SAMPLE CONTENT

CUET (UG)

COMMON UNIVERSITY ENTRANCE TEST

Based on notified syllabus prescribed by NTA

1226 MCQs

LOADED WITH AMAZING FEATURES

- Caution
- Connections
- Topic Test
- Shortcuts
- Subtopic wise MCQs

MATHEMATICS

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Target Publications® Pvt. Ltd.

For Common University Entrance Test CUET (UG) based on syllabus prescribed by Testing Agency (NTA)

CUET (UG)

Mathematics

Salient Features

- **'1226'** MCQs for ample practice
- Synopsis to offer a crisp overview of the chapter
- Subtopic wise segregation of MCQs for extensive practice
- Detailed solutions provided for better understanding
- Connections, Cautions designed to impart holistic learning
- Shortcuts are provided wherever deemed necessary
- A list of formulae provided via Q.R. code for quick revision
- Topic Test provided for self-assessment at the end of each chapter
- Solution to Topic Test accessible via Q.R. code
- Includes Passage-based MCQs with Answers (Solution provided through Q.R. code)
- Includes relevant questions of CUCET 2021
- Includes Question Paper of CUET (UG) 2022–17th August (Slot - 1) (Solution provided through Q.R. Code)

Please scan the adjacent QR code in *Quill - The Padhai App* to access the list of formulae segregated chapter-wise

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PREFACE

Common University Entrance Test, CUET (UG) is a pivotal juncture in a student's academic journey. It is a single-window opportunity for the Students to seek admission in the premier higher education institutions.

Target Publications, with more than a decade of experience and expertise in the domain of competitive examination, offers **"CUET (UG) Mathematics"** for all the CUET (UG) aspirants. This book is compiled according to the notified syllabus prescribed by NTA for CUET (UG).

It is a complete preparation and practice book with the unmatched comprehensive amalgamation of theory, MCQs, and the tools that will be needed to clear the exam successfully.

The content of this book is arranged in a logical sequence to enable strategic learning. It provides the students with scientifically accurate context, several study techniques, and relevant supporting details essential for a better understanding of the concepts of Mathematics.

The chapter begins with a **Synopsis** to offer crisp revision to students in efficient form of pointers, tables, charts etc. followed by a variety of **Multiple Choice Questions (MCQs)** in order to familiarize the students with the pattern of competitive examination.

To aid students, Solutions are provided as pertinent. **'Shortcut'** helps students to save time while dealing with a lengthy solution of a question. **'Caution'** is added to make students watchful against commonly made mistakes. Also, **'Connections'** are furnished to enable students perceive the interlinking of concepts covered in different chapters and preparing them for possible coalition questions.

Topic Test is provided at the end of each chapter for self evaluation. Solution to Topic Test can be viewed by scanning the QR code provided at the end of each chapter.

A section of **Passage-based MCQs** covering a wide range of concepts is included at the end of the book. These passages are segregated chapter-wise and their solutions can be viewed through Q.R. code in a pdf format.

Question paper of CUET (UG) 2022–17th August (Slot - 1) is provided at the end of book and its solutions can be viewed though Q.R. code in a pdf format, as the exam papers offer students a glimpse of the complexity of questions asked in entrance examination. The paper has been split topic wise to let the students know which of the topics were more relevant in the latest examination.

All the features of this book would pave the path for a student to excel in their examination. The features are designed keeping the following elements in mind: Time management, easy memorization, revision, and non-conventional yet simple methods for MCQ solving.

We are confident that this book will cater to the needs of students across varied backgrounds and effectively assist them to achieve their goals.

We hope the book benefits the learner as we have envisioned.

Publisher

Edition: Second

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.

Please write to us on: mail@targetpublications.org

KEY FEATURES

Synopsis

‘**Synopsis**’ presents crisp summary of a chapter in efficient form of pointers, tables, charts.

Connections

‘**Connections**’ enable students to interlink concepts covered in different chapters.

Caution

‘**Caution**’ apprises students about commonly made mistakes.

Shortcut

‘**Shortcut**’ section provides the students with tricks to arrive at the correct answer in a more non-conventional yet simple way.

Clock Symbol

‘**Clock Symbol**’ instruct students that the given MCQ can be solved apace by applying Shortcut.

Q.R. Code

‘**QR code**’ provides:

i. Solution to Topic Test of each chapter.
ii. Access to the list of formulae segregated chapter-wise, beneficial for last minute revision.
iii. Solutions to Passage-based MCQs.
iv. Solution to CUET (UG) 2022–17th August (Slot - 1)

CONTENTS

Note: Symbol along with the question indicates that there exists use of Shortcut for solving that MCQ in the Hint.

'**Caution**' makes students watchful against commonly made mistakes.

'**Connections'** to interlink concepts covered in different chapters.

Broad features of CUET (UG)

Mode of Examination: Computer Based Test (CBT) mode

Sections	Subjects/ Tests	Questions to be Attempted	Marks per Question	Total Marks	Question Type	Duration
Section IA - Languages	There are 13 different languages. Any of these languages may be chosen.	40 questions out of 50 in each language	5	200	• Language to be tested through Reading Comprehension based on different types of passages– Factual, Literary and Narrative, [Literary Aptitude and Vocabulary] • MCQ Based Questions	45 Minutes for each language
Section IB - Languages	There are 20 Languages. Any other language apart from those offered in Section I A may be chosen.					
Section II - Domain	There are 27 Domains specific Subjects being offered under this Section. A candidate may choose a maximum of Six Domains as desired by the applicable University/ Universities.	40 questions out of 50 in each subject	5	200	• Input text can be used for MCQ Based Questions • MCQs based on syllabus given on NTA website	45 Minutes for each Domain Specific Subjects
Section III General Test	For any such undergraduate programme/ programmes being offered by Universities where a General Test is being used for admission.	60 questions out of 75	5	300	• Input text can be used for MCQ Based Questions • General Knowledge, Current Affairs, General Mental Ability, Numerical Ability, Quantitative Reasoning (Simple application of basic mathematical arithmetic/ algebra geometry/ mensuration /stat taught till Grade 8), Logical and Analytical Reasoning	60 Minutes

Note:
- One mark will be deducted for a wrong answer.
- Unanswered/Marked for Review will be given no mark (0).

Candidates are advised to visit the NTA CUET (UG) official website ***https://cuet.samarth.ac.in/*** *for latest updates regarding the Examination.*

Note: In Section II, there are some Domain Specific Subjects which are multidisciplinary. In those subjects there will be two Sections, wherein Section A will be Compulsory for all. Section B may have more than one Sub Sections out of which a candidate can choose one or more than one Sections depending on the eligibility conditions of the Programme/ University they are applying for.

Mathematics Question Paper **(Section II)** will contain Two Sub-sections - Section A and Section B (B1 and B2, out of which only one Sub Section is to be attended.)
Section II-A will have 15 questions covering both i.e. Mathematics/Applied Mathematics which will be compulsory for all candidates.
Section II-B1 will have 35 questions from Mathematics out of which 25 questions need to be attempted.
Section II-B2 will have 35 questions purely from Applied Mathematics out of which 25 questions will be attempted.

In this book, we will be focusing on Section II-A and Section II-B1.
Following table maps the topics mentioned in the syllabus to the subtopics mentioned in the book.

