

SAMPLE CONTENT

Exam Experts

21

YEARS

2004 - 2024

2900+ MCQs



PREVIOUS SOLVED PAPERS

MHT-CET

CHAPTER-WISE & TOPIC-WISE

MATHEMATICS

► Quick Review

► Smart Keys

► Statistical analysis of all the shifts of 2024

Target Publications® Pvt. Ltd.

PREVIOUS SOLVED PAPERS

MATHEMATICS
Chapter-wise & Topic-wise

Salient Features

- **Unique Questions:**
 - A vast repository of 2900+ unique and authentic MCQs of 21 years (2004-2024) to enhance your preparation.
 - For the years 2004 – 2020, only questions relevant to the current syllabus have been included.
- **Organized Learning:** Questions are meticulously categorized by chapter and topic.
- **Solutions That Simplify:** Clear, detailed solutions for even the trickiest questions, making complex concepts easy to grasp.
- **The Chapters Include:**
 - **Quick Reviews:** For concept revision
 - **MCQs:** Arranged in a year-wise flow for each topic
 - **Solutions:** Provided wherever required, with solutions from 2004 to 2021 available via QR codes at the end of each chapter
- **Includes Smart Keys for Holistic Learning:**
 - *Thinking Hatke*
 - *Caution*
 - *Shortcuts*
- **2024 Trend Analysis:**
Gain valuable insights with:
 - Graphs showing difficulty levels across shifts
 - Chapter-wise analysis tables for all shifts
- **QR Codes Provide:** Solutions to MCQs from 2004 to 2021

Printed at: Prabodhan Prakashan Pvt. Ltd., Navi Mumbai

© Target Publications Pvt. Ltd.

No part of this book may be reproduced or transmitted in any form or by any means, C.D. ROM/Audio Video Cassettes or electronic, mechanical including photocopying; recording or by any information storage and retrieval system without permission in writing from the Publisher.

PREFACE

Target's 'MHT-CET Mathematics : Previous Solved Papers (PSP)' is a compilation of past 21 years' (2004-2024) questions asked in the MHT-CET examinations conducted by State Common Entrance Test Cell, Maharashtra State. This book is curated as per the **latest MHT-CET syllabus**.

The book features chapter-wise categorization of questions, with each chapter following a topic-wise flow. All questions related to a topic are arranged year-wise, ending with the most recent year. A special topic, **Concept fusion** is drafted at the end of the MCQ section to cover multifarious questions. Answers for all questions from 2004 to 2024 are provided, with solutions from 2004 to 2021 accessible via QR codes and 2022 to 2024 solutions in the book. The solutions will serve as valuable learning tools in understanding the concepts.

Selection of **unique MCQs** is prioritized while making this book to prevent the recurrence of identical questions. This will enable students to save time spent on repetitive questions.

We have infused several **Smart Keys** such as **Cautions, Thinking Hatke and Shortcuts**. These Important Study Techniques are created to help students with key objectives such as time management, easy memorization, revision and non-conventional yet simple methods for MCQ solving. To ensure adequate revision, each chapter begins with a **Quick review**.

A statistical analysis of the number of questions asked per chapter in each shift of MHT-CET 2024 examination is offered in tabular form. This analysis would help students understand the weightage allotted to each chapter. A graphical representation of analysis of all the papers (16 papers of PCM group) is also included at the start of the book to elaborate on the breakdown of the difficulty level of questions asked in the examination. Studying these representations should undoubtedly aid students in planning their study strategy for the examination. *There is a possibility that the weightage to a chapter and the level of difficulty of the question paper in the future examination may vary.*

This book would provide students with confidence regarding their exam preparedness. We are confident that this book will comprehensively cater to the needs of students and effectively assist them to achieve their goal.

Publisher

Edition: Second

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.

Please write to us at : mail@targetpublications.org

A book affects eternity; one can never tell where its influence stops.

Disclaimer

This reference book is transformative work based on the latest Textbooks of Std. XI and XII Mathematics published by the Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune. We the publishers are making this reference book which constitutes as fair use of textual contents which are transformed by adding and elaborating, with a view to simplify the same to enable the students to understand, memorize and reproduce the same in examinations.

This work is purely inspired upon the course work as prescribed by the State Council of Educational Research and Training, Maharashtra. Every care has been taken in the publication of this reference book by the Authors while creating the contents. The Authors and the Publishers shall not be responsible for any loss or damages caused to any person on account of errors or omissions which might have crept in or disagreement of any third party on the point of view expressed in the reference book.

© reserved with the Publisher for all the contents created by our Authors.

