SAMPLE CONTENT

MHT-CET TEST SERIES

MATHEMATICS

WITH ANSWER KEY & SOLUTIONS

1530 MCQs

- **24** Topic Tests
- **08** Revision Tests
- **05** Model Test Papers

Target Publications® Pvt. Ltd.

Based on **Latest Paper Pattern** of MHT-CET prescribed by State Common Entrance Test Cell, Government of Maharashtra

MHT-CET

Mathematics TEST SERIES

With Answer Key & Solutions

Salient Features:

- Contains 24 Topic Tests and 8 Revision Tests covering MCQs from multiple different topics for efficient practice of MCQs.
- 5 Model Test Papers at the end for self-evaluation.
- Includes '1530' MCQs for practice in the form of Topic Test, Revision Test and Model Test Papers as per latest paper pattern.
- Answers provided to all the questions and Solutions provided wherever required.

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PREFACE

Target's **'MHT-CET Mathematics Test Series'** is a complete practice book, extremely handy for the preparation of MHT-CET examinations. This book would act as a go-to tool for preparation and practice at the same time.

The core objective of the book is to help students gauge their preparedness to appear for MHT-CET examination, as it includes a beautiful assortment of MCQs in the form of Topic Tests and Revision Tests along with Model Test Papers as per latest paper pattern.

Topic Tests are provided for powerful concept building. Revision Tests develop confidence in the students, as it includes MCQs from three different topics. Model Test Papers help students analyse their strengths and area of improvement to yield better results.

All Test Papers in this book have been created in line with the examination pattern and touches upon all the conceptual nodes of the subject.

We have provided answers to all the questions along with detailed solutions for difficult questions.

We are sure that, these question papers would provide ample practice to students in a systematic manner and would boost their confidence to face the challenges posed in examinations.

We wish the students all the best for their examinations!

\- Publisher
Edition: First

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.

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A book affects eternity; one can never tell where its influence stops.

Disclaimer

This reference book is transformative work based on Std. XI Mathematics Part – I & II; Second Reprint: 2021 and Std. XII Mathematics Part – I & II; First Reprint: 2021, published by the Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune. We the publishers are making this book which constitutes as fair use of textual contents which are transformed in the form of Multiple Choice Questions and their relevant solutions; with a view to enable the students to understand memorize and reproduce the same in MHT-CET examination.

This work is purely inspired by the paper pattern prescribed by State Common Entrance Test Cell, Government of Maharashtra. Every care has been taken in the publication of this reference book by the Authors while creating the contents. The Authors and the Publishers shall not be responsible for any loss or damages caused to any person on account of errors or omissions which might have crept in or disagreement of any third party on the point of view expressed in the reference book.

NEW PAPER PATTERN

- There will be three papers of Multiple Choice Questions (MCQs) in 'Mathematics', 'Physics and Chemistry' and 'Biology' of 100 marks each. Duration of each paper will be 90 minutes.
- Questions will be based on the syllabus prescribed by Maharashtra State Board of Secondary and Higher Secondary Education with approximately 20% weightage given to Std. XI and 80% weightage will be given to Std. XII curriculum.
- Difficulty level of questions will be at par with JEE (Main) for Mathematics, Physics, Chemistry and at par with NEET for Biology.
- There will be no negative marking.
- Questions will be mainly application based.
- Details of the papers are as given below:

Paper	Subject(s)	No. of MCQs based on		Mark(s) Per Question	Total Marks	Duration in Minutes
		Std XI	Std XII			
Paper I	Mathematics	10	40	2	100	90
Paper II	Physics	10	40	1	100	90
	Chemistry	10	40			
Paper III	Biology	20	80	1	100	90

- **Chapters / units from Std. XI curriculum:**

Sr.no	Subject	Chapters/Units of Std. XI
1	Physics	Motion in a Plane, Laws of Motion, Gravitation, Thermal Properties of Matter, Sound, Optics, Electrostatics, Semiconductors
2	Chemistry	Some Basic Concepts of Chemistry, Structure of Atom, Chemical Bonding, Redox Reactions, Elements of Group 1 and Group 2, States of Matter (Gaseous and Liquid States), Adsorption and Colloids (Surface Chemistry), Hydrocarbons, Basic Principles of Organic Chemistry
3	Mathematics	Trigonometry II, Straight Line, Circle, Measures of Dispersion, Probability, Complex Numbers, Permutations and Combinations, Functions, Limits, Continuity
4	Biology	Biomolecules, Respiration and Energy Transfer, Human Nutrition, Excretion and Osmoregulation

- **Language of Question Paper:**
  The medium for examination shall be English / Marathi / Urdu for Physics, Chemistry and Biology. Mathematics paper shall be in English only.
- **Duration of Examination:**
  The duration of the examination for PCB is 180 minutes and PCM is 180 minutes.

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Topic Test 01

Trigonometry II

Time: 75 min | **Total Marks: 80**

1. $\dfrac{1+\cos\theta+\sin\theta}{1-\cos\theta+\sin\theta}$ =

(A) $\tan\left(\dfrac{\theta}{2}\right)$ (B) $\cot\left(\dfrac{\theta}{2}\right)$
(C) $\tan 2\theta$ (D) $\cot 2\theta$

2. cos 10° + cos 20° + cos 30° + … + cos 180° =

(A) 0 (B) 1
(C) – 1 (D) 2

3. $\dfrac{\cos 9^\circ-\sin 9^\circ}{\cos 9^\circ+\sin 9^\circ}$ =

(A) tan 9° (B) tan 36°
(C) cot 36° (D) cot 9°

4. sin 10° sin 30° sin 50° sin 70° =

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{4}$
(C) $\dfrac{1}{8}$ (D) $\dfrac{1}{16}$

5. $\dfrac{3\cos A+\cos 3A}{3\sin A-\sin 3A}$ =

(A) tan 3A (B) cot 3A
(C) $\cot^3 A$ (D) $\tan^3 A$

6. In ΔABC, if tan A + tan B + tan C = 6 and tan A tan B = 2, then tan C =

(A) 1 (B) 2
(C) 3 (D) 4

7. If P and Q are supplementary angles, then $\sin^2\dfrac{P}{2}+\sin^2\dfrac{Q}{2}$ =

(A) $\dfrac{1}{3}$ (B) 1
(C) $\dfrac{1}{2}$ (D) 0

8. sin (45° + A) . sin (45° – A) =

(A) $\dfrac{1}{2}$ sin 2A (B) $\dfrac{1}{2}$ cos 2A
(C) $\dfrac{1}{2}$ sin A (D) $\dfrac{1}{2}$ cos A

9. If $\sin\left(\theta+\dfrac{\pi}{6}\right) = 4\cos\left(\theta-\dfrac{\pi}{3}\right)$, then tan θ =

(A) $\sqrt{3}$ (B) $-\sqrt{3}$
(C) $\dfrac{1}{\sqrt{3}}$ (D) $-\dfrac{1}{\sqrt{3}}$