How This Book Covers the Entire Syllabus of CUET (UG) Mathematics

SECTION II-A

CUET Syllabus for Section II-A	Chapter No.	Subtopic No.	Subtopic name
ALGEBRA			
Chapter 3 : Matrices			
Matrices and types of Matrices	3	3.2, 3.3	Matrix, Types of Matrices
Equality of Matrices, transpose of a Matrix, Symmetric and Skew Symmetric Matrix	3	3.3, 3.5, 3.6	Types of Matrices, Transpose of a Matrix, Symmetric and Skew Symmetric Matrices
Algebra of Matrices	3	3.4	Operations on Matrices
Chapter 4 : Determinants			
Determinants	4	4.2, 4.3	Determinant, Properties of Determinants
Inverse of a Matrix	4	4.5, 4.6	Minors and Cofactors, Adjoint and Inverse of a Matrix
Solving of simultaneous equations using Matrix Method	4	4.7	Applications of Determinants and Matrices
CALCULUS			
Chapter 5 : Continuity and Differentiability			
Higher order derivatives	5	5.7	Second Order Derivative
Chapter 6 : Application of Derivatives			
Tangents and Normals	6	6.4	Tangents and Normals
Increasing and Decreasing Functions	6	6.3	Increasing and Decreasing Functions
Maxima and Minima	6	6.6	Maxima and Minima
INTEGRATION AND ITS APPLICATIONS			
Chapter 7 : Integrals			
Indefinite integrals of simple functions	7	7.3, 7.4	Methods of Integration, Integrals of some Particular Functions
Evaluation of indefinite integrals	7	7.5, 7.6	Integration by Partial Fractions, Integration by Parts
Definite Integrals	7	7.7	Definite Integral
Chapter 8 : Application of Integrals			
Application of Integration as area under the curve	8	8.2	Area under Simple Curves
DIFFERENTIAL EQUATIONS			
Chapter 9 : Differential Equations			
Order and degree of differential equations	9	9.2	Basic Concepts
Formulating and solving of differential equations with variable separable	9	9.4, 9.5	Formation of a Differential Equation whose General Solution is given, Methods of Solving First order, First Degree Differential Equations
PROBABILITY DISTRIBUTIONS			
Chapter 13 : Probability			
Random variables and its probability distribution	13	13.6	Random Variables and its Probability Distributions
Expected value of a random variable			
Variance and Standard Deviation of a random variable			
Binomial Distribution	13	13.7	Bernoulli Trials and Binomial Distribution

CUET Syllabus for Section II-A	Chapter No.	Subtopic No.	Subtopic name
LINEAR PROGRAMMING			
Chapter 12 : Linear Programming			
Mathematical formulation of Linear Programming Problem	12	12.2, 12.3	Linear Programming Problem and its Mathematical Formulation, Different Types of Linear Programming Problems
Graphical method of solution for problems in two variables Feasible and infeasible regions Optimal feasible solution	12	12.2	Linear Programming Problem and its Mathematical Formulation

SECTION II-B1

CUET Syllabus for Section II-B1	Chapter No.	Subtopic No.	Topic Name
UNIT 1 - RELATIONS AND FUNCTIONS			
Chapter 1 : Relations and Functions			
Types of relations : Reflexive, symmetric, transitive and equivalence relations	1	1.2	Types of Relations
One to one and onto functions	1	1.3	Types of Functions
Composite functions, inverse of a function	1	1.4	Composition of Functions and Invertible Function
Binary operations	1	1.5	Binary Operations
Chapter 2 : Inverse Trigonometric Functions			
Definition, range, domain, principal value branches.	2	2.2	Basic Concepts
Graphs of inverse trigonometric functions, Elementary properties of inverse trigonometric functions	2	2.3	Properties of Inverse Trigonometric Functions
UNIT 2 - ALGEBRA			
Chapter 3 : Matrices			
Concept, notation, order, equality	3	3.2	Matrix
Types of matrices, zero matrix,	3	3.3	Types of Matrices
Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2)	3	3.4	Operations on Matrices
Transpose of a matrix	3	3.5	Transpose of a Matrix
Symmetric and skew symmetric matrices	3	3.6	Symmetric and Skew Symmetric Matrices
Concept of elementary row and column operations	3	3.7	Elementary Operation (Transformation) of a Matrix
Invertible matrices and proof of the uniqueness of inverse, if it exists;(Here all matrices will have real entries)	3	3.8	Invertible Matrices
Chapter 4 : Determinants			
Determinant of a square matrix (upto 3×3 matrices)	4	4.2	Determinant
Properties of determinants	4	4.3	Properties of Determinants
Applications of determinants in finding the area of a triangle	4	4.4	Area of a Triangle
Minors, cofactors	4	4.5	Minors and Cofactors
Adjoint and inverse of a square matrix	4	4.6	Adjoint and Inverse of a Matrix

CUET Syllabus for Section II-B1	Chapter No.	Subtopic No.	Subtopic name
Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix	4	4.7	Applications of Determinants and Matrices
UNIT 3 - CALCULUS			
Chapter 5 : Continuity and Differentiability			
Continuity	5	5.2	Continuity
Differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit function	5	5.3	Differentiability
Concepts of exponential, logarithmic functions	5	5.4	Exponential and Logarithmic Functions
Derivatives of log x and e^x, Logarithmic differentiation	5	5.5	Logarithmic Differentiation
Derivative of functions expressed in parametric forms	5	5.6	Derivatives of Functions in Parametric Forms
Second-order derivatives	5	5.7	Second Order Derivative
Rolle's and Lagrange's Mean Value Theorems(without proof) and their geometric interpretations	5	5.8	Mean Value Theorem
Chapter 6 : Application of Derivatives			
Applications of derivatives: Rate of change	6	6.2	Rate of Change of Quantities
Increasing / decreasing functions	6	6.3	Increasing and Decreasing Functions
Tangents and Normals	6	6.4	Tangents and Normals
Approximation	6	6.5	Approximations
Maxima and minima(first derivative test motivated geometrically and second derivative test given as a provable tool).Simple problems(that illustrate basic principles and understanding of the subject as well as real-life situations)	6	6.6	Maxima and Minima
Chapter 7 : Integrals			
Integration as inverse process of differentiation	7	7.2	Integration as an Inverse Process of Differentiation
Integration of a variety of functions by substitution	7	7.3	Methods of Integration
Integration of simple integrals of the type – $\int \frac{dx}{x^2 \pm a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{dx}{\sqrt{a^2 - x^2}}$, $\int \frac{dx}{ax^2+bx+c}$, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$, $\int \frac{px+q}{ax^2+bx+c}dx$, $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$, $\int \sqrt{a^2 \pm x^2}\,dx$, $\int \sqrt{x^2 - a^2}\,dx$, $\int \sqrt{ax^2+bx+c}\,dx$, $\int (px+q)\sqrt{ax^2+bx+c}\,dx$	7	7.4	Integrals of some Particular Functions
Integration of a variety of functions by partial fractions	7	7.5	Integration by Partial Fractions
Integration of a variety of functions by parts	7	7.6	Integration by Parts
Definite integrals as a limit of a sum.	7	7.7	Definite Integral
Fundamental Theorem of Calculus(without proof)	7	7.8	Fundamental Theorem of Calculus
Evaluation of definite integrals	7	7.9	Evaluation of Definite Integrals by Substitution
Basic properties of definite integrals	7	7.10	Some Properties of Definite Integrals