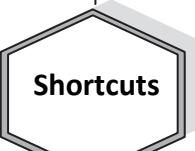
No copyright is claimed in the textual contents which are presented as part of fair dealing with a view to provide best supplementary study material for the benefit of students.

FEATURES

Quick Review includes tables/charts to summarize the key points of important concepts in the chapter. This is our attempt to help students to reinforce key concepts.



Shortcuts incorporate important theoretical or formula based short tricks, beneficial in solving MCQs.



MCQs are **segregated topic-wise** in each chapter. This is our attempt to cater to individualistic pace and preferences of studying a chapter in students and enable easy assimilation of questions based on the specific concept.



Concept Fusion topic encompasses questions whose solutions require knowledge of concepts covered in different topics from same chapter or from different chapters.



Thinking Hatke reveals quick witted approach to crack the specific question.



Caution apprises students about mistakes often made while solving MCQs.



QR Codes include solutions to MCQs from 2004 to 2021



INDEX

Sr. No.	Textbook Chapter No.	Chapter Name	Page No.
Std. XI			
1	3	Trigonometry - II	1
2	5	Straight Line	16
3	6	Circle	30
4	8	Measures of Dispersion	40
5	9	Probability	47
6	1	Complex Numbers	58
7	3	Permutations and Combinations	73
8	6	Functions	82
9	7	Limits	93
10	8	Continuity	106
Std. XII			
11	1	Mathematical Logic	119
12	2	Matrices	138
13	3	Trigonometric Functions	154
14	4	Pair of Straight Lines	188
15	5	Vectors	203
16	6	Line and Plane	254
17	7	Linear Programming	293
18	1	Differentiation	314
19	2	Applications of Derivatives	350
20	3	Indefinite Integration	388
21	4	Definite Integration	438
22	5	Application of Definite Integration	462
23	6	Differential Equations	475
24	7	Probability Distributions	518
25	8	Binomial Distribution	534

Evaluating your grasp of the content through chapter-specific tests is the most effective method for gauging your readiness with each topic.

Scan the adjacent QR code to know more about our "**MHT-CET Mathematics Test Series with Answer Key & Solutions**" book for the MHT-CET Entrance examination.



Practice test Papers are the only way to assess your preparedness for the Exams.

Scan the adjacent QR code to know more about our "**MHT-CET 22 Question Paper Set (PCM Group)**" book for the MHT-CET Entrance examination.



A competitive exam book should contain comprehensive subject coverage, practice questions and effective examination strategies.

Scan the adjacent QR code to know more about our "**MHT-CET Triumph Mathematics**" book for the MHT-CET Entrance examination.



MHT-CET PAPER PATTERN

- There will be three papers of Multiple Choice Questions (MCQs) in ‘Mathematics’, ‘Physics and Chemistry’ and ‘Biology’ of 100 marks each.
- Duration of each paper will be 90 minutes.
- Questions will be based on Syllabus of State Council of Educational Research and Training, Maharashtra with approximately 20% weightage given to Std. XI and 80% weightage will be given to Std. XII curriculum.
- Difficulty level of questions will be at par with JEE (Main) for Mathematics, Physics, Chemistry and at par with NEET for Biology.
- There will be no negative marking.
- Questions will be mainly application based.
- Details of the papers are as given below:

Paper	Subject	No. of MCQs based on		Mark(s) Per Question	Total Marks
		Std. XI	Std. XII		
Paper I	Mathematics	10	40	2	100
Paper II	Physics	10	40	1	100
	Chemistry	10	40		
Paper III	Biology	20	80	1	100

- Questions will be set on
 - i. the entire syllabus of Std. XII of Physics, Chemistry, Mathematics and Biology subjects prescribed by State Council of Educational Research and Training, Maharashtra and
 - ii. chapters / units from Std. XI curriculum prescribed by State Council of Educational Research and Training, Maharashtra as mentioned below:

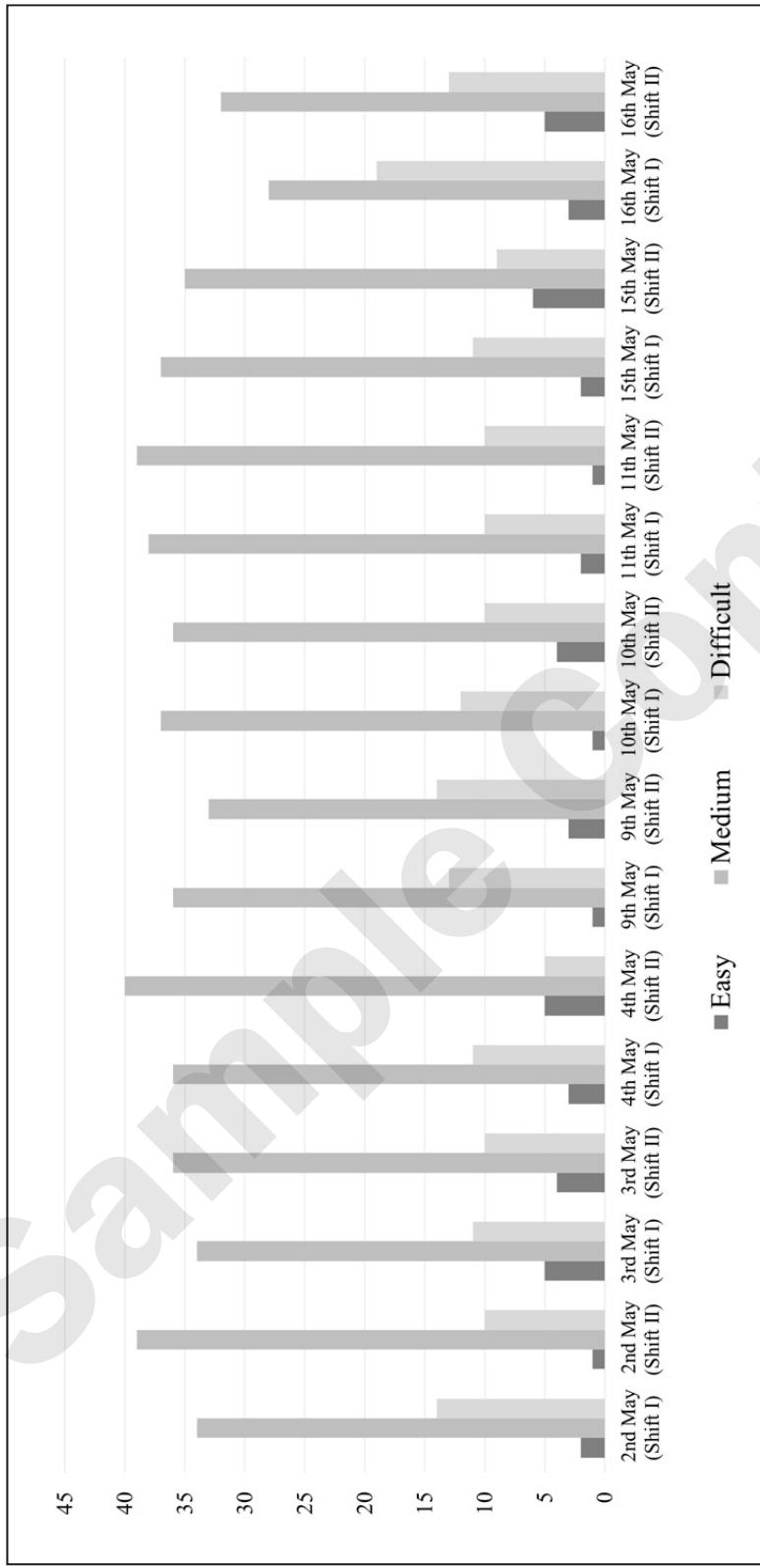
Sr. No.	Subject	Chapters / Units of Std. XI
1	Physics	Motion in a plane, Laws of motion, Gravitation, Thermal properties of matter, Sound, Optics, Electrostatics, Semiconductors
2	Chemistry	Some Basic Concepts of Chemistry, Structure of Atom, Chemical Bonding, Redox Reactions, Elements of Group 1 and Group 2, States of Matter: Gaseous and Liquid States, Basic Principles of Organic Chemistry, Adsorption and Colloids, Hydrocarbons
3	Mathematics	Trigonometry - II, Straight Line, Circle, Measures of Dispersion, Probability, Complex Numbers, Permutations and Combinations, Functions, Limits, Continuity
4	Biology	Biomolecules, Respiration and Energy Transfer, Human Nutrition, Excretion and osmoregulation

MATHEMATICS

Chapter-wise Analysis of MHT-CET 2024 Exam Papers

MATHEMATICS

Difficulty level-wise Analysis of MHT-CET 2024 Exam Papers



E – Easy: Questions whose answers can be directly and easily answered by the information given in Std. XI and XII Textbooks.

M – Medium: These questions require students to identify and apply the appropriate concepts which they studied from Std. XI and XII Textbooks.