10. cos 182° + cos 62° + cos 58° =

(A) 0 (B) 1
(C) 2 (D) $\dfrac{1}{2}$

11. The value of

$e^{\log_{10}\tan 1^\circ+\log_{10}\tan 2^\circ+\log_{10}\tan 3^\circ+.....+\log_{10}\tan 89^\circ}$

is

(A) 0 (B) e
(C) $\dfrac{1}{e}$ (D) 1

12. If $\tan x = \dfrac{5}{12}$ and x lies in the third quadrant, then $\cos\left(\dfrac{x}{2}\right)$ is equal to

(A) $\dfrac{5}{\sqrt{13}}$ (B) $-\sqrt{\dfrac{1}{26}}$
(C) $\dfrac{5}{\sqrt{26}}$ (D) $-\dfrac{5}{13}$

13. If $\sin x + \sin y = \dfrac{1}{4}$ and $\cos x + \cos y = 2$, then $\tan(x + y)$ =

(A) $\dfrac{-16}{63}$ (B) $\dfrac{16}{63}$
(C) $\dfrac{15}{16}$ (D) $\dfrac{-15}{16}$

14. $\dfrac{1}{4}\left(\sqrt{3}\cos 28^\circ-\sin 28^\circ\right)$ =

(A) $\dfrac{1}{2}$ cos 58°

(B) $\dfrac{1}{2}$ sin 58°

(C) $\dfrac{1}{4}$ cos 58°

(D) $\dfrac{1}{4}$ sin 58°

15. $\sin^2(5^\circ)+\sin^2(10^\circ)+\sin^2(15^\circ)+\ldots+\sin^2(80^\circ)+\sin^2(85^\circ)+\sin^2(90^\circ)$ =

(A) $\dfrac{19}{2}$ (B) $\dfrac{21}{2}$
(C) $\dfrac{31}{2}$ (D) 35

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Revision Test 01

Topic Test – 01, 02, 03

Time: 75 min **Total Marks: 80**

1. If $4x^2 + 2\lambda xy + 4y^2 + (8 - \lambda)x + (3\lambda - 8)y - 56 = 0$ is the equation of a circle, then its radius is
   (A) $\sqrt{14}$ (B) $2\sqrt{14}$
   (C) 4 (D) $4\sqrt{2}$

2. If $A + B = \frac{\pi}{4}$, then $(\cot A - 1)(\cot B - 1) =$
   (A) 0 (B) 2
   (C) 1 (D) 4

3. If the line $y = mx + c$ passes through the points (5, 3) and (– 5, – 7), then
   (A) m = 1, c = 1
   (B) m = 2, c = – 1
   (C) m = – 2, c = 2
   (D) m = 1, c = – 2

4. The circles $x^2 + y^2 + 6x + 6y = 0$ and $x^2 + y^2 - 12x - 12y = 0$
   (A) touch each other externally
   (B) touch each other internally
   (C) intersect at two points
   (D) none of these

5. If sin 2A + cos 2A = 1, then sin 4A is equal to
   (A) 0 (B) 1
   (C) 2 (D) $\frac{1}{2}$

6. The X – axis touches the circle whose centre is (0, 2) .The equation of the tangent to the circle at (2, 2) is
   (A) $x + 2 = 0$ (B) $x - 4 = 0$
   (C) $x - 2 = 0$ (D) $x + y - 4 = 0$

7. $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} =$
   (A) $2\sin\theta$ (B) $2\cos\theta$
   (C) $\sin 2\theta$ (D) $\cos 2\theta$

8. The y-intercept of the line passing through (–2, 6) and perpendicular to the line $3x - 4y = 25$ is
   (A) $\frac{1}{3}$ (B) $\frac{4}{3}$
   (C) $\frac{5}{3}$ (D) $\frac{10}{3}$

9. If a circle passes through the points (0, 0), (3, 0), (0, 4), then its centre is
   (A) $\left(2, \frac{3}{2}\right)$ (B) $\left(\frac{3}{2}, \frac{1}{2}\right)$
   (C) $\left(\frac{3}{2}, 2\right)$ (D) $\left(\frac{-3}{2}, -2\right)$

10. $\sin(25° + \theta) . \cos(25° - \phi) - \cos(25° + \theta) . \sin(25° - \phi) =$
   (A) $\sin(\theta + \phi)$ (B) $\cos(\theta + \phi)$
   (C) $2\cos\theta$ (D) $2\sin\phi$

11. The points P(–a, –b), Q(0, 0), R(a, b) and S(a^2, ab) are
   (A) vertices of a square
   (B) vertices of a parallelogram
   (C) vertices of a rectangle
   (D) collinear

12. The equations of the sides of a square are $x - 5 = 0$, $x + 4 = 0$, $y - 5 = 0$, $y + 4 = 0$. The equation of the circle drawn on the diagonal passing through the origin as its diameter is
   (A) $x^2 + y^2 - x - y + 40 = 0$
   (B) $x^2 + y^2 - x - y - 40 = 0$
   (C) $x^2 + y^2 + x + y + 40 = 0$
   (D) $x^2 + y^2 + x + y - 40 = 0$

13. $\sin^2 22.5° + \sin^2 67.5° =$
   (A) $\cos^2 90°$ (B) $\sin^2 45°$
   (C) $\cos^2 30°$ (D) $\tan^2 45°$

14. Equation of locus of a point, so that the segment joining the points (1, 2) and (3, 0) subtends a right angle at that point, is
   (A) $x^2 + y^2 - 4x + 2y + 3 = 0$
   (B) $x^2 - y^2 + 2x - 3y + 6 = 0$
   (C) $x^2 + y^2 - 4x - 2y + 3 = 0$
   (D) $x^2 + y^2 - 2x - 2y + 6 = 0$

15. The equation of a line passing through (4, 3) and making an angle 120° with positive X–axis is
   (A) $\sqrt{3}x + y + 3 - 4\sqrt{3} = 0$
   (B) $\sqrt{3}x + y - 3 - 4\sqrt{3} = 0$
   (C) $\sqrt{3}x + 2y + 3 + 4\sqrt{3} = 0$
   (D) $\sqrt{3}x + 2y - 3 - 4\sqrt{3} = 0$

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Model Test Paper - 01

Time: 90 min **Total Marks: 100**

1. $\int_{\frac{\pi}{18}}^{\frac{4\pi}{9}} \frac{2\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx =$

   (A) $\frac{5\pi}{18}$ (B) $\frac{7\pi}{18}$ (C) $\frac{5\pi}{36}$ (D) $\frac{7\pi}{36}$

2. Which of the following statements is true:
   (A) adj (kA) = k^n (adjA)
   (B) Adjoint of a diagonal matrix of order 3×3 need not be a diagonal matrix
   (C) adj (adj A) = $|A|^{n-1}$. A
   (D) adj (AB) = adj (B) adj (A)

3. If the vectors $\bar{a} = \hat{i} + \hat{j} + \hat{k}$, $\bar{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\bar{c} = x\hat{i} + (x-2)\,\hat{j} - \hat{k}$ are coplanar, then $x =$
   (A) 1 (B) 2
   (C) 0 (D) –2

