. (A) 2. (C) 3. (C) 4. (B)
5. (C)

Hints:

1. Required area = $\int_1^3 y\,dx = \int_1^3 x^2\,dx$

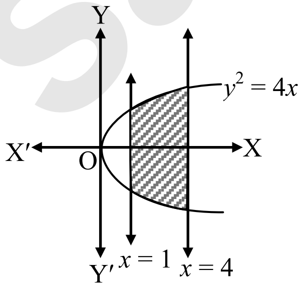
$$= \left[\frac{x^3}{3}\right]_1^3$$

$$= \frac{1}{3}(3^3 - 1^3)$$

$$= \frac{1}{3}(27 - 1)$$

$$= \frac{26}{3} \text{ sq.units}$$

2.



Required area = $2\int_1^4 y\,dx$

$$= 2\int_1^4 2\sqrt{x}\,dx$$

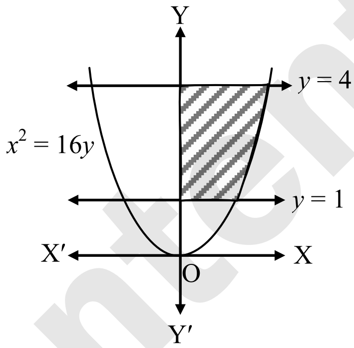
$$= 4\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$$

$$= \frac{8}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$$

$$= \frac{8}{3}(8 - 1)$$

$$= \frac{56}{3} \text{ sq. units}$$

3.



Required area = $\int_1^4 x\,dy$

$$= \int_1^4 4\sqrt{y}\,dy$$

$$= 4\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$$

$$= \frac{8}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$$

$$= \frac{8}{3}(8 - 1) = \frac{56}{3} \text{ sq. units}$$

4. *Refer Exercise 7.1 Q.1 (i).*

5. *Refer Exercise 7.1 Q.3.*

II. Fill in the blanks.

1. Area of the region bounded by $y = x^4$, $x = 1$, $x = 5$ and the X-axis is _______.

2. Using definite integration, area of the circle $x^2 + y^2 = 49$ is _______.

3. Area of the region bounded by $x^2 = 16y$, $y = 1$, $y = 4$ and the Y-axis, lying in the first quadrant is _______.

4. The area of the region bounded by the curve $x^2 = y$, the X-axis and the lines $x = 3$ and $x = 9$ is _______.

5. The area of the region bounded by $y^2 = 4x$, the X-axis and the lines $x = 1$ and $x = 4$ is _______.

Answers:

1. $\frac{3124}{5}$ sq. units
2. 49π sq. units
3. $\frac{56}{3}$ sq. units
4. 234 sq. units
5. $\frac{56}{3}$ sq. units

Hints:

1. *Refer Exercise 7.1 Q.1 (i).*

---

2. Area of the circle $x^2 + y^2 = r^2$ is πr^2 sq. units.
Here, $r^2 = 49$

$\therefore$ Required area = 49π sq.units

---

3. *Refer Miscellaneous Exercise 7. Q.I (3).*

---

4. Required area = $\int_3^9 y \, dx$

$$= \int_3^9 x^2 \, dx$$

$$= \left[\frac{x^3}{3}\right]_3^9$$

$$= \frac{1}{3}(9^3 - 3^3)$$

$$= \frac{1}{3}(729 - 27)$$

$$= \frac{702}{3} = 234 \text{ sq.units}$$

[**Note:** Answer given in the textbook is $\frac{70}{3}$ sq.units. However, as per our calculation it is 234 sq.units.]

---

5. *Refer Miscellaneous Exercise Q.I (2)*

[**Note:** Answer given in the textbook is $\frac{28}{3}$ sq.units. However, as per our calculation it is $\frac{56}{3}$ sq. units.]

III. State whether each of the following is True or False.

1. The area bounded by the curve $x = g(y)$, Y-axis and bounded between the lines $y = c$ and $y = d$ is given by $\int_c^d x \, dy = \int_{y=c}^{y=d} g(y) \, dy$.

2. The area bounded by the two curves $y = f(x)$, $y = g(x)$ and X-axis is $\left|\int_a^b f(x)\,dx - \int_b^a g(x)\,dx\right|$.