D – Difficult: The most Challenging Questions that require application of various concepts and encourage students to think beyond the information given in the textbooks.

Analysis

➤ **Analysis of questions by difficulty level:** While the distribution of easy, medium, and difficult questions varies among the sixteen papers, a notable trend is the prevalence of medium-level questions, with a smaller number of both difficult and easy questions.

This suggests that the entrance exam places a strong emphasis on the comprehension and practical application of concepts. Students are encouraged to approach their preparation by meticulously studying the chapters, with a particular focus on effectively applying formulas and concepts in order to excel in the entrance exam.

3 Trigonometry - II

3.1	Trigonometric functions of sum and difference of angles	3.3	Trigonometric functions of multiple angles
3.2	Trigonometric functions of allied angles	3.4	Factorization formulae

3.5	Trigonometric functions of angles of a triangle
-----	-------------------------------------------------

Quick Review



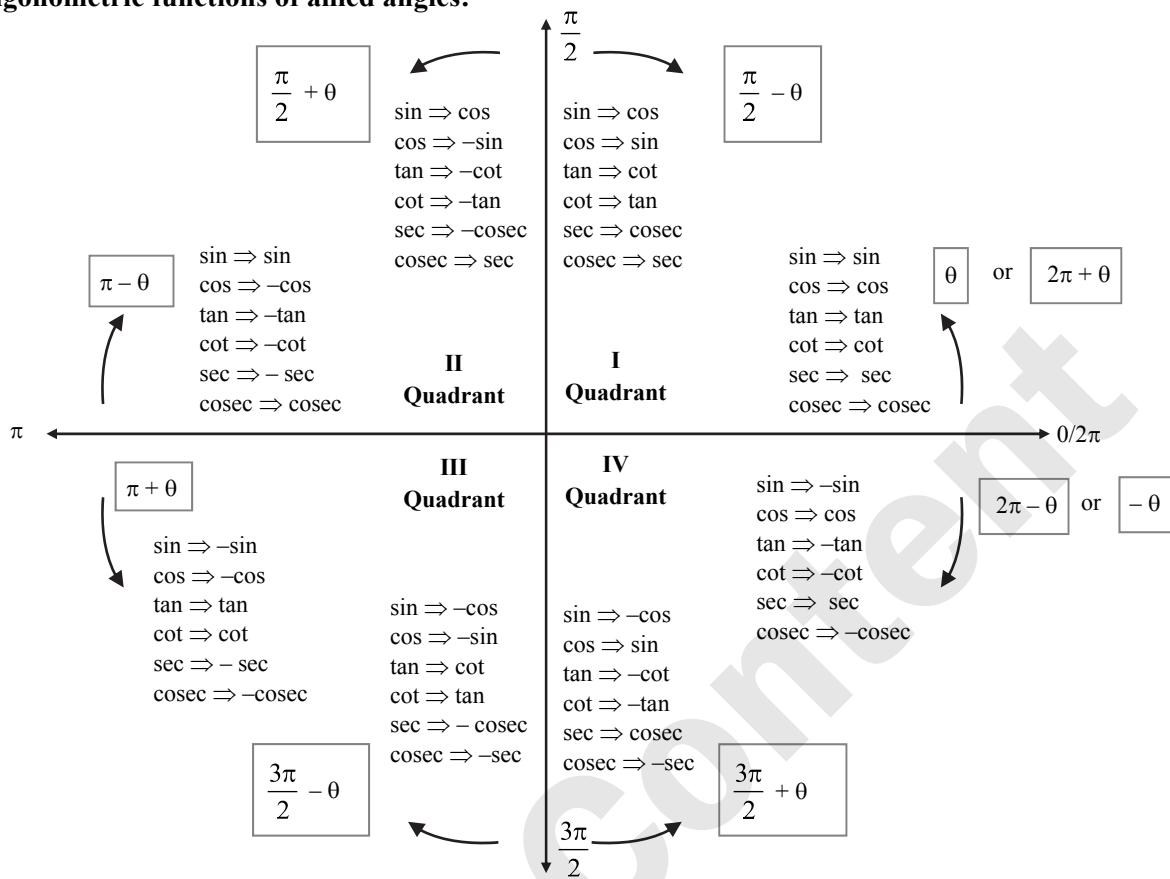
	Trigonometric functions of sum and difference of two angles	Formulae
i.	$\sin(A + B)$	$\sin A \cos B + \cos A \sin B$
ii.	$\sin(A - B)$	$\sin A \cos B - \cos A \sin B$
iii.	$\cos(A + B)$	$\cos A \cos B - \sin A \sin B$
iv.	$\cos(A - B)$	$\cos A \cos B + \sin A \sin B$
v.	$\tan(A + B)$	$\frac{\tan A + \tan B}{1 - \tan A \tan B}$
vi.	$\tan(A - B)$	$\frac{\tan A - \tan B}{1 + \tan A \tan B}$
vii.	$\cot(A + B)$	$\frac{\cot A \cot B - 1}{\cot A + \cot B}$
viii.	$\cot(A - B)$	$\frac{\cot A \cot B + 1}{\cot B - \cot A}$
ix.	$\sin(A + B) \sin(A - B)$	$= \sin^2 A - \sin^2 B$ $= \cos^2 B - \cos^2 A$
x.	$\cos(A + B) \cos(A - B)$	$= \cos^2 A - \sin^2 B$ $= \cos^2 B - \sin^2 A$