CUET Syllabus for Section II-B1	Chapter No.	Subtopic No.	Topic Name
Chapter 8 : Application of Integrals			
Applications in finding the area under simple curves, especially lines, arcs of circles/parabolas/ellipses(in standard form only)	8	8.2	Area under Simple Curves
Area between the two above said curves(the region should be clearly identifiable)	8	8.3	Area between Two Curves
Chapter 9 : Differential Equations			
Definition, order and degree of a differential equation	9	9.2	Basic Concepts
General and particular solutions of a differential equation	9	9.3	General and Particular Solutions of a Differential Equation
Formation of differential equation whose general solution is given	9	9.4	Formation of a Differential Equation whose General Solution is given
Solution of differential equations by method of separation of variables, homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type – $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x or constant $\frac{dx}{dy} + Px = Q$, where P and Q are functions of y or constant	9	9.5	Methods of Solving First order, First Degree Differential Equations
UNIT 4 - VECTORS AND THREE-DIMENSIONAL GEOMETRY			
Chapter 10 : Vectors			
Vectors and scalars, magnitude and direction of a vector. Direction cosines/ratios of vectors.	10	10.2	Some Basic Concepts
Types of vectors(equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector	10	10.3	Types of Vectors
Addition of vectors	10	10.4	Addition of Vectors
Multiplication of a vector by a scalar	10	10.5	Multiplication of a Vector by a Scalar
Position vector of a point dividing a line segment in a given ratio. Scalar (dot) product of vectors, projection of a vector on a line. Vector (cross) product of vectors, scalar triple product.	10	10.6	Product of Two Vectors
Chapter 11 : Three Dimensional Geometry			
Direction cosines/ratios of a line joining two points	11	11.2	Direction Cosines and Direction Ratios of a Line
Cartesian and vector equation of a line	11	11.3	Equation of a Line in Space
Angle between Two Lines	11	11.4	Angle between Two Lines
Shortest distance between two lines, skew lines	11	11.5	Shortest Distance between Two Lines
Cartesian and vector equation of a plane	11	11.6	Plane
Coplanar lines	11	11.7	Coplanarity of Two Lines
Angle between Two Planes	11	11.8	Angle between Two Planes
Distance of a Point from a Plane	11	11.9	Distance of a Point from a Plane
Angle between a Line and a Plane	11	11.10	Angle between a Line and a Plane

CUET Syllabus for Section II-B1	Chapter No.	Subtopic No.	Topic Name
UNIT 5 - LINEAR PROGRAMMING			
Chapter 12 : Linear Programming			
Introduction, related terminology such as constraints, objective function, optimization, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints)	12	12.2	Linear Programming Problem and its Mathematical Formulation
Different types of linear programming (L.P.) problems	12	12.3	Different Types of Linear Programming Problems
UNIT 6 - PROBABILITY			
Chapter 13 : Probability			
Conditional probability	13	13.2	Conditional Probability
Multiplications theorem on probability	13	13.3	Multiplication Theorem on Probability
Independent events	13	13.4	Independent Events
Total probability, Baye’s theorem	13	13.5	Bayes' Theorem
Random variable and its probability distribution, mean and variance of haphazard variable.	13	13.6	Random Variables and its Probability Distributions
Repeated independent (Bernoulli) trials and Binomial distribution.	13	13.7	Bernoulli Trials and Binomial Distribution

01 Relations and Functions

Content and Concepts

Synopsis

1. Relations from a set A to a set B:
A relation (or binary relation) R, from a non-empty set A to another non-empty set B, is a subset of A × B.

2. Number of possible relations from A to B:
If A has m elements and B has n elements, then A × B has m × n elements and total number of possible relations from A to B is 2^{mn}.

3. Domain, Range and Co-domain of a relation:
If R is a relation from A to B, then

i. the set of first elements of ordered pairs in R is called the **domain** of R.
Domain of R = {a : (a, b) ∈ R}

ii. the whole set B is called the **co-domain** of the relation R.

iii. the set of second elements of ordered pairs in R is called the **range** of R.

∴ Range of R = {b : (a, b) ∈ R}
Note that range is always a subset of codomain.

4. Types of relations:
Let A be a non-empty set, then a relation R on A is said to be

i. Empty Relation:
if no element of A is related to any element of A, i.e., R = $\phi \subset$ A × A.

ii. Universal relation:
if each element of A is related to every element of A, i.e., R = A × A.

iii. Reflexive relation:
If aRa ∀ a ∈ A
i.e., (a, a) ∈ R ∀ a ∈ A

iv. Symmetric relation:
If aRb ⇒ bRa ∀ a, b ∈ A
i.e., if (a, b) ∈ R ⇒ (b, a) ∈ R ∀ a, b ∈ A

v. Transitive relation:
If aRb and bRc ⇒ aRc ∀ a, b, c ∈ A
i.e., if (a, b) ∈ R and (b, c) ∈ R
⇒ (a, c) ∈ R ∀ a, b, c ∈ A.

vi. Equivalence relation:
A relation R on a set A is said to be an equivalence relation on A iff R is Reflexive, Symmetric and Transitive.

5. Functions:
Let A and B be any two non-empty sets. If to each element x ∈ A ∃ a unique element y ∈ B under a rule f, then this relation is called function from A into B and is written as f : A → B.

6. Domain, Co-domain and Range:
If f is a function from A to B, then

i. the set A is called the **domain** of f.
i.e., all possible values of x for which f(x) exists.

ii. the set B is called the **co-domain** of f.

iii. the set of all f - images of the elements of A is called the **range** of function f.
i.e., all possible values of f(x), for all values of x.

∴ Range of f = {f(x) : x ∈ A}
Note that range is always a subset of codomain.

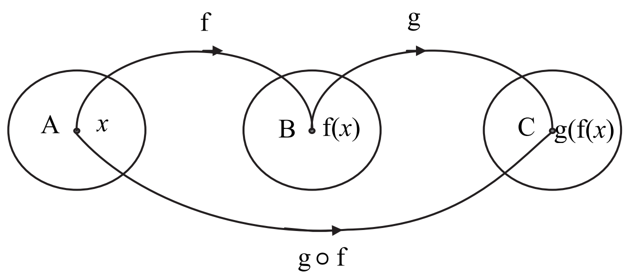
7. Types of Functions:

i. One-one function:
A function f : A → B is said to be one-one if different elements of A have different images in B, i.e., if $x_1, x_2 \in$ A, then $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ and $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.
Such a mapping is also known as **injective mapping** or an **injection** or **monomorphism**.

ii. Onto function:
Let f : A → B, if every element in B has at least one pre-image in A, then f is said to be **onto function** or **surjective mapping** or **surjection**.

iii. Bijection (one-one onto function):
A function f : A → B is a **bijection** or **bijective**, if it is one-one as well as onto.

8. Composite function:
Let f : A → B be defined by b = f(a) and
g : B → C be defined by c = g(b), then
g ∘ f : A → C be defined by g ∘ f (a) = g[f(a)] ∀ x ∈ A is called composite function.



9. Properties of Composite functions:

i. If $f : A \to B$ and $g : B \to C$ are one-one, then $g \circ f : A \to C$ is also one-one

ii. If $f : A \to B$ and $g : B \to C$ are onto, then $g \circ f : A \to C$ is also onto.

Note: Converse of above stated results need not be true.

iii. Let $f : A \to B$ and $g : B \to C$ be the given functions such that $g \circ f$ is one-one. Then f is one-one.

iv. Let $f : A \to B$ and $g : B \to C$ be the given functions such that $g \circ f$ is onto. Then g is onto.

v. If $f : X \to Y$, $g : Y \to Z$ and $h : Z \to S$ are functions, then $h \circ (g \circ f) = (h \circ g) \circ f$.

10. Invertible function:

i. A function $f : X \to Y$ is defined to be invertible, if there exists a function $g : Y \to X$ such that $g \circ f = I_x$ and $f \circ g = I_Y$. The function g is called the inverse of f and is denoted by f^{-1}.

ii. A function $f : X \to Y$ is invertible if and only if f is a bijective function.

iii. Let $f : X \to Y$ and $g : Y \to Z$ be two invertible functions. Then $g \circ f$ is also invertible with $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

11. Binary Operations:

i. A binary operation * on a set A is a function $* : A \times A \to A$. We denote * (a, b) by a * b.

ii. A binary operation * on the set X is called commutative, if $a*b = b*a \ \forall\ a, b \in X$.

iii. A binary operation $*: A \times A \to A$ is said to be associative if $(a*b)*c = a*(b*c)$, $\forall\ a, b, c \in A$.

iv. Given a binary operation $*: A \times A \to A$, an element $e \in A$, if it exists, is called identity for the operation *, if $a*e = a = e*a$, $\forall\ a \in A$.