3. The area bounded by the curve $y = f(x)$, X-axis and lines $x = a$ and $x = b$ is $\left|\int_a^b f(x)\,dx\right|$.

4. If the curve, under consideration, is below the X-axis, then the area bounded by curve, X-axis and lines $x = a$, $x = b$ is positive.

5. The area of the portion lying above the X-axis is positive.

Answers:

1. True
2. False
3. True
4. True
5. True

Justification:

2. The area bounded by two curves $y = f(x)$, $y = g(x)$ and X-axis is $\left|\int_a^b f(x)\,dx - \int_a^b g(x)\,dx\right|$.

---

4. Area is always positive.
[**Note:** Answer given in the textbook is 'False'. However, we found that it is 'True'.]

IV. Solve the following.

1. Find the area of the region bounded by the curve $xy = c^2$, the X-axis, and the lines $x = c$, $x = 2c$.

Solution:

Given equation of the curve is $xy = c^2$

$\therefore$ $y = \frac{c^2}{x}$

$\therefore$ Required area $= \int_c^{2c} y \, dx = \int_c^{2c} \frac{c^2}{x} \cdot dx$

$$= c^2 \int_c^{2c} \left(\frac{1}{x}\right) dx = c^2 \left[\log x\right]_c^{2c}$$

$$= c^2(\log 2c - \log c)$$

$$= c^2 \log\left(\frac{2c}{c}\right)$$

$$= c^2 \log 2 \text{ sq. units.}$$

---

2. Find the area between the parabolas $y^2 = 7x$ and $x^2 = 7y$.

Solution:

Given equations of the parabolas are $y^2 = 7x$ …(i)

and $x^2 = 7y$.

$\therefore$ $y = \frac{x^2}{7}$ …(ii)

From (i), we get

$y = \sqrt{7x}$ …(iii)[$\because$ In first quadrant, $y > 0$]

Find the points of intersection of $y^2 = 7x$ and $x^2 = 7y$.

Substituting (ii) in (i), we get

$$\left(\frac{x^2}{7}\right)^2 = 7x$$

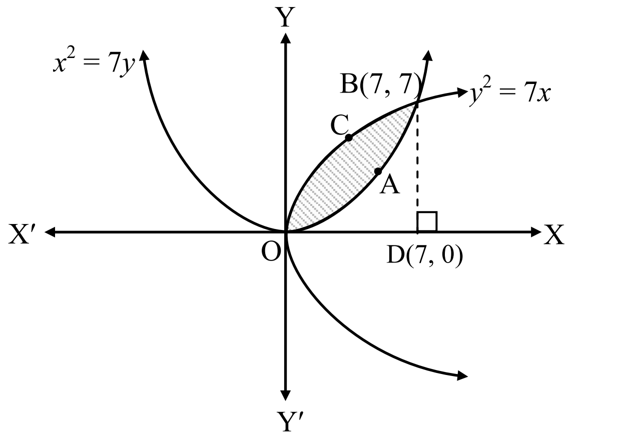
$\therefore$ $x^4 = 343x$

$\therefore$ $x^4 - 343x = 0$

$\therefore \quad x(x^3 - 343) = 0$

$\therefore \quad x = 0$ or $x^3 = 343 = 7^3$

$\therefore \quad x = 0$ or $x = 7$



When $x = 0$, $y = 0$ and when $x = 7$, $y = 7$

$\therefore$ The points of intersection of $y^2 = 7x$ and $x^2 = 7y$ are O(0, 0) and B (7, 7).

Draw BD ⊥ OX

Required area = area of the region OABCO
= area of the region ODBCO
– area of the region ODBAO

= area under the parabola $y^2 = 7x$
– area under the parabola $x^2 = 7y$

$= \int_0^7 \sqrt{7x}\, dx - \int_0^7 \frac{x^2}{7}\, dx$ …[From (iii) and (ii)]

$= \sqrt{7}\int_0^7 x^{\frac{1}{2}}\, dx - \frac{1}{7}\int_0^7 x^2\, dx$

$= \sqrt{7}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^7 - \frac{1}{7}\left[\frac{x^3}{3}\right]_0^7 = \frac{2\sqrt{7}}{3}\left[(7)^{\frac{3}{2}} - 0\right]$

$- \frac{1}{21}\left[(7)^3 - 0\right]$

$= \frac{2\sqrt{7}}{3}\left(7\sqrt{7}\right) - \frac{1}{21}(343)$

$= \frac{98}{3} - \frac{49}{3} = \frac{49}{3}$ sq. units

Smart Check

The area of the region bounded by $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16ab}{3}$ sq. units.