➤ Trigonometric functions of sum and difference of three angles:

- i. $\sin(A + B + C) = \sin A \cos B \cos C + \cos A \sin B \cos C + \cos A \cos B \sin C - \sin A \sin B \sin C$
or
 $\sin(A + B + C) = \cos A \cos B \cos C (\tan A + \tan B + \tan C - \tan A \tan B \tan C)$
- ii. $\cos(A + B + C) = \cos A \cos B \cos C - \sin A \sin B \cos C - \sin A \cos B \sin C - \cos A \sin B \sin C$
or
 $\cos(A + B + C) = \cos A \cos B \cos C (1 - \tan A \tan B - \tan B \tan C - \tan C \tan A)$
- iii. $\tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$
- iv. $\cot(A + B + C) = \frac{\cot A \cot B \cot C - \cot A - \cot B - \cot C}{\cot A \cot B + \cot B \cot C + \cot C \cot A - 1}$



➤ Trigonometric functions of allied angles:



➤ Trigonometric functions of multiple angles:

Trigonometric functions of multiple angles

$\sin \theta$	$\cos \theta$	$\tan \theta$
$\rightarrow \sin 2\theta = 2 \sin \theta \cos \theta$ $= \frac{2\tan \theta}{1 + \tan^2 \theta}$	$\rightarrow \cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $= 1 - 2 \sin^2 \theta$ $= 2 \cos^2 \theta - 1$ $= \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$	$\rightarrow \tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta}$ $\rightarrow \tan 3\theta = \frac{3\tan \theta - \tan^3 \theta}{1 - 3\tan^2 \theta}$
$\rightarrow \sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$	$\rightarrow \cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$	Note: $1 + \cos 2\theta = 2 \cos^2 \theta$ $1 - \cos 2\theta = 2 \sin^2 \theta$

➤ Trigonometric functions of half angles:

Trigonometric functions of half angles

$\sin \theta$	$\cos \theta$	$\tan \theta$
$\rightarrow \sin \theta = 2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right)$ $\rightarrow \sin \theta = \frac{2 \tan\left(\frac{\theta}{2}\right)}{1 + \tan^2\left(\frac{\theta}{2}\right)}$	$\rightarrow \cos \theta = \cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)$ $= 1 - 2 \sin^2\left(\frac{\theta}{2}\right)$ $= 2 \cos^2\left(\frac{\theta}{2}\right) - 1$ $= \frac{1 - \tan^2\left(\frac{\theta}{2}\right)}{1 + \tan^2\left(\frac{\theta}{2}\right)}$	$\rightarrow \tan \theta = \frac{2 \tan\left(\frac{\theta}{2}\right)}{1 - \tan^2\left(\frac{\theta}{2}\right)}$
Note: $1 + \cos \theta = 2 \cos^2\left(\frac{\theta}{2}\right)$ $1 - \cos \theta = 2 \sin^2\left(\frac{\theta}{2}\right)$		



➤ **Formulae to convert sum or difference into product:**

i.	sin C + sin D	$2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$
ii.	sin C – sin D	$2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$
iii.	cos C + cos D	$2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$
iv.	cos C – cos D	$= 2 \sin \frac{C+D}{2} \sin \frac{D-C}{2}$ $= -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$

➤ **Formulae to convert product into sum or difference:**

i.	2 sin A cos B	$\sin(A+B) + \sin(A-B)$
ii.	2 cos A sin B	$\sin(A+B) - \sin(A-B)$
iii.	2 cos A cos B	$\cos(A+B) + \cos(A-B)$
iv.	2 sin A sin B	$\cos(A-B) - \cos(A+B)$