4. If $\tan 5x = 1$ (n ∈ I), then $x =$
   (A) $x = n\pi + \frac{\pi}{4}$ (B) $x = \frac{n\pi}{3} + \frac{\pi}{12}$
   (C) $x = \frac{n\pi}{15} + \frac{\pi}{20}$ (D) $x = \frac{n\pi}{5} + \frac{\pi}{20}$

5. If $\frac{x-1}{l} = \frac{y-2}{m} = \frac{z+1}{n}$ is the equation of the line through (1, 2, –1) and (–1, 0, 1), then (l, m, n) is
   (A) (–1, 0, 1) (B) (1, 1, –1)
   (C) (1, 2, –1) (D) (0, 1, 0)

6. $\int \frac{x}{16+x^4}\,dx =$
   (A) $\frac{1}{4}\tan^{-1}\left(\frac{x^2}{2}\right) + c$ (B) $\frac{1}{4}\tan^{-1}(x^2) + c$
   (C) $\frac{1}{8}\tan^{-1}\left(\frac{x^2}{4}\right) + c$ (D) $\frac{1}{8}\tan^{-1}(x^2) + c$

7. The area bounded by the parabola $x = 9 - y^2$ and Y-axis is
   (A) 18 sq. units (B) 27 sq. units
   (C) 36 sq. units (D) 45 sq. units

8. The sides in ΔABC are a = 4, b = 3 and c = 5, then $\sin\frac{A}{2} + \cos\frac{A}{2} =$
   (A) $\frac{\sqrt{5}+1}{2}$ (B) $\frac{\sqrt{3}+1}{2}$
   (C) $\frac{\sqrt{3}-1}{2}$ (D) $\frac{1+2}{\sqrt{5}}$

9. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A) =$
   (A) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (B) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$
   (C) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (D) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

10. If $|\bar{a}+\bar{b}| > |\bar{a}-\bar{b}|$, then the angle between $\bar{a}$ and $\bar{b}$ is
    (A) acute (B) obtuse
    (C) $\frac{\pi}{2}$ (D) π

11. If $\int \frac{1}{(x-1)(x^2+1)}\,dx = \frac{1}{4}\log|f(x)| - \frac{1}{2}\tan^{-1} x + c$, then f($x$) =
    (A) $\frac{x-1}{x^2+1}$ (B) $\frac{(x-1)^2}{x^2+1}$
    (C) $\frac{x-1}{(x^2+1)^2}$ (D) $\frac{x^2+1}{x-1}$

12. If $\int_0^{\pi/4} \tan^3 x \sec x\,dx = \frac{k}{3}$, then k =
    (A) 2 (B) $\sqrt{2}$
    (C) $2-\sqrt{2}$ (D) $\sqrt{2}-2$

13. The value of tan 7A – tan 5A – tan 2A is equal to
    (A) – tan 7A tan 5A tan 2A
    (B) tan 7A tan 5A tan 2A
    (C) 0
    (D) 1

14. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer k is equal to
    (A) 5 (B) 2
    (C) –2 (D) –5

15. If $\bar{u}$ and $\bar{v}$ are unit vectors and θ is the acute angle between them, then $2\bar{u} \times 3\bar{v}$ is a unit vector for
    (A) no value of θ
    (B) exactly one value of θ
    (C) exactly two values of θ
    (D) more than two values of θ

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Topic Test - 01

1. **(B)**

$\frac{1+\cos\theta+\sin\theta}{1-\cos\theta+\sin\theta}$

$= \frac{2\cos^2\left(\frac{\theta}{2}\right)+2\sin\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{\theta}{2}\right)}{2\sin^2\left(\frac{\theta}{2}\right)+2\sin\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{\theta}{2}\right)}$

$= \frac{2\cos\left(\frac{\theta}{2}\right)\left[\cos\left(\frac{\theta}{2}\right)+\sin\left(\frac{\theta}{2}\right)\right]}{2\sin\left(\frac{\theta}{2}\right)\left[\sin\left(\frac{\theta}{2}\right)+\cos\left(\frac{\theta}{2}\right)\right]}$

$= \cot\left(\frac{\theta}{2}\right)$

2. **(C)**

$\cos 10° + \cos 20° + \cos 30° + \ldots + \cos 180°$

$= (\cos 10° + \cos 170°) + (\cos 20° + \cos 160°) + \ldots + (\cos 80° + \cos 100°) + (\cos 90° + \cos 180°)$

$= -1$ ...[$\because \cos(180° - \theta) = -\cos\theta$]

3. **(B)**

$\frac{\cos 9° - \sin 9°}{\cos 9° + \sin 9°} = \frac{1-\tan 9°}{1+\tan 9°}$

$= \frac{\tan 45° - \tan 9°}{1+\tan 45° \tan 9°}$

$= \tan(45° - 9°)$

$= \tan 36°$

4. **(D)**

$\sin 10° \sin 30° \sin 50° \sin 70°$

$= \frac{1}{2}\sin 10° \sin(60° - 10°)\sin(60° + 10°)$

$= \frac{1}{2}.\frac{1}{4}\sin(3(10°))$

...[$\because \sin\theta \sin(60° - \theta)\sin(60° + \theta) = \frac{1}{4}\sin 3\theta$]

$= \frac{1}{8}\sin 30°$

$= \frac{1}{8}.\frac{1}{2}$

$= \frac{1}{16}$

5. **(C)**

$\frac{3\cos A + \cos 3A}{3\sin A - \sin 3A}$

$= \frac{3\cos A + (4\cos^3 A - 3\cos A)}{3\sin A - (3\sin A - 4\sin^3 A)}$

$= \frac{3\cos A + 4\cos^3 A - 3\cos A}{3\sin A - 3\sin A + 4\sin^3 A}$

$= \frac{4\cos^3 A}{4\sin^3 A}$

$= \cot^3 A$

6. **(C)**

Given, $\tan A + \tan B + \tan C = 6$

$\Rightarrow \tan A \tan B \tan C = 6$

...[In ΔABC, $\tan A + \tan B + \tan C = \tan A \tan B \tan C$]

$\Rightarrow 2\tan C = 6$ ...[$\because \tan A \tan B = 2$(given)]

$\Rightarrow \tan C = 3$

7. **(B)**

P and Q are supplementary angles.