Multiple Choice Questions

1.2 Types of Relations

1. If R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}, then R is
(A) reflexive (B) transitive
(C) not symmetric (D) a function

2. $x^2 = xy$ is a relation which is
(A) symmetric (B) reflexive
(C) transitive (D) none of these

3. For real numbers x and y, $x \mathrel{R} y \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. The relation R is
(A) reflexive (B) symmetric
(C) transitive (D) none of these

4. The relation "less than" in the set of natural numbers is
(A) only symmetric
(B) only transitive
(C) only reflexive
(D) equivalence relation

5. With reference to a universal set, the inclusion of a subset in another, is relation, which is
(A) symmetric only
(B) an equivalence relation
(C) reflexive only
(D) not symmetric

6. Let R be a relation over the set of integers such that mRn iff m is a multiple of n, then R is
(A) reflexive and transitive
(B) symmetric
(C) only transitive
(D) an equivalance relation

7. The relation S = {(3, 3), (4, 4)} on the set A = {3, 4, 5} is _______.
(A) an equivalence relation
(B) reflexive only
(C) not reflexive but symmetric and transitive
(D) symmetric only

8. Let R be the relation on the set R of all real numbers defined by a R b iff $|a - b| \le 1$. Then R is
(A) reflexive and symmetric
(B) symmetric only
(C) transitive only
(D) anti-symmetric only

9. Let ρ be a relation defined on N, the set of natural numbers, as
$\rho = \{(x, y) \in N \times N: 2x + y = 41\}$. Then
(A) ρ is an equivalence relation
(B) ρ is only reflexive relation
(C) ρ is only symmetric relation
(D) ρ is not transitive

10. Let L be the set of all straight lines in the Euclidean plane and R be the relation defined by the rule $l_1 \mathrel{R} l_2$ iff $l_1 \perp l_2$. Then relation R is
(A) reflexive (B) symmetric
(C) transitive (D) not symmetric

11. On the set R of real numbers we define $x\mathrm{P}y$ if and only if $xy \ge 0$. Then the relation P is
(A) reflexive but not symmetric
(B) symmetric but not reflexive
(C) transitive but not reflexive
(D) reflexive and symmetric but not transitive

12. Let S be the set of all real numbers. Then the relation R = {(a, b) : 1 + ab > 0} on S is
(A) reflexive and symmetric, but not transitive.
(B) reflexive and transitive, but not symmetric.
(C) reflexive, symmetric and transitive.
(D) symmetric and transitive, but not reflexive.

13. Let W denote the words in English dictionary. Define the relation R by R = $\{(x, y) \in W \times W$: the words x and y have at least one letter in common$\}$, then R is
(A) reflexive, not symmetric and transitive
(B) not reflexive, symmetric and transitive
(C) reflexive, symmetric and not transitive
(D) reflexive, symmetric and transitive

14. Let P be any non-empty set containing p elements. Then, what is the number of relations on P? **[CUCET 2021]**
(A) 2p (B) 2^{p^2} (C) p^2 (D) p^p

15. The number of reflexive relations of a set with four elements is equal to
(A) 2^{16} (B) 2^{12}
(C) 2^8 (D) 2^4

1.3 Types of Functions

1. The function f : N → N, where N is the set of natural numbers, defined by $f(x) = 2x + 3$, is
(A) surjective (B) bijective
(C) injective (D) none of these

2. Let A and B be two finite sets having m and n elements respectively. If m ≤ n, then total number of injective functions from A to B is
(A) m^n (B) n^m
(C) $\dfrac{n!}{(n-m)!}$ (D) n!

3. If $x, y \in R$ and $x, y \neq 0$; $f(x, y) \to \dfrac{x}{y}$, then the function is a/an
(A) surjective (B) bijective
(C) one-one (D) none of these

4. The number of surjections from A = {1, 2,…., n}, n ≥ 2, onto B = {a, b} is
(A) ${}^nP_2$ (B) $2^n - 2$
(C) $2^n - 1$ (D) 2^n

5. The number of onto functions from {1, 2, 3} onto {p, q} is
(A) 7 (B) 5 (C) 6 (D) 4

6. If A = {a, b, c}, then f = {(a, b), (b, c), (c, a)} is
(A) not a function from A to A
(B) a bijection from A to A
(C) one-one but not onto
(D) none of these

7. A function f from the set of natural numbers to integers defined by
$$f(n) = \begin{cases} \dfrac{n-1}{2}, & \text{when n is odd} \\ -\dfrac{n}{2}, & \text{when n is even} \end{cases}, \text{ is}$$
(A) one-one but not onto
(B) onto but not one-one
(C) one-one and onto both
(D) neither one-one nor onto

8. If $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$

and $g(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ x, & \text{if } x \text{ is irrational} \end{cases}$,

then f – g is
(A) one-one and onto
(B) only one-one
(C) only onto
(D) neither one-one nor onto

9. If f : [0, ∞) → [0, 2] be defined by $f(x) = \dfrac{2x}{1+x}$, then f is
(A) one-one but not onto
(B) onto but not one-one
(C) both one-one and onto
(D) neither one-one nor onto

10. The function f : R → R, $f(x) = x^2$ is
(A) injective but not surjective
(B) surjective but not injective
(C) injective as well as surjective
(D) neither injective nor surjective

11. Let f : R → R be defined by $f(x) = x^4$, then
(A) f is one-one and onto
(B) f may be one-one and onto
(C) f is one-one but not onto
(D) f is neither one-one nor onto

12. Let f: R → R be a function defined by $f(x) = \dfrac{x^2-8}{x^2+2}$, then f is
(A) one-one but not onto
(B) one-one and onto
(C) onto but not one-one
(D) neither one-one nor onto

13. If R denotes the set of all real numbers, then the function f : R → R defined by $f(x) = [x]$ is
(A) one-one only
(B) onto only
(C) both one-one and onto
(D) neither one-one nor onto

14. Function f : R → R, $f(x) = x^2 + x$, is
(A) one-one only
(B) onto only
(C) both one-one and onto
(D) neither one-one nor onto

1.4 Composition of Functions and Invertible Function

1. If for two functions g and f, gof is both injective and surjective, then which of the following is true?
   (A) g and f should be injective and surjective
   (B) g should be injective and surjective
   (C) f should be injective and surjective
   (D) none of them may be surjective and injective

2. $f : (-\infty, 0] \to [0, \infty)$ is defined as $f(x) = x^2$. The domain and range of its inverse is
   (A) Domain of $(f^{-1}) = [0, \infty)$, Range of $(f^{-1}) = (-\infty, 0]$
   (B) Domain of $(f^{-1}) = [0, \infty)$, Range of $(f^{-1}) = (-\infty,\infty)$
   (C) Domain of $(f^{-1}) = [0, \infty)$, Range of $(f^{-1}) = [0,\infty)$
   (D) f^{-1} does not exist

3. If $f : A \to B$ is a bijection and $g : B \to A$ is the inverse of f, then fog is equal to
   (A) I_A (B) I_B (C) f (D) g

4. The composite map fog of the functions $f : R \to R$, $f(x) = \sin x$ and $g : R \to R$, $g(x) = x^2$ is
   (A) $(\sin x)^2$ (B) $\sin x^2$
   (C) x^2 (D) $x^2(\sin x)$