➤ **Trigonometric functions of angles of a triangle:**

- i. If A, B, C are the angles of a triangle ABC, then $A + B + C = \pi$
- a. $\sin(B+C) = \sin(\pi-A) = \sin A$
 $\sin(C+A) = \sin B$
 $\sin(A+B) = \sin C$
- b. $\cos(B+C) = \cos(\pi-A) = -\cos A$
 $\cos(C+A) = -\cos B$
 $\cos(A+B) = -\cos C$

- c. $\tan(B+C) = \tan(\pi-A) = -\tan A$
 $\tan(C+A) = -\tan B$
 $\tan(A+B) = -\tan C$
- ii. If $A + B + C = \pi$, then $\frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$,
 $\frac{C+A}{2} = \frac{\pi}{2} - \frac{B}{2}$ and $\frac{B+C}{2} = \frac{\pi}{2} - \frac{A}{2}$
- a. $\sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cos\frac{C}{2}$
 $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$
 $\sin\left(\frac{C+A}{2}\right) = \cos\frac{B}{2}$
- b. $\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$
 $\cos\left(\frac{B+C}{2}\right) = \sin\frac{A}{2}$
 $\cos\left(\frac{C+A}{2}\right) = \sin\frac{B}{2}$

➤ **Some Important results:**

- i. $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$
- ii. $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$
- iii. $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$
- iv. $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$

Shortcuts

- 1. $\sin n\pi = 0, \cos n\pi = (-1)^n$
- 2. i. $\sin(n\pi + \theta) = (-1)^n \sin \theta$
ii. $\cos(n\pi + \theta) = (-1)^n \cos \theta$
iii. $\sin(n\pi - \theta) = (-1)^{n-1} \sin \theta$
iv. $\cos(n\pi - \theta) = (-1)^n \cos \theta$
- 3. $\sin\left(\frac{n\pi}{2} + \theta\right) = (-1)^{\frac{n-1}{2}} \cos \theta$, if n is odd
 $= (-1)^{\frac{n}{2}} \sin \theta$, if n is even
- 4. $\cos\left(\frac{n\pi}{2} + \theta\right) = (-1)^{\frac{n+1}{2}} \sin \theta$, if n is odd
 $= (-1)^{\frac{n}{2}} \cos \theta$, if n is even

- 5. $\left| \sin \frac{A}{2} + \cos \frac{A}{2} \right| = \sqrt{1 + \sin A}$
or $\sin \frac{A}{2} + \cos \frac{A}{2} = \pm \sqrt{1 + \sin A}$
i.e., $\begin{cases} +ve, & \text{if } 2n\pi - \frac{\pi}{4} \leq \frac{A}{2} \leq 2n\pi + \frac{3\pi}{4} \\ -ve, & \text{otherwise} \end{cases}$
- 6. $\left| \sin \frac{A}{2} - \cos \frac{A}{2} \right| = \sqrt{1 - \sin A}$
or $\sin \frac{A}{2} - \cos \frac{A}{2} = \pm \sqrt{1 - \sin A}$
i.e., $\begin{cases} +ve, & \text{if } 2n\pi + \frac{\pi}{4} \leq \frac{A}{2} \leq 2n\pi + \frac{5\pi}{4} \\ -ve, & \text{otherwise} \end{cases}$

7. $\tan x \cdot \tan 2x \cdot \tan 3x = \tan 3x - \tan 2x - \tan x$

8. $\tan 2\alpha \cdot \tan 3\alpha \cdot \tan 5\alpha = \tan 5\alpha - \tan 3\alpha - \tan 2\alpha$

9. $\frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$, where $\theta \neq (2n+1)\pi$

10. $\frac{1+\cos\theta}{\sin\theta} = \cot\frac{\theta}{2}$, where $\theta \neq 2n\pi$

11. $\frac{1-\cos\theta}{1+\cos\theta} = \tan^2\frac{\theta}{2}$, where $\theta \neq (2n+1)\pi$

12. $\frac{1+\cos\theta}{1-\cos\theta} = \cot^2\frac{\theta}{2}$, where $\theta \neq 2n\pi$

13. $\cos \alpha \cdot \cos 2\alpha \cdot \cos 2^2\alpha \cdot \cos 2^3\alpha \dots \cos 2^{n-1}\alpha$
 $= \frac{\sin 2^n\alpha}{2^n \sin \alpha}$, if $\alpha \neq n\pi$
 $= 1$, if $\alpha = 2n\pi$
 $= -1$, if $\alpha = (2n+1)\pi$