$\therefore$ $P + Q = 180° \Rightarrow Q = 180° - P$

$\therefore$ $\sin^2\frac{P}{2} + \sin^2\frac{Q}{2} = \sin^2\frac{P}{2} + \sin^2\left(90° - \frac{P}{2}\right)$

$= \sin^2\frac{P}{2} + \cos^2\frac{P}{2}$

$= 1$

8. **(B)**

$\sin(45° + A) . \sin(45° - A)$

$= (\sin 45° \cos A + \cos 45° \sin A) . (\sin 45° \cos A - \cos 45° \sin A)$

$= \left(\frac{1}{\sqrt{2}}\cos A + \frac{1}{\sqrt{2}}\sin A\right)\cdot\left(\frac{1}{\sqrt{2}}\cos A - \frac{1}{\sqrt{2}}\sin A\right)$

$= \frac{1}{\sqrt{2}}(\cos A + \sin A) \times \frac{1}{\sqrt{2}}(\cos A - \sin A)$

$= \frac{1}{2}(\cos^2 A - \sin^2 A) = \frac{1}{2}\cos 2A$

9. **(D)**

$\sin\left(\theta + \frac{\pi}{6}\right) = 4\cos\left(\theta - \frac{\pi}{3}\right)$

$\therefore$ $\sin\theta\cdot\cos\frac{\pi}{6} + \cos\theta\cdot\sin\frac{\pi}{6} = 4\left(\cos\theta\cdot\cos\frac{\pi}{3} + \sin\theta\cdot\sin\frac{\pi}{3}\right)$

$\Rightarrow \frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta = 4\left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right)$

$\Rightarrow \cos\theta + \sqrt{3}\sin\theta = 0$

$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$

10. **(A)**

$(\cos 182° + \cos 62°) + \cos 58°$

$= 2\cos 122° \cos 60° + \cos 58°$

$= 2\cos 122° . \frac{1}{2} + \cos 58°$

$= \cos 122° + \cos 58°$

$= 2\cos 90° \cos 32°$

$= 0$

11. **(D)**

$e^{\log_{10}\tan 1°+\log_{10}\tan 2°+\log_{10}\tan 3°+.....+\log_{10}\tan 89°}$

$= e^{\log_{10}(\tan 1°.\ \tan 2°.\ \tan 3°.......\tan 89°)}$

$= e^{\log_{10}[\{\tan 1°.\tan(90°-1°)\}\{\tan 2°.\tan(90°-2°)\}...\tan 45°]}$

$= e^{\log_{10}(\tan 1°.\ \cot 1°.\ \tan 2°.\ \cot 2°......\tan 45°)}$

$= e^{\log_{10} 1}$

$= e^0 = 1$

12. **(B)**

x lies in IIIrd quadrant

$\Rightarrow \frac{x}{2}$ lies in IInd quadrant

$\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{5}{12}\right)^2$

$\Rightarrow \sec^2 x = \frac{169}{144}$

$\Rightarrow \sec x = \frac{-13}{12}$ …[∵ x lies in IIIrd quadrant]

$\Rightarrow \cos x = \frac{-12}{13}$

$\cos x = 2\cos^2\left(\frac{x}{2}\right) - 1$

$\Rightarrow \cos^2\left(\frac{x}{2}\right) = \frac{1+\cos x}{2} = \frac{1-\frac{12}{13}}{2}$

$\Rightarrow \cos^2\left(\frac{x}{2}\right) = \frac{1}{26}$

$\Rightarrow \cos\left(\frac{x}{2}\right) = -\sqrt{\frac{1}{26}}$

…$\left[\because \frac{x}{2}\text{ lies in II}^{nd}\text{ quadrant}\right]$

13. **(B)**

$\sin x + \sin y = \frac{1}{4}$

$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{1}{4}$ …(i)

$\cos x + \cos y = 2$

$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = 2$ …(ii)

Dividing (i) by (ii), we get

$\tan\left(\frac{x+y}{2}\right) = \frac{1}{8}$

Now, $\tan(x+y) = \frac{2\tan\left(\frac{x+y}{2}\right)}{1-\tan^2\left(\frac{x+y}{2}\right)}$

$= \frac{2\left(\frac{1}{8}\right)}{1-\frac{1}{64}} = \frac{16}{63}$

14. **(A)**

$\frac{1}{4}\left(\sqrt{3}\cos 28° - \sin 28°\right) = \frac{1}{2}\left(\frac{\sqrt{3}}{2}\cos 28° - \frac{1}{2}\sin 28°\right)$

$= \frac{1}{2}$ (cos 30° cos 28° – sin 30° sin 28°)

$= \frac{1}{2}$ cos (30° + 28°)

$= \frac{1}{2}$ cos 58°

15. **(A)**

$\sin^2(5°) + \sin^2(10°) + \sin^2(15°) + \ldots + \sin^2(80°) + \sin^2(85°) + \sin^2(90°)$

$= \sin^2(5°) + \sin^2(10°) + \sin^2(15°) + \ldots + \cos^2(10°) + \cos^2(5°) + \sin^2(90°)$

…[∵ sin (90° – θ) = cos θ]

$= [\sin^2(5°) + \cos^2(5°)] + \ldots + [\sin^2(40°) + \cos^2(50°)] + \sin^2(30°) + \sin^2(45°) + \sin^2(60°) + \sin^2(90°)$

$= (1 + 1 + \ldots + 1) + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$

$= 7 + \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1$

$= \frac{19}{2}$

16. **(C)**

$\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x}$

$= \frac{\sin 5x + \sin x - 2\sin 3x}{\cos 5x - \cos x}$

$= \frac{2\sin 3x\cos 2x - 2\sin 3x}{-2\sin 3x\sin 2x}$

$= \frac{2\sin 3x(\cos 2x - 1)}{-2\sin 3x\sin 2x}$

$= \frac{1-\cos 2x}{\sin 2x}$

$= \frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x}$

$= \text{cosec } 2x - \cot 2x$

17. **(B)**

$\cos\left(\frac{13\pi}{18}\right) = \cos\left(\pi - \frac{5\pi}{18}\right) = -\cos\left(\frac{5\pi}{18}\right)$

$\therefore \cos^2\left(\frac{13\pi}{18}\right) = \cos^2\left(\frac{5\pi}{18}\right)$

$\cos\left(\frac{7\pi}{9}\right) = \cos\left(\pi - \frac{2\pi}{9}\right) = -\cos\left(\frac{2\pi}{9}\right)$

$\therefore \cos^2\left(\frac{7\pi}{9}\right) = \cos^2\left(\frac{2\pi}{9}\right)$

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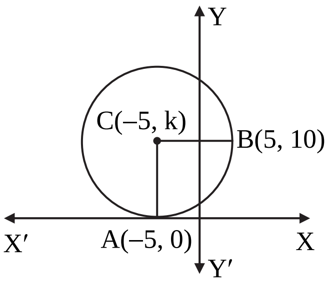
To see complete chapter buy **Target Notes** or **Target E-Notes**

39. **(C)**



Since the circle touches the X-axis at (–5, 0).

$\therefore$ Centre $\equiv (-5, k)$

Now, $AC^2 = BC^2$

$\Rightarrow (-5 + 5)^2 + (k - 0)^2 = (-5 - 5)^2 + (k - 10)^2$

$\Rightarrow k^2 = 100 + k^2 - 20k + 100$

$\Rightarrow 20k = 200$

$\Rightarrow k = 10$

$\therefore$ Centre $\equiv (-5, 10)$

Radius = 10

$\therefore$ Equation of circle is

$(x + 5)^2 + (y - 10)^2 = 10^2$

$\Rightarrow x^2 + y^2 + 10x - 20y + 25 = 0$

Only option (C) satisfies the above equation.