5. If $f(x) = x^2$ and $g(x) = \sqrt{x}$, then
   (A) $(gof)(-2) = 2$ (B) $(fog)(2) = 4$
   (C) $(gof)(2) = 4$ (D) $(fog)(3) = 6$

6. The inverse of the function $y = 2x - 3$ is
   (A) $\frac{x+3}{2}$ (B) $\frac{x-3}{2}$
   (C) $\frac{1}{2x-3}$ (D) $\frac{1}{2x+3}$

7. If $f(x) = (25 - x^4)^{1/4}$ for $0 < x < \sqrt{5}$, then $f\left(f\left(\frac{1}{2}\right)\right) =$
   (A) 2^{-4} (B) 2^{-3} (C) 2^{-2} (D) 2^{-1}

8. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x^2}$, then $(fog)(x)$ equals
   (A) $-f(x)$ (B) $3\ f(x)$
   (C) $[f(x)]^3$ (D) $2\ f(x)$

9. The inverse of the function $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 2$ is
   (A) $\log_e\left(\frac{x-2}{x-1}\right)^{\frac{1}{2}}$ (B) $\log_e\left(\frac{x-1}{3-x}\right)^{\frac{1}{2}}$
   (C) $\log_e\left(\frac{x}{2-x}\right)^{\frac{1}{2}}$ (D) $\log_e\left(\frac{x-1}{x+1}\right)^{-2}$

10. If $f(x) = \sin^2 x$ and the composite function $g(f(x)) = |\sin x|$, then $g(x)$ is equal to
    (A) $\sqrt{x-1}$ (B) $\sqrt{x+1}$
    (C) $\sqrt{x}$ (D) $-\sqrt{x}$

11. If $g(x) = x^2 + x - 2$ and $\frac{1}{2}(gof)(x) = 2x^2 - 5x + 2$, then $f(x)$ is equal to
    (A) $2x + 3$ (B) $2x - 3$
    (C) $2x^2 + 3x + 1$ (D) $2x^2 - 3x - 1$

12. Let $f : R - \left\{\frac{5}{4}\right\} \to R$ be a function defined as $f(x) = \frac{5x}{4x+5}$. The inverse of f is the map $g : \text{Range } f \to R - \left\{\frac{5}{4}\right\}$ given by
    (A) $g(y) = \frac{y}{5-4y}$ (B) $g(y) = \frac{5y}{5+4y}$
    (C) $g(y) = \frac{5y}{5-4y}$ (D) None of these

13. Two functions $f : R \to R$ and $g : R \to R$ are defined as follows:
    $$f(x) = \begin{cases} 0; & (x \text{ rational}) \\ 1; & (x \text{ irrational}) \end{cases}$$
    $$g(x) = \begin{cases} -1; & (x \text{ rational}) \\ 0; & (x \text{ irrational}), \end{cases}$$
    then $(gof)(e) + (fog)(\pi) =$
    (A) -1 (B) 0 (C) 1 (D) 2

14. If $f : N \to N$, $f(x) = x + 3$, then $f^{-1}(x) =$ _______
    (A) $x + 3$ (B) does not exists
    (C) $x - 3$ (D) $3 - x$

15. If $f(x) = \frac{x}{\sqrt{1+x^2}}$, then $(fofof)(x) =$
    (A) $\frac{x}{\sqrt{1+3x^2}}$ (B) $\frac{x}{\sqrt{1+x^2}}$
    (C) $\frac{x}{\sqrt{1+2x^2}}$ (D) $f\left(\frac{1-x}{1+x}\right)$

1.5 Binary Operations

1. Which of the following is not a binary operation on N?
   (A) Addition (B) Subtraction
   (C) Multiplication (D) None of these

2. Which of the following is a binary operation?
   (A) on R, define * by a * b = ab^2
   (B) on Z^+, define * by a * b = $|a - b|$
   (C) on Z^+, define * by a * b = a
   (D) All of these

3. The binary operation defined on Q, as a * b = ab + 1, is
(A) commutative
(B) associative
(C) both commutative and associative
(D) neither commutative nor associative

4. The binary operation * defined on Q, as $a * b = \frac{ab}{2}$, is
(A) commutative
(B) associative
(C) both commutative and associative
(D) neither commutative nor associative

5. The binary operation * defined on R – {–1} as $a * b = \frac{a}{b+1}$, is
(A) commutative
(B) associative
(C) both commutative and associative
(D) neither commutative nor associative

6. Binary operation * defined on set {1, 2, 3, 4, 5} as a * b = H.C.F.(a, b), then the value of 3 * (2 * 5) =
(A) 1 (B) 3 (C) 2 (D) 5

7. Which of the following operations defined on the set Q of rational numbers has identity?
(A) a * b = a + ab (B) $a * b = (a – b)^2$
(C) $a * b = \frac{ab}{4}$ (D) $a * b = ab^2$

8. Let * be the binary operation on R × R defined as (a, b) * (c, d) = (a + c, b + d), then which of the following is not true?
(A) (–2, 4) * (–3, –5) = (–5, –1)
(B) (–1, 9) * (3.4, –6.7) = (2.4, 2.3)
(C) (–7.2, 3.4) * (8.9, –4.9) = (1.7, 1.5)
(D) (–6.1, 2.3) * (–1.8, –3.8) = (–7.9, –1.5)

9. Binary operation * defined on the set R as $a * b = \frac{a+b}{2}$ $\forall$ a, b $\in$ R. Then 11 * (5 * 9) =
(A) 12.5 (B) 9 (C) 7.5 (D) 8.5

Answers to MCQs

1.2 :	1.	(C)	2.	(B)	3.	(A)	4.	(B)	5.	(D)	6.	(A)	7.	(C)	8.	(A)	9.	(D)	10.	(B)
	11.	(D)	12.	(A)	13.	(C)	14.	(B)	15.	(B)										
1.3 :	1.	(C)	2.	(C)	3.	(A)	4.	(B)	5.	(C)	6.	(B)	7.	(C)	8.	(A)	9.	(A)	10.	(D)
	11.	(D)	12.	(D)	13.	(D)	14.	(D)												
1.4 :	1.	(A)	2.	(A)	3.	(B)	4.	(B)	5.	(A)	6.	(A)	7.	(D)	8.	(B)	9.	(B)	10.	(C)
	11.	(B)	12.	(C)	13.	(A)	14.	(B)	15.	(A)										
1.5 :	1.	(B)	2.	(D)	3.	(A)	4.	(C)	5.	(D)	6.	(A)	7.	(C)	8.	(C)	9.	(B)		

Solutions to MCQs

1.2 Types of Relations

1. (C)
Given, A = {1, 2, 3, 4}
R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)}
(2, 3) $\in$ R, but (3, 2) $\notin$ R
$\therefore$ R is not symmetric
R is not reflexive as (1, 1) $\notin$ R,
R is not a function as (2, 4) $\in$ R and (2, 3) $\in$ R,
R is not transitive as (1, 3) $\in$ R and (3, 1) $\in$ R, but (1, 1) $\notin$ R.

2. (B)

3. (A)
For any $x \in$ R, we have $x - x + \sqrt{2} = \sqrt{2}$ which is an irrational number
$\Rightarrow x$ R x for all x.
So, R is reflexive.
But, R is not symmetric, because $\sqrt{2}$ R 1 but $1 \not R \sqrt{2}$.
And R is not transitive, because $\sqrt{2}$ R 1 and 1 R $2\sqrt{2}$, but $\sqrt{2} \not R 2\sqrt{2}$.