Here, $a = \frac{7}{4}$, $b = \frac{7}{4}$

$\therefore$ Required area $= \frac{16\left(\frac{7}{4}\right)\left(\frac{7}{4}\right)}{3} = \frac{49}{3}$ sq. units

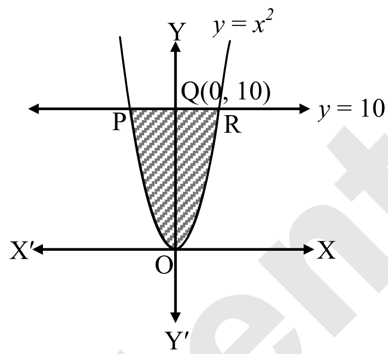
3. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 10$.

Solution:

Given equation of the curve is

$y = x^2$

$\therefore \quad x = \sqrt{y}$ …[$\because$ In first quadrant, $x > 0$]



Required area = area of the region ORQPO

$= 2$ (area of the region ORQO)

$= 2\int_0^{10} x\, dy$

$= 2\int_0^{10} \sqrt{y}\, dy$

$= 2\int_0^{10} y^{\frac{1}{2}}\, dy$

$= 2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{10}$

$= \frac{4}{3}\left[(10)^{\frac{3}{2}} - 0\right]$

$= \frac{4}{3}\left(10\sqrt{10}\right)$

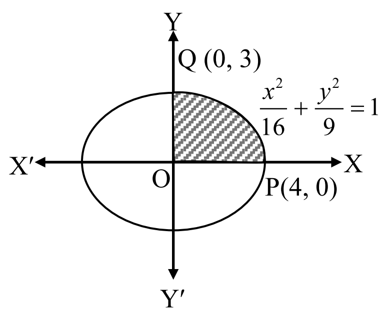
$= \frac{40\sqrt{10}}{3}$ sq.units

4. Find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

Solution:

By the symmetry of the ellipse, required area of the ellipse is 4 times the area of the region OPQO.

For the region OPQO, the limits of integration are $x = 0$ and $x = 4$.



Given equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$\therefore \quad \frac{y^2}{9} = 1 - \frac{x^2}{16}$

$\therefore \quad y^2 = 9\left(1 - \frac{x^2}{16}\right)$

$= \frac{9}{16}(16 - x^2)$

$\therefore \quad y = \pm\frac{3}{4}\sqrt{16 - x^2}$

$\therefore \quad y = \frac{3}{4}\sqrt{16 - x^2}$ …[$\because$ In first quadrant, $y > 0$]

$\therefore$ Required area = 4(area of the region OPQO)

$= 4\int_0^4 y\,dx$

$= 4\int_0^4 \frac{3}{4}\sqrt{16 - x^2}\,dx$

$= 3\int_0^4 \sqrt{(4)^2 - x^2}\,dx$

$= 3\left[\frac{x}{2}\sqrt{(4)^2 - x^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$

$= 3\left\{\left[\frac{4}{2}\sqrt{(4)^2 - (4)^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{4}{4}\right)\right] - \left[\frac{0}{2}\sqrt{(4)^2 - (0)^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{0}{4}\right)\right]\right\}$

$= 3\{[0 + 8\sin^{-1}(1)] - [0 + 0]\}$

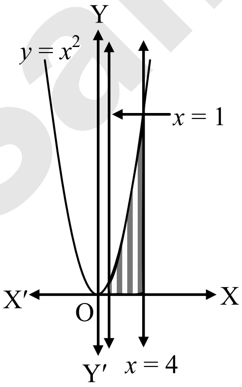
$= 3\left(8 \times \frac{\pi}{2}\right)$

$= 12\pi$ sq. units

---

5. Find the area of the region bounded by $y = x^2$, the X-axis and $x = 1$, $x = 4$.

Solution:



Required area $= \int_1^4 y\,dx$

$= \int_1^4 x^2\,dx$

$= \left[\frac{x^3}{3}\right]_1^4$

$= \frac{1}{3}(4^3 - 1^3)$

$= \frac{1}{3}(64 - 1)$

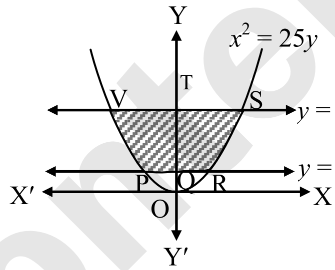
$= \frac{1}{3}(63)$

$= 21$ sq. units

---

6. Find the area of the region bounded by the curve $x^2 = 25y$, $y = 1$, $y = 4$ and the Y-axis.

Solution:



Given equation of the curve is $x^2 = 25y$

$\therefore \quad x = 5\sqrt{y}$ …[$\because$ In first quadrant, $x > 0$]

Required area = area of the region PRSVP

= 2 (area of the region QRSTQ)

$= 2\int_1^4 x\,dy$

$= 2\int_1^4 5\sqrt{y}\,dy$

$= 10\int_1^4 y^{\frac{1}{2}}\,dy$

$= 10\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$

$= \frac{20}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$

$= \frac{20}{3}(8 - 1)$

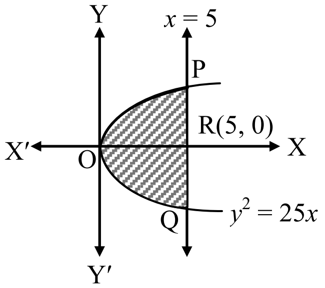
$= \frac{20}{3}(7)$

$= \frac{140}{3}$ sq. units

[**Note:** Answer given in the textbook is $\frac{70}{3}$ sq. units. However, as per our calculation it is $\frac{140}{3}$ sq. units.]

7. Find the area of the region bounded by the parabola $y^2 = 25x$ and the line $x = 5$.

Solution:



Given equation of the parabola is $y^2 = 25x$

$\therefore$ $y = 5\sqrt{x}$ …[$\because$ In first quadrant, $y > 0$]

Required area = area of the region OQRPO

= 2 (area of the region ORPO)

$$= 2\int_0^5 y\ dx$$

$$= 2\int_0^5 5\sqrt{x}\ dx$$

$$= 10\int_0^5 x^{\frac{1}{2}}\ dx$$

$$= 10\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^5$$

$$= \frac{20}{3}\left[(5)^{\frac{3}{2}} - 0\right]$$

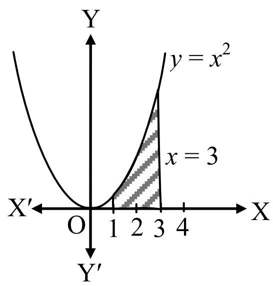
$$= \frac{20}{3}\left(5\sqrt{5}\right)$$

$$= \frac{100\sqrt{5}}{3}\text{ sq. units}$$

Activities for Practice

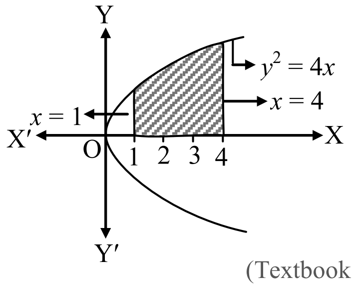
From the following information find the area of the shaded region.

1.



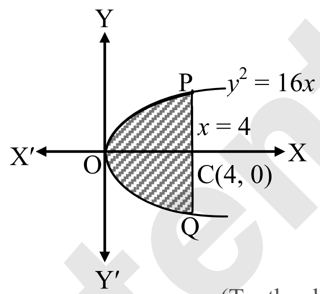
(Textbook page no. 158)

2.



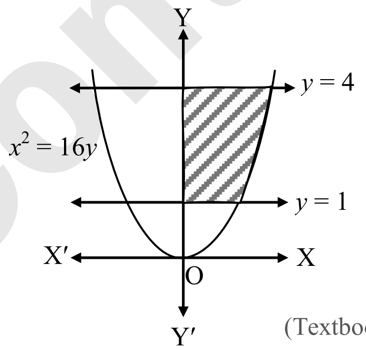
(Textbook page no. 158)

3.