40. **(B)**

Given equation of circle is

$x^2 + y^2 = 1$

$\therefore$ radius = 1

$\therefore$ P = $(\cos\alpha, \sin\alpha)$

Q = $(\cos\beta, \sin\beta)$

R = $(\cos\gamma, \sin\gamma)$

$\therefore$ $AP = \sqrt{(\cos\alpha + 1)^2 + (\sin\alpha - 0)^2}$

$= \sqrt{\cos^2\alpha + 2\cos\alpha + 1 + \sin^2\alpha}$

$= \sqrt{2(1 + \cos\alpha)}$

$= \sqrt{2.2\cos^2\frac{\alpha}{2}} = 2\cos\frac{\alpha}{2}$

Similarly, $AQ = 2\cos\frac{\beta}{2}$, $AR = 2\cos\frac{\gamma}{2}$

Since AP, AQ, AR are in G.P.

$\therefore$ $AQ^2 = AP.AR$

$\Rightarrow \left(2\cos\frac{\beta}{2}\right)^2 = 2\cos\frac{\alpha}{2}\cdot 2\cos\frac{\gamma}{2}$

$\Rightarrow \left(\cos\frac{\beta}{2}\right)^2 = \cos\frac{\alpha}{2}\cos\frac{\gamma}{2}$

$\therefore$ $\cos\frac{\alpha}{2}$, $\cos\frac{\beta}{2}$, $\cos\frac{\gamma}{2}$ are in G.P.

Revision Test - 01

1. **(C)**

Given equation of circle is

$4x^2 + 2\lambda xy + 4y^2 + (8 - \lambda)x + (3\lambda - 8)y - 56 = 0$

$\therefore$ $2\lambda = 0$

$\Rightarrow \lambda = 0$

$\therefore$ The equation of circle is

$4x^2 + 4y^2 + 8x - 8y - 56 = 0$

$\Rightarrow x^2 + y^2 + 2x - 2y - 14 = 0$

$\therefore$ Radius = $\sqrt{1^2 + (-1)^2 + 14}$

$= \sqrt{16} = 4$

2. **(B)**

$A + B = \frac{\pi}{4}$

$\Rightarrow \tan(A + B) = 1$

$\Rightarrow \tan A + \tan B = 1 - \tan A \tan B$

$\Rightarrow \frac{1}{\cot A} + \frac{1}{\cot B} = 1 - \frac{1}{\cot A \cot B}$

$\Rightarrow \cot A + \cot B = \cot A \cot B - 1$

$\Rightarrow \cot A \cot B - \cot A - \cot B + 1 = 2$

$\Rightarrow (\cot A - 1)(\cot B - 1) = 2$

3. **(D)**

The equation of the line passing through (5, 3) and (– 5, – 7) is

$\frac{y - 3}{3 - (-7)} = \frac{x - 5}{5 - (-5)}$

$\Rightarrow \frac{y - 3}{10} = \frac{x - 5}{10}$

$\Rightarrow y = x - 2$

$\therefore$ m = 1 and c = – 2

4. **(A)**

$x^2 + y^2 + 6x + 6y = 0$

$C_1 = (-3, -3)$, $r_1 = \sqrt{3^2 + 3^2 - 0} = 3\sqrt{2}$

$x^2 + y^2 - 12x - 12y = 0$

$C_2 = (6, 6)$, $r_2 = \sqrt{(-6)^2 + (-6)^2 - 0} = 6\sqrt{2}$

$C_1C_2 = \sqrt{[6 - (-3)]^2 + [6 - (-3)]^2}$

$= 9\sqrt{2}$

$C_1C_2 = r_1 + r_2$

$\therefore$ The given circles touch each other externally.

5. **(A)**

Given, sin 2A + cos 2A = 1

Squaring on both sides, we get

$(\sin 2A + \cos 2A)^2 = 1$

$\Rightarrow 1 + \sin 4A = 1$

$\Rightarrow \sin 4A = 0$

6. **(C)**

The equation of the circle with centre (0, 2) is $x^2 + (y - 2)^2 = r^2$

It passes through the point (2, 2) .

$\therefore$ $2^2 + (2 - 2)^2 = r^2$

$\Rightarrow$ Radius(r) = 2

$\therefore$ The equation of the circle is $x^2 + y^2 - 4y = 0$.
The equation of the tangent at (2, 2) is
$2x + 2y - 2(y + 2) = 0$
$\Rightarrow x - 2 = 0$

7. **(B)**
$2 + 2\cos 8\theta = 2(1 + \cos 8\theta)$
$= 2.\ 2\cos^2 4\theta$
$= 4\cos^2 4\theta$
$\therefore$ $\sqrt{2+2\cos 8\theta} = 2\cos 4\theta$
Now,
$\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = \sqrt{2+\sqrt{2+2\cos 4\theta}}$
$= \sqrt{2+\sqrt{2(1+\cos 4\theta)}}$
$= \sqrt{2+2\cos 2\theta}$
$= \sqrt{2(1+\cos 2\theta)}$
$= \sqrt{2\times 2\cos^2\theta}$
$= 2\cos\theta$

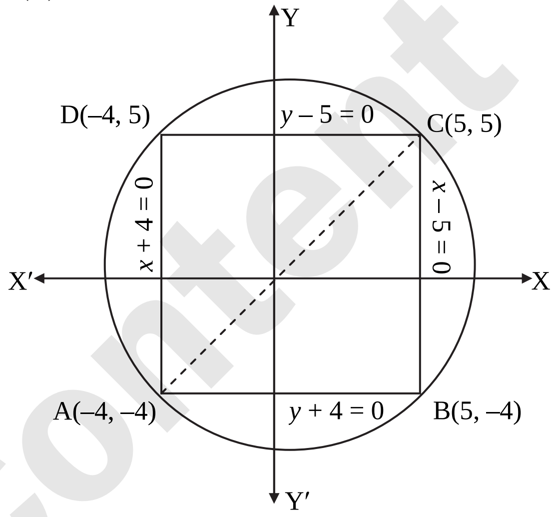
8. **(D)**
Slope of $3x - 4y = 25$ is $\frac{3}{4}$.
$\therefore$ Slope of line perpendicular to $3x - 4y = 25$ is $\frac{-4}{3}$.
$\therefore$ Equation of line passing through (– 2, 6) and having slope $\frac{-4}{3}$ is
$y - 6 = \frac{-4}{3}(x + 2)$
$\Rightarrow 4x + 3y - 10 = 0$
For y intercept, put $x = 0$
$\therefore$ $0 + 3y - 10 = 0$
$\Rightarrow y = \frac{10}{3}$

9. **(C)**
Let the equation of circle be
$x^2 + y^2 + 2gx + 2fy + c = 0$.
Now on passing through the given points, we get three equations
$c = 0$ ….(i)
$9 + 6g + c = 0$ ….(ii)
$16 + 8f + c = 0$ ….(iii)
Solving equations (i), (ii) and (iii), we get
$g = \frac{-3}{2}$, $f = -2$
Hence, the centre is $\left(\frac{3}{2}, 2\right)$.

10. **(A)**
Let $25° + \theta = A$ and $25° - \phi = B$
$\therefore$ $\sin(25° + \theta).\cos(25° - \phi)$
$- \cos(25° + \theta)\sin(25° - \phi)$
$= \sin A\cos B - \cos A\sin B$
$= \sin(A - B)$
$= \sin[(25° + \theta) - (25° - \phi)]$
$= \sin(25° + \theta - 25° + \phi)$
$= \sin(\theta + \phi)$

11. **(D)**
Slope of PQ = Slope of QR = Slope of RS = $\frac{b}{a}$
$\therefore$ The points P, Q, R, S are collinear.