4. (B)
Let A be the set of natural nos.
$\therefore$ R = {(x, y) / $x, y \in$ A and $x < y$}
Since, $x < y$, $y < z \Rightarrow x < z$ $\forall$ $x, y, z \in$ N
$\therefore$ x R y and y R z $\Rightarrow x$ R z
$\therefore$ Relation is transitive.
Also, $x < y$ does not give $y < x$
$\therefore$ Relation is not symmetric and $x < x$ does not hold.
$\therefore$ Relation is not reflexive.

5. (D)
Since, A $\subseteq$ A
$\therefore$ Relation '$\subseteq$' is reflexive.

Since, $A \subseteq B$, $B \subseteq C \Rightarrow A \subseteq C$

$\therefore$ Relation '$\subseteq$' is transitive.

But, $A \subseteq B \not\Rightarrow B \subseteq A$

$\therefore$ Relation is not symmetric.

6. **(A)**

mRm, as m is a multiple of m

$\therefore$ R is reflexive.

mRn $\not\Rightarrow$ nRm

$\therefore$ R is not symmetric.

mRn and nRp $\Rightarrow$ mRp

$\therefore$ R is transitive.

Hence, R is reflexive and transitive.

7. **(C)**

Since $(5, 5) \notin S$.

$\therefore$ The relation S is not reflexive.

It is symmetric and transitive.

CAUTION

Even though we cannot see any transitive mapping in the relation, relation is transitive.

8. **(A)**

$|a - a| = 0 < 1$

$\Rightarrow a\ R\ a\ \forall\ a \in R$

$\therefore$ R is reflexive.

Now, $aRb \Rightarrow |a - b| \le 1 \Rightarrow |b - a| \le 1 \Rightarrow bRa$

$\therefore$ R is symmetric.

9. **(D)**

$x\rho y$, $y\rho z \Rightarrow 2x + y = 41$ and $2y + z = 41$ which do not imply $2x + z = 41$

$\therefore$ ρ is not transitive.

10. **(B)**

As a line cannot be perpendicular to itself.

$\therefore$ R is not reflexive.

R is symmetric as $l_1 \perp l_2 \Rightarrow l_2 \perp l_1$

Also, R is not transitive as $l_1 \perp l_2$ and $l_2 \perp l_3$ $\Rightarrow l_1 \parallel l_3$

i.e., $l_1 \not\perp l_2$

11. **(D)**

Clearly, P is reflexive and symmetric.

$(-1, 0)$ and $(0, 2)$ satisfies the relation $xy \ge 0$.

But $(-1, 2)$ does not satisfy the relation $xy \ge 0$.

$\therefore$ P is not transitive.

12. **(A)**

Since, $1 + a.a = 1 + a^2 > 0\ \forall\ a \in S$

$\therefore$ $(a, a) \in R$

$\Rightarrow$ R is reflexive.

Also, $(a, b) \in R \Rightarrow 1 + ab > 0$

$\Rightarrow 1 + ba > 0$

$\Rightarrow (b, a) \in R$

Hence, R is symmetric.

R is not transitive as $(a, b) \in R$ and $(b, c) \in R$ need not imply $(a, c) \in R$.

13. **(C)**

Here, $R = \{(x, y) \in W \times W$: the words x and y have at least one letter in common$\}$

R is reflexive as the words x and x have all letters in common.

Hence, R is reflexive.

Also, if $(x, y) \in R$ i.e., x and y have a common letter, then y and x also have a letter in common

$\therefore$ R is symmetric.

R is not transitive as $(x, y) \in R$ and $(y, z) \in R$ need not imply $(x, z) \in R$

For example, let x = CANE, y = NEST and z = WITH

then $(x, y) \in R$ and $(y, z) \in R$, but $(x, z) \notin R$

$\therefore$ R is reflexive and symmetric but not transitive.

14. **(B)**

Shortcut

Number of relations on A with n elements = 2^{n^2}

15. **(B)**

Total number of reflexive relations of a set with 4 elements = $2^{16-4} = 2^{12}$

Shortcut

Total number of reflexive relations in a set with n elements = $2^{n^2 - n}$

1.3 Types of Functions

1. **(C)**

$f(x_1) = 2x_1 + 3$, $f(x_2) = 2x_2 + 3$

Let $f(x_1) = f(x_2)$

$\Rightarrow 2x_1 + 3 = 2x_2 + 3$

$\Rightarrow x_1 = x_2$

$\Rightarrow$ f is injective.

2. **(C)**

Since $m \le n$, injective functions from A to B are defined and the total number of such functions is ${}^nP_m = \dfrac{n!}{(n-m)!}$

3. **(A)**

4. **(B)**

Number of surjections from A to B, where $o(A) = m$, $o(B) = n$ and $m \ge n$ is

$\sum_{r=1}^{n} (-1)^{n-r}\ {}^nC_r\ (r)^m$

$= \sum_{r=1}^{2} (-1)^{2-r}\ {}^2C_r\ r^n$

$= (-1)^1\ {}^2C_1\ (1)^n + (-1)^0\ {}^2C_2\ 2^n$

$= 2^n - 2$

5. **(C)**

The number of onto functions from {1, 2, 3} onto {p, q} is

$= 2^3 - 2 = 6$

$\ldots\left[\begin{array}{l}\because \text{ The number of onto functions} \\ \text{defined from a finite set A, containing n} \\ \text{elements onto a finite set B, containing} \\ \text{2 elements} = 2^n - 2\end{array}\right]$

6. **(B)**

Here, f(a) = b, f(b) = c, f(c) = a

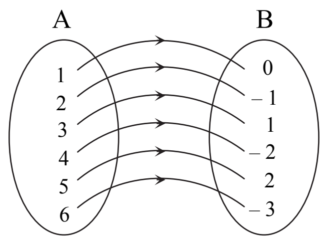
$\therefore$ f is one-one and onto

$\therefore$ f is a bijection from A to A.

7. **(C)**

Here, $f : N \to I$

Now, f(1) = 0, f(2) = –1, f(3) = 1, f(4) = –2, f(5) = 2 and f(6) = –3 so on.



$\because$ Every element of set A has unique image in set B and there is no element left in set B.

Hence, f is one-one and onto function.

8. **(A)**

Here, $(f - g)(x) = f(x) - g(x)$

$\therefore$ $(f - g)(x) = \begin{cases} x - 0 = x, & \text{if } x \text{ is rational} \\ 0 - x = -x, & \text{if } x \text{ is irrational} \end{cases}$

Let k = f – g

Let x, y be any two distinct real numbers.

Then, $x \neq y$

$\Rightarrow -x \neq -y$

Now, $x \neq y$

$\Rightarrow k(x) \neq k(y)$

$\Rightarrow (f - g)(x) \neq (f - g)(y)$

$\Rightarrow$ f – g is one-one.

Let y be any real number

If y is a rational number, then

$k(y) = y$

$\Rightarrow (f - g)(y) = y$

If y is an irrational number , then

$k(-y) = y$

$\Rightarrow (f - g)(-y) = y$

Thus, every $y \in R$ (co-domain) has its pre-image in R (domain)

$\therefore$ $f - g : R \to R$ is onto.

Hence, f – g is one-one and onto.

9. **(A)**

Given, $f(x) = \dfrac{2x}{1+x}$, $D_f = [0, \infty)$

Let $x_1, x_2 \in D_f$ and $f(x_1) = f(x_2)$

$\Rightarrow \dfrac{2x_1}{1+x_1} = \dfrac{2x_2}{1+x_2}$

$\Rightarrow 2x_1 + 2x_1x_2 = 2x_2 + 2x_1x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one.