14. i. $\tan(45^\circ + \theta) = \frac{1 + \tan\theta}{1 - \tan\theta} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$
ii. $\tan(45^\circ - \theta) = \frac{1 - \tan\theta}{1 + \tan\theta} = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

15. Maximum and minimum values of
 $a \cos \theta + b \sin \theta$ are $\sqrt{a^2 + b^2}$ and $-\sqrt{a^2 + b^2}$
i.e., $-\sqrt{a^2 + b^2} \leq a \cos \theta + b \sin \theta \leq \sqrt{a^2 + b^2}$

16. $\sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta)$
 $+ \dots + \sin[\alpha + (n-1)\beta]$
 $= \frac{\sin\left[\alpha + \left(\frac{n-1}{2}\right)\beta\right]}{\sin\frac{\beta}{2}} \cdot \sin\frac{n\beta}{2}$

If $\beta = \alpha$, then

$$\begin{aligned} & \sin \alpha + \sin 2\alpha + \sin 3\alpha + \dots + \sin n\alpha \\ &= \frac{\sin\left(\frac{n+1}{2}\right)\alpha \cdot \sin\frac{n\alpha}{2}}{\sin\left(\frac{\alpha}{2}\right)} \end{aligned}$$

17. $\cos \alpha + \cos (\alpha + \beta) + \cos (\alpha + 2\beta) + \dots + \cos [\alpha + (n-1)\beta]$

$$= \frac{\cos \left[\alpha + (n-1)\frac{\beta}{2} \right] \cdot \sin \left(\frac{n\beta}{2} \right)}{\sin \frac{\beta}{2}}$$

If $\beta = \alpha$, then

$$\cos \alpha + \cos 2\alpha + \cos 3\alpha + \dots + \cos n\alpha$$

$$= \frac{\cos \left(\frac{n+1}{2} \right) \alpha \cdot \sin \left(\frac{n\alpha}{2} \right)}{\sin \left(\frac{\alpha}{2} \right)}$$

18. $\sin \theta \sin(60^\circ - \theta) \sin(60^\circ + \theta) = \frac{1}{4} \sin 3\theta$

19. $\cos \theta \cos(60^\circ - \theta) \cos(60^\circ + \theta) = \frac{1}{4} \cos 3\theta$

20. $\tan \theta \tan(60^\circ - \theta) \tan(60^\circ + \theta) = \tan 3\theta$

21. If $A + B + C = 180^\circ$, then

 - $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$
 - $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$
 - $\cos 2A + \cos 2B - \cos 2C = 1 - 4 \sin A \sin B \cos C$
 - $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$
 - $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$
 - $\cos A + \cos B - \cos C = -1 + 4 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$
 - $\tan A + \tan B + \tan C = \tan A \tan B \tan C$
 - $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$
 - $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$
 - $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2}$

Multiple Choice Questions

3.1 Trigonometric functions of sum and difference of angles

3. The value of $(\sqrt{3} \sin 75^\circ - \cos 75^\circ)$ is [2019]

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$
 (C) $2 \cos 15^\circ$ (D) $2 \sin 15^\circ$