12. **(B)**



The vertices of the square are
A(– 4, – 4), B(5, – 4), C(5, 5), D(– 4, 5)
Diagonal AC passes through the origin.
$\therefore$ Equation of the circle is
$(x + 4)(x - 5) + (y + 4)(y - 5) = 0$
$\Rightarrow x^2 + y^2 - x - y - 40 = 0$

13. **(D)**
$\sin^2 22.5° + \sin^2 67.5°$
$= \sin^2 22.5° + [\sin(90° - 22.5°)]^2$
$= \sin^2 22.5° + \cos^2 22.5°$
$= 1 = \tan^2 45°$

14. **(C)**
Let P(x, y) be any point on the locus and A ≡ (1, 2) and B ≡ (3, 0), then ∠APB = 90°
$\therefore$ By Pythagoras theorem,
$AP^2 + BP^2 = AB^2$
$\Rightarrow (x - 1)^2 + (y - 2)^2 + (x - 3)^2 + (y - 0)^2 = (3 - 1)^2 + (0 - 2)^2$
$\Rightarrow 2x^2 + 2y^2 - 8x - 4y + 6 = 0$
$\Rightarrow x^2 + y^2 - 4x - 2y + 3 = 0$

15. **(B)**
Slope of the line(m) = $\tan 120° = -\sqrt{3}$
$\therefore$ The equation of the line having slope m and passing through (4, 3) is
$y - 3 = -\sqrt{3}(x - 4)$
$\Rightarrow \sqrt{3}x + y - 3 - 4\sqrt{3} = 0$

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To see complete chapter buy **Target Notes** or **Target E-Notes**

Model Test Paper - 01

1. **(B)**

$$\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}\,dx = \frac{1}{2}(b-a)$$

$\therefore$ $$\int_{\frac{\pi}{18}}^{\frac{4\pi}{9}} \frac{2\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

$$= 2\int_{\frac{\pi}{18}}^{\frac{4\pi}{9}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

$$= 2\left[\frac{1}{2}\left(\frac{4\pi}{9}-\frac{\pi}{18}\right)\right]$$

$$= \frac{7\pi}{18}$$

2. **(D)**

3. **(D)**

Since $\bar{a} = \hat{i} + \hat{j} + \hat{k}$, $\bar{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\bar{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$ are coplanar vectors,

$\left[\bar{a}\ \bar{b}\ \bar{c}\right] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$$

$\Rightarrow 1\,[1 - 2(x-2)] - 1\,(-1-2x) + 1(x-2+x) = 0$

$\Rightarrow 1 - 2x + 4 + 1 + 2x + 2x - 2 = 0$

$\Rightarrow 2x = -4$

$\Rightarrow x = -2$

4. **(D)**

$\tan 5x = 1$

$\therefore$ $\tan 5x = \tan\frac{\pi}{4} \Rightarrow 5x = n\pi + \frac{\pi}{4}$

$$\ldots\left[\begin{array}{l}\because \tan\theta = \tan\alpha \\ \Rightarrow \theta = n\pi + \alpha\end{array}\right]$$

$\therefore$ $x = \frac{n\pi}{5} + \frac{\pi}{20}$, $n \in I$

5. **(B)**

Given equation is $\frac{x-1}{l} = \frac{y-2}{m} = \frac{z+1}{n}$

The equation of line passing through $(1, 2, -1)$ and $(-1, 0, 1)$ is

$$\frac{x-1}{-1-1} = \frac{y-2}{0-2} = \frac{z+1}{1+1}$$

$$\Rightarrow \frac{x-1}{-2} = \frac{y-2}{-2} = \frac{z+1}{2}$$

$$\Rightarrow \frac{x-1}{1} = \frac{y-2}{1} = \frac{z+1}{-1} \quad \ldots\text{(i)}$$

Comparing (i) with given equation, we get

$l = 1$, $m = 1$, $n = -1$

6. **(C)**

Put $x^2 = t \Rightarrow 2x\,dx = dt$

$\therefore$ $$\int \frac{x}{16+x^4}\,dx = \frac{1}{2}\int \frac{dt}{4^2+t^2}$$

$$= \frac{1}{2}\cdot\frac{1}{4}\tan^{-1}\left(\frac{t}{4}\right) + c$$

$$= \frac{1}{8}\tan^{-1}\left(\frac{x^2}{4}\right) + c$$

7. **(C)**

Required area

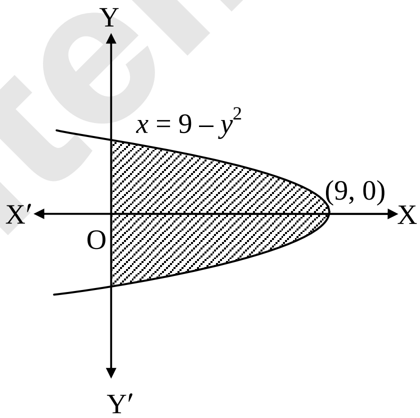
$$= 2\int_0^9 \sqrt{9-x}\,dx$$

$$= 2\left[\frac{-(9-x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^9$$

$$= \frac{-4}{3}\left(0 - 9^{\frac{3}{2}}\right)$$

$$= \frac{-4}{3}(-27)$$

$= 36$ sq. units



8. **(D)**

$$s = \frac{a+b+c}{2} = \frac{12}{2} = 6$$

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}} = \sqrt{\frac{3\times1}{15}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$$

$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}} = \sqrt{\frac{6\times2}{15}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$\therefore$ $$\sin\frac{A}{2} + \cos\frac{A}{2} = \frac{1+2}{\sqrt{5}}$$

9. **(C)**

$$A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$$

$$3A^2 = 3\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix} = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix}$$

$$3A^2 + 12A = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$\therefore$ $$\text{adj}\,(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$$

10. **(A)**

$|\bar{a}+\bar{b}| > |\bar{a}-\bar{b}|$

Squaring both sides, we get

$\left|\bar{a}\right|^2 + \left|\bar{b}\right|^2 + 2\bar{a}\cdot\bar{b} > \left|\bar{a}\right|^2 + \left|\bar{b}\right|^2 - 2\bar{a}\cdot\bar{b}$

$\Rightarrow\ 4\bar{a}\cdot\bar{b} > 0$

$\Rightarrow\ \cos\theta > 0$

Hence, $\theta < 90°$ (acute).