Let $y \in [0, 2]$ be arbitrary

Then, $y = f(x) = \dfrac{2x}{1+x}$

$\Rightarrow y + xy = 2x$

$\Rightarrow x(2 - y) = y$

$\Rightarrow x = \dfrac{y}{2-y}$, where $y \neq 2$

This means $y = 2$ is not the image of any element of $[0, \infty)$

$\therefore$ f is not onto.

10. **(D)**

Since, f(–1) = f(1) = 1

$\therefore$ f is not injective.

To each $4 \in R$, $\exists -2 \in R$ and $2 \in R$ such that f(–2) = 4 and f(2) = 4

$\therefore$ f is not surjective.

11. **(D)**

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow x_1 = \pm x_2$

$\therefore$ $f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$

$\therefore$ f is not one-one.

Consider an element 2 in the co-domain R.

There does not exist any x in domain R such that $f(x) = 2$.

$\therefore$ f is not onto.

12. **(D)**

Here, $f(x) = f(-x)$

$\therefore$ f is not one-one.

Let $y \in R$, then $f(x) = y$

$\Rightarrow y = \dfrac{x^2 - 8}{x^2 + 2}$

$\Rightarrow x^2 = \dfrac{8 + 2y}{1 - y}$

For x to be real,

$(8 + 2y)(1 - y) \geq 0$ and $1 - y \neq 0$

$\Rightarrow (y + 4)(y - 1) \leq 0$ and $y \neq 1$

$\Rightarrow -4 \leq y < 1$

$\therefore$ Range of $f = [-4, 1) \subset R$

$\therefore$ f is not onto. $\ldots[\because R_f \neq \text{co-domain of f}]$

13. **(D)**

Let $f(x_1) = f(x_2)$

$\Rightarrow [x_1] = [x_2]$

$\not\Rightarrow x_1 = x_2$

$\therefore$ f is not one-one

Also, f is not onto as its range I (set of integers) is a proper subset of its co-domain R.

Hence, f is neither one-one nor onto.

14. **(D)**

Let $x_1 = 0$ and $x_2 = -1$

Here, $x_1 \neq x_2$

but $f(0) = f(-1) = 0$

$\Rightarrow f(x_1) = f(x_2)$

$\therefore$ f is not one-one

Also, there is no pre-image of -1

$\therefore$ $f(x)$ is not onto function

1.4 Composition of Functions and Invertible Function

1. **(A)**

If f, g are one-one onto, then gof is also one-one, onto.

2. **(A)**

$f(x) = x^2$

$\Rightarrow y = x^2$

$\Rightarrow x = \pm\sqrt{y}$

$\Rightarrow x = -\sqrt{y}$

$\Rightarrow f^{-1}(y) = -\sqrt{y}$

$\Rightarrow f^{-1}(x) = -\sqrt{x}$

$\therefore$ domain of $f^{-1} = [0, \infty)$,

Range of $f^{-1} = (-\infty, 0]$

3. **(B)**

4. **(B)**

$(fog)(x) = f(g(x)) = f(x^2) = \sin x^2$

5. **(A)**

$(gof)(-2) = g(f(-2)) = g((-2)^2) = g(4)$

$= \sqrt{4}$

$= 2$

6. **(A)**

Let $y = f(x) = 2x - 3$

$\Rightarrow x = \dfrac{y+3}{2}$

$\Rightarrow f^{-1}(y) = \dfrac{y+3}{2}$

$\Rightarrow f^{-1}(x) = \dfrac{x+3}{2}$

7. **(D)**

$$f\left(\frac{1}{2}\right) = \left(25 - \frac{1}{16}\right)^{\frac{1}{4}} = \left(\frac{399}{16}\right)^{\frac{1}{4}}$$

$$\Rightarrow f\left[f\left(\frac{1}{2}\right)\right] = f\left(\left(\frac{399}{16}\right)^{\frac{1}{4}}\right)$$

$$= \left(25 - \frac{399}{16}\right)^{\frac{1}{4}}$$

$$= \left(\frac{1}{16}\right)^{\frac{1}{4}}$$

$$= \frac{1}{2} = 2^{-1}$$

8. **(B)**

$(fog)(x) = f(g(x))$

$$= f\left(\frac{3x + x^3}{1 + 3x^2}\right)$$

$$= \log\left(\frac{1 + \frac{3x + x^3}{1 + 3x^2}}{1 - \frac{3x + x^3}{1 + 3x^2}}\right)$$

$$= \log\left(\frac{1 + 3x^2 + 3x + x^3}{1 + 3x^2 - 3x - x^3}\right)$$

$$= \log\frac{(1+x)^3}{(1-x)^3}$$

$$= \log\left[\frac{1+x}{1-x}\right]^3$$

$$= 3\,f(x)$$

9. **(B)**

Let $y = f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} + 2$

$\therefore$ $y - 2 = \dfrac{e^{2x} - 1}{e^{2x} + 1}$

$\Rightarrow (y - 2)\, e^{2x} + y - 2 = e^{2x} - 1$

$\Rightarrow e^{2x} = \dfrac{1 - y}{y - 3} = \dfrac{y - 1}{3 - y}$

$\Rightarrow 2x = \log_e\left(\dfrac{y-1}{3-y}\right)$

$\Rightarrow x = \dfrac{1}{2}\log_e\left(\dfrac{y-1}{3-y}\right)$

$\Rightarrow f^{-1}(y) = \dfrac{1}{2}\log_e\left(\dfrac{y-1}{3-y}\right)$

$\Rightarrow f^{-1}(x) = \log_e\left(\dfrac{x-1}{3-x}\right)^{\frac{1}{2}}$

10. **(C)**

Here, $g(f(x)) = |\sin x|$

$\Rightarrow g(f(x)) = \sqrt{\sin^2 x}$

$\Rightarrow g(\sin^2 x) = \sqrt{\sin^2 x}$

$\Rightarrow g(x) = \sqrt{x}$

11. **(B)**

Given, $g(x) = x^2 + x - 2$

and $\frac{1}{2}(gof)(x) = 2x^2 - 5x + 2$

$\Rightarrow g(f(x)) = 4x^2 - 10x + 4$

$\Rightarrow (f(x))^2 + f(x) - 2 = 4x^2 - 10x + 4$

$\Rightarrow (f(x))^2 + f(x) - (4x^2 - 10x + 6) = 0$

$$\Rightarrow f(x) = \frac{-1 \pm \sqrt{1 + 16x^2 - 40x + 24}}{2}$$

$$= \frac{-1 \pm (4x - 5)}{2}$$

$= 2x - 3, -2x + 2$

12. **(C)**

We have, $f(x) = \frac{5x}{4x+5}, x \in R - \left\{\frac{5}{4}\right\}$

Let $f(x) = y$

$\Rightarrow x = f^{-1}(y)$

$y = \frac{5x}{4x+5}$

$\Rightarrow 4xy + 5y = 5x$

$\Rightarrow 5y = 5x - 4xy = x(5 - 4y)$

$\Rightarrow x = \frac{5y}{5-4y}$

$g(y) = f^{-1}(y) = \frac{5y}{5-4y}, \ y \in R - \left\{\frac{5}{4}\right\}$

13. **(A)**

$(gof)(e) + (fog)(\pi) = g(f(e)) + f(g(\pi))$

$= g(1) + f(0)$

$= -1 + 0$

$= -1$

14. **(B)**

For $1 \in N$ in co-domain we cannot find any $x \in N$ in domain such that $f(x) = 1$

$\therefore$ function is into $\Rightarrow f^{-1}(x)$ does not exist.