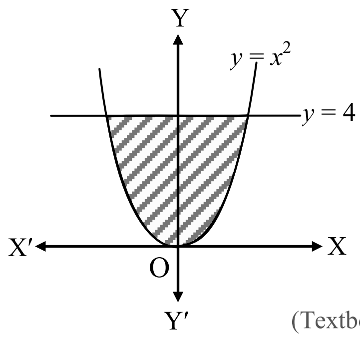
(Textbook page no. 159)

4.



(Textbook page no. 159)

5.



(Textbook page no. 159)

6. By filling the boxes, find the area of the circle $x^2 + y^2 = 9$ using integration.

By the symmetry of the circle, required area of the circle is ☐ times the area of the region OPQO.

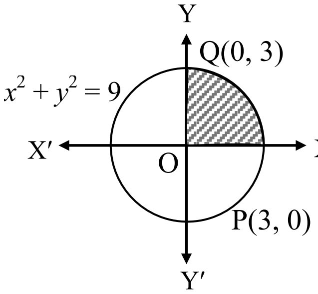
For the region OPQO, the limits of integration are $x = 0$ and $x =$ ☐.

$\therefore$ Required area = 4(area of the region OPQO)

$$= 4\int_0^{\square} y\ dx$$

$= 4\int_0^{\square} \square \, dx$

$= \square$ sq. units



Multiple Choice Questions

1. The area of the region (in square units) bounded by the curve $x^2 = 4y$, line $x = 2$ and X-axis is

   (A) 1 (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{8}{3}$

2. The area of the region bounded by $y^2 = 4x$, $x = 0$, $x = 4$ and the X-axis in the first quadrant is

   (A) 16 (B) $\frac{16}{3}$
   (C) 32 (D) $\frac{32}{3}$

3. Area bounded by the parabola $y^2 = 2x$ and the ordinates $x = 1$, $x = 4$ is

   (A) $\frac{4\sqrt{2}}{3}$ sq.units (B) $\frac{28\sqrt{2}}{3}$ sq.units
   (C) $\frac{56}{3}$ sq.units (D) $\frac{4}{3}$ sq. units

4. The area of the region bounded by the parabola $y^2 = 4ax$ and the line $y = mx$ is

   (A) $\frac{8a^2}{3m^3}$ (B) $\frac{8m^2}{3a^3}$
   (C) $\frac{8a^2}{3}$ (D) $\frac{8a^2m^3}{3}$

5. The area bounded by the curves $y^2 - x = 0$ and $y - x^2 = 0$ is

   (A) $\frac{7}{3}$ (B) $\frac{1}{3}$ (C) $\frac{5}{3}$ (D) 1

6. The area enclosed by $y = 3x - 5$, $y = 0$, $x = 3$ and $x = 5$ is

   (A) 12 sq. units (B) 13 sq. units
   (C) $13\frac{1}{2}$ sq. units (D) 14 sq. units

7. Area enclosed between the curve $y = x^{\frac{1}{3}}$, the Y-axis and the lines $y = -1$, $y = 1$ is

   (A) 0 (B) $\frac{2}{3}$
   (C) $\frac{3}{2}$ (D) $\frac{1}{2}$

8. Area bounded by the curve $y^2 = 16x$ and line $y = mx$ is $\frac{2}{3}$, then m =

   (A) 3 (B) 4
   (C) 1 (D) 2

9. Area inside the parabola $y^2 = 4ax$ between the lines $x = a$ and $x = 4a$ is equal to

   (A) $4a^2$ (B) $8a^2$
   (C) $56\frac{a^2}{3}$ (D) $35\frac{a^2}{3}$

10. Area bounded by the lines $y = x$, $x = -1$, $x = 2$ and X-axis is

    (A) $\frac{5}{2}$ sq. units (B) $\frac{3}{2}$ sq. units
    (C) $\frac{1}{2}$ sq. units (D) $\frac{2}{5}$ sq. units

Additional Problems for Practice

Based on Exercise 7.1

1. Find the area of the region bounded by the following curves, the X-axis and the given lines:

   i. $y = 6x$, $x = 1$, $x = 5$
   ii. $y = x^3$, $x = 1$, $x = 4$
   iii. $y = \sqrt{3x+4}$, $x = 0$, $x = 4$
   iv. $y - 1 = x$, $x = -2$, $x = 3$
   v. $y = 2\sqrt{1-x^2}$, $x = 0$, $x = 1$
   +vi. $y = -2x$, $x = -1$, $x = 2$