11. **(B)**

Let $\dfrac{1}{(x-1)(x^2+1)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+1}$

$\therefore$ $1 = A(x^2+1) + (Bx+C)(x-1)$ ...(i)

Putting $x = 1$ in (i), we get

$A = \dfrac{1}{2}$

Putting $x = 0$ in (i), we get

$A - C = 1 \Rightarrow C = -\dfrac{1}{2}$

Comparing the coefficient of x^2, we get

$A + B = 0 \Rightarrow B = -\dfrac{1}{2}$

$\therefore$ $\displaystyle\int \frac{1}{(x-1)(x^2+1)}dx = \int\left[\frac{1}{2(x-1)} - \frac{x+1}{2(x^2+1)}\right]dx$

$= \dfrac{1}{2}\displaystyle\int\frac{1}{x-1}dx - \frac{1}{4}\int\frac{2x}{x^2+1}dx - \frac{1}{2}\int\frac{1}{x^2+1}dx$

$= \dfrac{1}{2}\log|x-1| - \dfrac{1}{4}\log|x^2+1| - \dfrac{1}{2}\tan^{-1}x + c$

$= \dfrac{1}{4}\log\left|\dfrac{(x-1)^2}{x^2+1}\right| - \dfrac{1}{2}\tan^{-1}x + c$

$\therefore$ $f(x) = \dfrac{(x-1)^2}{x^2+1}$

12. **(C)**

Let $I = \displaystyle\int_0^{\pi/4}\tan^3 x\sec x\,dx$

$= \displaystyle\int_0^{\pi/4}(\sec^2 x - 1)\sec x\tan x\,dx$

Put $\sec x = t \Rightarrow \sec x\tan x\,dx = dt$

When $x = 0$, $t = 1$ and when $x = \dfrac{\pi}{4}$, $t = \sqrt{2}$

$\therefore$ $I = \displaystyle\int_1^{\sqrt{2}}(t^2-1)dt$

$= \left[\dfrac{t^3}{3} - t\right]_1^{\sqrt{2}}$

$= \dfrac{2\sqrt{2}-1}{3} - (\sqrt{2}-1)$

$= \dfrac{2-\sqrt{2}}{3}$

$\therefore$ $k = 2-\sqrt{2}$

13. **(B)**

$\tan 7A = \tan(5A + 2A)$

$\therefore$ $\tan 7A = \dfrac{\tan 5A + \tan 2A}{1 - \tan 5A\cdot\tan 2A}$

$\therefore$ $\tan 7A - \tan 7A\tan 5A\tan 2A = \tan 5A + \tan 2A$

$\therefore$ $\tan 7A - \tan 5A - \tan 2A = \tan 7A\tan 5A\tan 2A$

14. **(D)**

Since the given lines intersect each other,

$\begin{vmatrix} 2-1 & 3-2 & 1-3 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$

$\Rightarrow 1(4-3k) - 1(2k-9) - 2(k^2-6) = 0$

$\Rightarrow 2k^2 + 5k - 25 = 0$

$\Rightarrow k = \dfrac{5}{2}, -5$

15. **(B)**

Since, $|\bar{u}| = |\bar{v}| = 1$ and θ is the acute angle between $\bar{u}$ and $\bar{v}$.

$\therefore$ $|\bar{u}\times\bar{v}| = \sin\theta$ …(i)

Now, $2\bar{u}\times 3\bar{v}$ will be a unit vector, if

$|2\bar{u}\times 3\bar{v}| = 1$

$\Rightarrow 6|\bar{u}\times\bar{v}| = 1$

$\Rightarrow 6\sin\theta = 1$ …[From (i)]

$\Rightarrow \sin\theta = \dfrac{1}{6}$

As θ is an acute angle. So, there is only one value of θ for which $2\bar{u}\times 3\bar{v}$ is a unit vector.

16. **(D)**

$\cos\theta = 2x^2 - 1$

$\therefore$ $\cos\theta = 2\sin^2 35° - 1$ …[$\because \sin 35° = x$]

$= -(1 - 2\sin^2 35°)$

$= -\cos(2\times 35°) = -\cos 70°$

$\therefore$ $\cos\theta = \cos(180° + 70°) = \cos 250^{o}$

and $\cos\theta = \cos(180° - 70°) = \cos 110^{o}$

$\therefore$ $\theta = 110°$ and $250°$

17. **(C)**

Given equation of pair of lines is

$3y^2 - 8xy + px^2 - 29x + 3y - 18 = 0$

$\therefore$ $a = p,\ b = 3,\ c = -18,\ f = \dfrac{3}{2},\ g = \dfrac{-29}{2},\ h = -4$

The given equation represents pair of straight lines if $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow p(3)(-18) + 2\left(\dfrac{3}{2}\right)\left(\dfrac{-29}{2}\right)(-4) - p\left(\dfrac{3}{2}\right)^2 - 3\left(\dfrac{-29}{2}\right)^2 + 288 = 0$

$\Rightarrow\ p = -3$

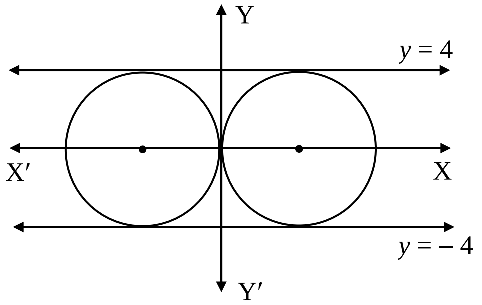
18. (B)

$$\int e^{3x}\left(\frac{1}{x}-\frac{1}{3x^2}\right)dx = \frac{e^{3x}}{3x} + c$$

$$\ldots\left[\because \int e^{mx}\left[f(x)+\frac{f'(x)}{m}\right]dx = \frac{e^{mx}f(x)}{m}+c\right]$$

19. (B)

Two circles can be drawn.



20. (D)

The d.r.s. of line are 2, –1, 1 and

the d.r.s. of normal to the plane are –3, 4, 1

∴ The angle between line and plane is

$$\sin\theta = \left|\frac{-6-4+1}{\sqrt{4+1+1}\sqrt{9+16+1}}\right| = \left|\frac{-9}{\sqrt{156}}\right| = \frac{9}{\sqrt{156}}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{9}{\sqrt{156}}\right)$$

$$= \sin^{-1}\left(\frac{9}{2\sqrt{39}}\right)$$

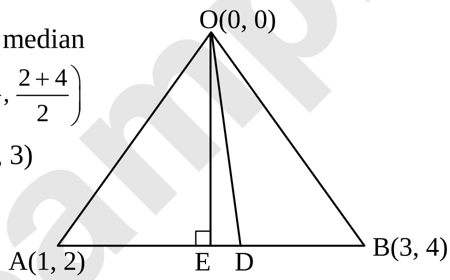
$$= \cos^{-1}\left(\frac{5}{2\sqrt{13}}\right)$$

21. (D)

OD is the median

∴ $D \equiv \left(\frac{1+3}{2}, \frac{2+4}{2}\right)$

$\Rightarrow D \equiv (2, 3)$



Equation of OD is $y = mx$

$\Rightarrow y = \frac{3}{2}x \Rightarrow 3x - 2y = 0$

Slope of line AB = $\frac{2}{2} = 1$

Given, OE ⊥ AB

∴ Slope of OE = –1

Equation of OE is $y = mx$

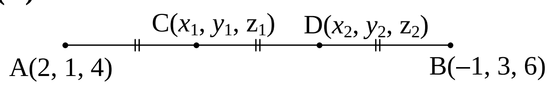
$\Rightarrow y = -x \Rightarrow x + y = 0$

∴ Joint equation of median and altitude is

$(3x - 2y)(x + y) = 0$

$\Rightarrow 3x^2 + xy - 2y^2 = 0$

22. (D)



C divides AB internally in the ratio 1 : 2 and

D divides AB internally in the ratio 2 : 1.