15. **(A)**

$(fofof)(x) = (fof)(f(x))$

$$= (fof)\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$= f\left(f\left(\frac{x}{\sqrt{1+x^2}}\right)\right)$$

$$= f\left[\frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\frac{x^2}{1+x^2}}}\right]$$

$$= f\left(\frac{x}{\sqrt{1+2x^2}}\right)$$

$$= \frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{1+\frac{x^2}{1+2x^2}}}$$

$$= \frac{x}{\sqrt{1+3x^2}}$$

1.5 Binary Operations

1. **(B)**

Consider 'Subtraction' operation $-: N \times N \to N$ given by $(a, b) \to a - b$.

Consider (2, 4) under '–'

Here, $2 - 4 = -2 \notin N$

$\therefore$ Subtraction is not a binary operation on N.

CAUTION

In such examples, be careful about the domain. As subtraction is binary operation on R but it is not a binary operation on N.

2. **(D)**

3. **(A)**

Consider, $b * a = ba + 1$

$= ab + 1 = a * b$

$\therefore$ * is commutative.

Consider $(a * b) * c = (ab + 1) * c$

$= abc + c + 1$

Also, $a * (b * c) = a * (bc + 1)$

$= abc + a + 1$

Here, $(a * b) * c \neq a * (b * c)$

$\therefore$ * is not associative.

4. **(C)**

Consider, $b * a = \frac{ba}{2} = \frac{ab}{2} = a * b$

Consider, $a * (b * c) = a + \left(\frac{bc}{2}\right) = \frac{abc}{4}$

Also, $(a * b) * c = \left(\frac{ab}{2}\right) * c = \frac{abc}{4}$

Here, $a * (b * c) = (a * b) * c$

$\therefore$ * is associative.

5. **(D)**

Consider b * a = $\frac{b}{a+1} \neq \frac{a}{b+1}$

$\therefore$ b * a $\neq$ a * b

$\therefore$ * is not commutative.

Consider, a * (b * a) = a * $\left(\frac{b}{c+1}\right)$

$$= \frac{a}{\left(\frac{b}{c+1}\right)+1}$$

Also, (a * b) * c = $\left(\frac{a}{b+1}\right)$ * c

$$= \frac{\left(\frac{a}{b+1}\right)}{c+1}$$

Here, a * (b * c) = (a * b) * c

$\therefore$ * is not associative.

6. **(A)**

3 * (2 * 5)

= 3 * H.C.F.(2, 5)

= 3 * 1

= H.C.F. (3, 1) = 1

7. **(C)**

Consider option (C),

Let 'e' be the identity.

Consider a * e = $\frac{ae}{4} = \frac{a}{4}$

Also, e * a = $\frac{ea}{4} = \frac{a}{4}$

Here, we get a * e = a = e * a

$\therefore$ Identity exists.

8. **(C)**

Consider option (C)

(–7.2, 3.4) * (8.9, –4.9)

= (–7.2 + 8.9, 3.4 – 4.9) = (1.7, –1.5)

$\therefore$ Option (C) is not true.

9. **(B)**

11 * (5 * 9) = 11 * $\left(\frac{5+9}{2}\right)$

= 11 * 7

= $\frac{18}{2}$

= 9

Topic Test

1. If f(x) = e^{2x} and g(x) = $\log\sqrt{x}$ (x > 0), then fog(x) is equal to
   (A) e^{2x} (B) x
   (C) 0 (D) $\log\sqrt{x}$

2. Set A has 3 elements and set B has 4 elements. The number of injection that can be defined from A to B is
   (A) 144 (B) 12
   (C) 24 (D) 64

3. If P = {(x, y) / $x^2 + y^2 = 1$, (x, y) $\in$ R}, then P is
   (A) reflexive (B) symmetric
   (C) transitive (D) anti-symmetric

4. The inverse of the function $y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is
   (A) $\log_{10}(2 - x)$ (B) $\frac{1}{2}\log_{10}\frac{1+x}{1-x}$
   (C) $\frac{1}{2}\log_{10}(2x-1)$ (D) $\frac{1}{4}\log_{10}\frac{2x}{2-x}$

5. Which of the following is a bijective function on the set of real numbers?
   (A) $x^2 + 1$ (B) $2x - 5$
   (C) x^2 (D) $|x|$

6. Which of the following is not a binary operation on R?
   (A) Addition (B) Subtraction
   (C) Multiplication (D) Division

7. If g(f(x)) = |sin x | and f(g(x)) = $\left(\sin\sqrt{x}\right)^2$, then
   (A) f(x) = sin x, g(x) = | x |
   (B) f(x) = x^2, g(x) = $\sin\sqrt{x}$
   (C) f(x) = $\sin^2 x$, g(x) = $\sqrt{x}$
   (D) f and g cannot be determined

8. If f(x) = $x^3 + 5x + 1$ for real x, then
   (A) f is one-one and onto in R
   (B) f is one-one but not onto in R
   (C) f is onto in R but not one-one
   (D) f is neither one-one nor onto in R

9. If g(y) is inverse of function f : R $\to$ R given by f(x) = x + 3, then g(y) =
   (A) $y + 3$ (B) $y - 3$
   (C) $\frac{y}{3}$ (D) $3y$

10. On the set R of real numbers, the relation ρ is defined by $x\rho y$, (x, y) $\in$ R
    (A) If | x – y | < 2 then ρ is reflexive but neither symmetric nor transitive
    (B) If x – y < 2 then ρ is reflexive and symmetric but not transitive
    (C) If | x | $\geq$ y then ρ is reflexive and transitive but not symmetric
    (D) If x > | y | then ρ is transitive but neither reflexive nor symmetric

11. Let N be the set of all natural numbers, Z be the set of all integers and $\sigma : N \to Z$ defined by

$$\sigma(n) = \begin{cases} \dfrac{n}{2} & \text{, if n is even} \\ -\dfrac{n-1}{2} & \text{, if n is odd} \end{cases} \quad \text{then}$$

(A) σ is one-one but not onto
(B) σ is onto but not one-one
(C) σ is one-one and onto
(D) σ is neither one-one not onto

12. For the binary operation * defined on Z^+ as $a * b = 2^{ab}$, * is
(A) commutative
(B) associative
(C) both commutative and associative
(D) neither commutative nor associative

13. If the functions f, g, h are defined from the sets of real numbers R to R such that

$$f(x) = x^2 - 1,\ g(x) = \sqrt{x^2+1}, h(x) = \begin{cases} 0, \text{ if } x \le 0 \\ x, \text{ if } x > 0 \end{cases},$$

then the composite function (hofog) (x) =

(A) $\begin{cases} 0, & x = 0 \\ x^2, & x > 0 \\ -x^2, & x < 0 \end{cases}$ (B) $\begin{cases} 0, & x = 0 \\ x^2, & x \neq 0 \end{cases}$

(C) $\begin{cases} 0, & x \le 0 \\ x^2, & x > 0 \end{cases}$ (D) none of these

14. Consider the following statements on a set A = {1, 2, 3}:
(1) R = {(1, 1), (2, 2)} is a reflexive relation on A.
(2) R = {(3, 3)} is symmetric and transitive but not a reflexive relation on A.

Which of the following given above is/are correct?
(A) (1) only
(B) (2) only
(C) both (1) and (2)
(D) neither (1) nor (2)

15. A function $f : [0, \infty) \to [0, \infty)$ defined as $f(x) = \dfrac{x}{1+x}$ is
(A) one-one and onto
(B) one-one but not onto
(C) onto but not one-one
(D) neither one-one nor onto

Answers to Topic Test

1.	(B)	2.	(C)	3.	(B)	4.	(B)
5.	(B)	6.	(D)	7.	(C)	8.	(A)
9.	(B)	10.	(D)	11.	(C)	12.	(A)
13.	(B)	14.	(B)	15.	(B)		

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