+2. Find the area of the region bounded by the parabola $y^2 = 16x$ and the line $x = 4$. **[Mar 18]**

3. Find the area of the circle $x^2 + y^2 = 16$.

+4. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **[Mar 15]**

5. Find the area of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

Based on Miscellaneous Exercise – 7

1. Find the area of the region bounded by the curve $xy = 4$, the X-axis and the lines $x = 2$, $x = 4$.

2. Find the area between the parabolas $y^2 = 16x$ and $x^2 = 16y$.

3. Find the area of the region bounded by the curve $y = 4x^2$ and the line $y = 4$.

4. Find the area of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

5. Find the area of the region bounded by $3y = x^2$, the X-axis and $x = 3$, $x = 6$.

6. Find the area of the region bounded by the curve $x^2 = 36y$, $y = 1$, $y = 9$ and the Y-axis.

7. Find the area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$.

Topic Test

Time: 1 hour **Marks: 20**

Q.1. (A) Choose the correct alternative. **[2]**

i. Area of the region bounded by the curve $x^2 = y$, the X-axis and the lines $x = 1$ and $x = 3$ is _______.

(A) $\frac{26}{3}$ sq. units (B) $\frac{3}{26}$ sq. units (C) 26 sq. units (D) 3 sq. units

ii. The area enclosed by $y = 2x$, X-axis and lines $x = 0$ and $x = 3$ is

(A) 6 sq.units (B) 9 sq.units (C) 18 sq.units (D) 27 sq.units

(B) State whether the following statements are True or False. **[2]**

i. The area bounded by the two curves $y = \text{f}(x)$, $y = \text{g}(x)$ and X-axis is $\left|\int_a^b \text{f}(x)\,\text{d}x - \int_b^a \text{g}(x)\,\text{d}x\right|$.

ii. The area of the portion lying above the X-axis is positive.

(C) Fill in the blanks **[2]**

i. The area of the region bounded by $y = x$, the X-axis and the lines $x = 0$ and $x = 4$ is _______.

ii. The area of the region bounded by $x = y^2$, the Y-axis and the lines $y = 0$ and $y = 3$ is _______.

Q.2. Attempt the following. **[4]**

i. Find the area of the region bounded by the curve $y = x^4$, the X-axis and the lines $x = 1$, $x = 5$.

ii. Find the area of the region bounded by the curve $2y + x = 8$, the X-axis and the lines $x = 2$, $x = 4$.

Q.3. Attempt the following. **[6]**

i. Find the area of the region bounded by the curve $x^2 = 16y$, $y = 1$, $y = 4$, and the Y-axis lying in the first quadrant.

ii. Find the area of the region bounded by the curve $y = -2x$, the X-axis and the lines $x = -1$, $x = 2$.

Q.4. Attempt any one of the following. **[4]**

i. Find the area of ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$.

ii. Find the area between the parabolas $y^2 = 7x$ and $x^2 = 7y$.

Answers

Activities for practice

1. Required area = $\int_1^3 y \, dx$

$= \int_1^3 x^2 \, dx$

$= \left[\frac{x^3}{3}\right]_1^3$

$= \frac{1}{3}(3^3 - 1^3)$

$= \frac{1}{3}(27 - 1)$

$= \frac{26}{3}$ sq. units.

2. Required area = $\int_1^4 y \, dx$

$= \int_1^4 2\sqrt{x} \, dx$

$= 2\int_1^4 x^{\frac{1}{2}} \, dx$

$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$

$= \frac{4}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$

$= \frac{4}{3}(8 - 1)$

$= \frac{4}{3}(7)$

$= \frac{28}{3}$ sq. units

3. Required area = area of the region OQCPO

= 2 (area of the region OCPO)