∴ $z_1 + z_2 = \frac{1(6)+2(4)}{1+2} + \frac{2(6)+1(4)}{2+1}$

$= \frac{14}{3} + \frac{16}{3}$

$= \frac{30}{3}$

$= 10$

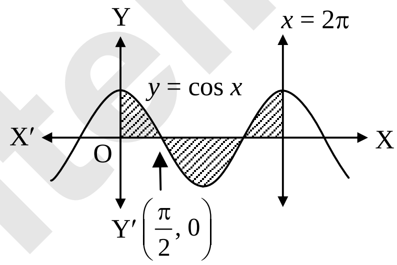
23. (D)

Required area

$= 4\int_0^{\pi/2} \cos x\, dx$

$= 4[\sin x]_0^{\pi/2}$

= 4 sq. units



24. (A)

Let $\theta = \tan^{-1}\left(\frac{2}{7}\right)$

∴ $\sin\left(2\tan^{-1}\left(\frac{2}{7}\right)\right)$

= sin 2θ

= 2 sin θ cos θ

$= 2\sin\left(\tan^{-1}\left(\frac{2}{7}\right)\right)\cos\left(\tan^{-1}\left(\frac{2}{7}\right)\right)$

$$= 2 \times \frac{\frac{2}{7}}{\sqrt{1+\frac{4}{49}}} \times \frac{1}{\sqrt{1+\frac{4}{49}}}$$

$$\ldots\left[\begin{array}{l}\sin\left(\tan^{-1}x\right) = \frac{x}{\sqrt{1+x^2}}, \text{ and} \\ \cos\left(\tan^{-1}x\right) = \frac{1}{\sqrt{1+x^2}}\end{array}\right]$$

$$= 2 \times \frac{2}{\sqrt{53}} \times \frac{1}{\sqrt{53}} = \frac{4}{53}$$

25. (B)

Point (2, 1, –2) lies in the plane

$x + 3y - \alpha z + \beta = 0$

∴ $2 + 3(1) - \alpha(-2) + \beta = 0$

$\Rightarrow 2\alpha + \beta = -5$ …(i)

Also, the d.r.s of the normal are perpendicular to the given plane.

∴ $3(1) + (-5)(3) + (2)(-\alpha) = 0$

$\Rightarrow 3 - 15 - 2\alpha = 0$

$\Rightarrow \alpha = -6$

Substituting value of α in equation (i), we get

$\beta = 7$

26. **(B)**

Applying L'Hospital's rule, we get

$$\lim_{x\to 5}\frac{e^x - e^5}{x-5} = \lim_{x\to 5}\frac{e^x}{1} = e^5$$

27. **(D)**

The probability distribution of X is

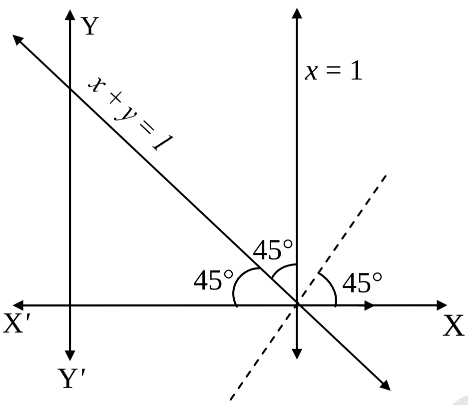
X	10	11	12
P (X)	3k	k	2k

Since $\sum_{x=10}^{12} P(X = x) = 1$,

3k + k + 2k = 1

$\Rightarrow$ 6k =1 $\Rightarrow$ k = $\frac{1}{6}$

28. **(B)**



Angle of incidence = 45°

$\therefore$ Angle of reflection = 45°

From geometry, reflected ray will travel along X-axis.

29. **(A)**

$y = ce^{\cos^{-1}x}$ …(i)

$\Rightarrow \frac{dy}{dx} = ce^{\cos^{-1}x}.\frac{-1}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{\sqrt{1-x^2}}$ …[From (i)]

30. **(B)**

$y = \tan^{-1}\left(\frac{8+7\tan x}{7-8\tan x}\right)$

$= \tan^{-1}\left(\frac{\frac{8}{7}+\tan x}{1-\frac{8}{7}\tan x}\right)$

$= \tan^{-1}\left(\frac{8}{7}\right) + \tan^{-1}(\tan x)$

$= \tan^{-1}\left(\frac{8}{7}\right) + x$

$\therefore \frac{dy}{dx} = 0 + 1 = 1$

31. **(A)**

$(x + yi)^{1/3} = u + vi$

$\Rightarrow (u + vi)^3 = x + yi$

$\Rightarrow u^3 - 3uv^2 + i\,(3u^2v - v^3) = x + y\,i$

$\Rightarrow u^3 - 3uv^2 = x$ and $3u^2v - v^3 = y$

$\Rightarrow \frac{x}{u} = u^2 - 3v^2$ and $\frac{y}{v} = 3u^2 - v^2$

$\Rightarrow \frac{x}{u} + \frac{y}{v} = 4\,(u^2 - v^2)$

32. **(C)**

p	q	~q	p ∨ ~q	~(p ∨~q)	~(p ∨ ~q) → p
T	T	F	T	F	T
T	F	T	T	F	T
F	T	F	F	T	F
F	F	T	T	F	T

33. **(C)**

$\frac{dy}{dx} = x^2 + 3\cos 3x$

Integrating on both sides, we get

$\int dy = \int\left(x^2 + 3\cos 3x\right)dx$

$\Rightarrow y = \frac{x^3}{3} + \sin 3x + c$

34. **(D)**

f(x) is continuous at x = 0.

$\therefore$ $f(0) = \lim_{x\to 0} f(x)$

$\Rightarrow a = \lim_{x\to 0}\frac{\cos 5x - \cos 2x}{x^2}$

Applying L'Hospital's rule on R.H.S., we get

$a = \lim_{x\to 0}\frac{-5\sin 5x + 2\sin 2x}{2x}$

Applying L'Hospital's rule on R.H.S., we get

$a = \lim_{x\to 0}\frac{-25\cos 5x + 4\cos 2x}{2}$

$\Rightarrow a = \frac{-25+4}{2} = \frac{-21}{2}$

35. **(B)**

Suppose x kg of food A and y kg of food B are consumed to form a weekly diet.

$\therefore$ $x \geq 0, y \geq 0$.

…[Since quantity of food cannot be negative]

Representing the given information in table form, we get

	Food A (x)	**Food B (y)**	**Minimum requirement**
Fats (units)	4	12	18
Carbohydrates (units)	16	4	24
Protein (units)	8	6	16
Cost (₹)	6	5	z

$\therefore$ Required LPP is formulated as

Minimize z = 6x + 5y subject to constraints,

$4x + 12y \geq 18$, $16x + 4y \geq 24$, $8x + 6y \geq 24$, $x \geq 0$, $y \geq 0$