$= 2\int_0^4 y \, dx$

$= 2\int_0^4 4\sqrt{x} \, dx$

$= 8\int_0^4 x^{\frac{1}{2}} \, dx$

$= 8\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$

$= \frac{16}{3}\left[(4)^{\frac{3}{2}} - 0\right]$

$= \frac{16}{3}(8)$

$= \frac{128}{3}$ sq. units

4. Required area = $\int_1^4 x \, dy$

$= \int_1^4 4\sqrt{y} \, dy$

$= 4\int_1^4 y^{\frac{1}{2}} \, dy$

$= 4\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$

$= \frac{8}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$

$= \frac{8}{3}(8 - 1)$

$= \frac{8}{3}(7)$

$= \frac{56}{3}$ sq. units

5. Required area = $2\int_0^4 x \, dy$

$= 2\int_0^4 \sqrt{y} \, dy$

$= 2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$

$= \frac{4}{3}\left[(4)^{\frac{3}{2}} - 0\right]$

$= \frac{4}{3}(8)$

$= \frac{32}{3}$ sq. units

6. i. 4 ii. 3
iii. 3 iv. 3
v. $\sqrt{9 - x^2}$ vi. 9π

Multiple Choice Questions

1.	(B)	2.	(D)	3.	(B)	4.	(A)
5.	(B)	6.	(D)	7.	(D)	8.	(B)
9.	(C)	10.	(A)				

Additional Problems for Practice

Based on Exercise 7.1

1. i. 72 sq. units ii. $\frac{255}{4}$ sq. units

   iii. $\frac{112}{9}$ sq. units iv. $\frac{15}{2}$ sq. units

   v. $\frac{\pi}{2}$ sq. units vi. 5 sq. units

2. $\frac{128}{3}$ sq. units
3. 16π sq. units
4. π ab sq. units
5. 6π sq. units

Based on Miscellaneous Exercise – 7

1. 4 log 2 sq. units
2. $\frac{256}{3}$ sq. units
3. $\frac{16}{3}$ sq. units
4. 20 π sq. units
5. 21sq. units
6. 208 sq. units
7. $\frac{32}{3}$ sq. units

Topic Test

1. (A)
   i. (A) ii. (B)

   (B)
   i. False ii. True

   (C)
   i. 8 sq.units
   ii. 9 sq.units
2. i. $\frac{3124}{5}$ sq. units
   ii. 5 sq. units
3. i. $\frac{56}{3}$ sq. units
   ii. 5 sq. units
4. i. 10π sq. units
   ii. $\frac{49}{3}$ sq. units

*Download the solutions of the Topic Test by scanning the given **Q.R. Code.***

AVAILABLE NOTES FOR STD. XI & XII:

SCIENCE

Perfect Series:

For students who want to excel in board exams and simultaneously study for entrance exams.

- Physics Vol. I
- Physics Vol. II
- Chemistry Vol. I
- Chemistry Vol. II
- Mathematics & Statistics Part - I
- Mathematics & Statistics Part - II
- Biology Vol. I
- Biology Vol. II

Precise Series:

For students who want to excel in board exams.

- Physics
- Chemistry
- Biology

Additional Books for Std. XII Sci. & Com.:

- A collection of Board Questions (PCMB)
- A collection of Board Questions with solutions (PCMB)
- 20 Model Question Papers with solutions (PCMB & E)
- Solution to HSC Board Question Bank (Science)
- Solution to HSC Board Question Bank (Commerce)

COMMERCE (ENG. & MAR. MED.)

Smart Notes:

- Book-Keeping and Accountancy
- Book Keeping and Accountancy (Practice)
- Economics
- Organisation of Commerce and Management
- Secretarial Practice
- Mathematics and Statistics - I
- Mathematics and Statistics - II
- Supplementary Questions (BK • ECO • OCM • SP)

ARTS (ENG. & MAR. MED.)

- History
- Geography
- Political Science
- Psychology
- Sociology

Books available for MHT-CET, NEET & JEE

Languages:

- English Yuvakbharati
- Hindi Yuvakbharati
- Marathi Yuvakbharati

Scan the QR code to buy e-book version of Target's Notes on Quill - The Padhai App

OUR PRODUCT RANGE

Children Books | School Section | Junior College
Degree College | Entrance Exams | Stationery

Explore our range of STATIONERY

Visit Our Website

Target Publications® Pvt. Ltd.
Transforming lives through learning.

Address:
2nd floor, Aroto Industrial Premises CHS,
Above Surya Eye Hospital, 63-A, P. K. Road,
Mulund (W), Mumbai 400 080
Tel: 88799 39712 / 13 / 14 / 15
Website: www.targetpublications.org
Email: mail@targetpublications.org