4. If $\sec x + \tan x = 3$, where $x \in \left(0, \frac{\pi}{2}\right)$, then
 $\sin x =$ [2020]

(A) $\frac{4}{5}$ (B) $\frac{3}{5}$
 (C) -1 (D) $\frac{1}{5}$



5. If $2 \cos^2 \theta + 3 \cos \theta = 2$, then permissible value of $\cos \theta$ is [2020]
- (A) 1 (B) $-\frac{1}{2}$
(C) $\frac{1}{2}$ (D) 0
6. If $\sin x + \operatorname{cosec} x = 3$, then value of $\sin^4 x + \operatorname{cosec}^4 x$ is [2020]
- (A) 49 (B) 47
(C) 7 (D) 74
7. If $\frac{1 - \tan \theta}{1 + \tan \theta} = \frac{1}{\sqrt{3}}$, where $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\theta =$ [2020]
- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{12}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$
8. If $\tan \theta + \sin \theta = a$ and $\tan \theta - \sin \theta = b$, then the values of $\cot \theta$ and $\operatorname{cosec} \theta$ are respectively [2020]
- (A) $\frac{2}{a+b}, \frac{2}{a-b}$ (B) $\frac{2}{a-b}, \frac{2}{a+b}$
(C) $\frac{1}{a+b}, \frac{1}{a-b}$ (D) $\frac{1}{a-b}, \frac{1}{a+b}$
9. $\sin\left(\frac{\pi}{3} + x\right) - \cos\left(\frac{\pi}{6} + x\right) =$ [2020]
- (A) $-\sin x$ (B) $-\cos x$
(C) $\sin x$ (D) $\cos x$
10. If $x = 3 \sin \theta$, $y = 3 \cos \theta \cos \phi$, $z = 3 \cos \theta \sin \phi$, then $x^2 + y^2 + z^2 =$ [2020]
- (A) 9 (B) 3
(C) 18 (D) 27
11. If $\sin x + \sin^2 x = 1$, then $\cos^8 x + 2 \cos^6 x + \cos^4 x$ is [2020]
- (A) 3 (B) 2
(C) 4 (D) 1
12. $\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \times \dots \times \tan 89^\circ =$ [2020]
- (A) 1 (B) $\sqrt{3}$
(C) 2 (D) $\sqrt{2}$
13. If A and B are two angles such that $A, B \in (0, \pi)$ and they are not supplementary angles such that $\sin A - \sin B = 0$, then [2020]
- (A) $A - B = \frac{\pi}{2}$ (B) $A - B = \frac{\pi}{3}$
(C) $A \neq B$ (D) $A = B$
14. For $\theta \in \left(0, \frac{\pi}{2}\right)$,
 $\tan 3\theta \cdot \tan 2\theta \cdot \tan \theta + \tan 2\theta + \tan \theta = 1$, then $\theta =$ [2020]
- (A) $\frac{\pi^c}{3}$ (B) $\frac{\pi^c}{6}$
(C) $\frac{\pi^c}{12}$ (D) $\frac{\pi^c}{4}$
15. If $\tan A = \frac{5}{6}$, $\tan B = \frac{1}{11}$, then $A + B =$ [2020]
- (A) $\frac{\pi}{3}$ (B) $-\frac{\pi}{3}$
(C) $-\frac{\pi}{4}$ (D) $\frac{\pi}{4}$
16. If $\operatorname{cosec} \theta + \cot \theta = 5$, then $\sin \theta =$ [2020]
- (A) $\frac{5}{26}$ (B) $\frac{5}{13}$
(C) $\frac{1}{13}$ (D) $\frac{1}{5}$
17. If $2\sin\left(\theta + \frac{\pi}{3}\right) = \cos\left(\theta - \frac{\pi}{6}\right)$, then $\tan \theta =$ [2021]
- (A) $-\frac{1}{\sqrt{3}}$ (B) $-\sqrt{3}$
(C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$
18. $\tan 3A \cdot \tan 2A \cdot \tan A =$ [2021]
- (A) $\tan 3A + \tan 2A - \tan A$
(B) $\tan 3A - \tan 2A - \tan A$
(C) $\tan 3A + \tan 2A + \tan A$
(D) $\tan 3A - \tan 2A + \tan A$
19. If $\theta + \phi = \alpha$ and $\tan \theta = k \tan \phi$ (where $k > 1$), then the value of $\sin(\theta - \phi)$ is [2021]
- (A) $k \tan \phi$ (B) $\sin \alpha$
(C) $\left(\frac{k-1}{k+1}\right) \sin \alpha$ (D) $k \cos \phi$
20. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \leq \alpha, \beta \leq \frac{\pi}{4}$, then $\tan 2\alpha =$ [2022]
- (A) $\frac{25}{16}$ (B) $\frac{19}{13}$
(C) $\frac{20}{7}$ (D) $\frac{56}{33}$
21. If $\cot \alpha = \frac{1}{2}$ and $\sec \beta = -\frac{5}{3}$ where $\alpha \in \left(\pi, \frac{3\pi}{2}\right)$ and $\beta \in \left(\frac{\pi}{2}, \pi\right)$, then $\tan(\alpha + \beta)$ has the value [2022]
- (A) $\frac{2}{11}$ (B) $\frac{3}{11}$
(C) $\frac{22}{9}$ (D) $\frac{9}{11}$

Page no. **6** to **8** are purposely left blank.

To see complete chapter buy **Target Notes** or **Target E-Notes**



- 10.** The value of $\cos 20^\circ + 2 \sin^2 55^\circ - \sqrt{2} \sin 65^\circ$ is [2024]
- (A) 0 (B) 1
(C) -1 (D) $\frac{1}{2}$
- 11.** The smallest positive value of x in degrees satisfying the equation $\tan(x+100^\circ) = \tan(x+50^\circ)\tan(x)\tan(x-50^\circ)$ is [2024]
- (A) 30° (B) 15°
(C) 45° (D) 60°
- 12.** The value of $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is [2024]
- (A) $\frac{1}{8}$ (B) $-\frac{1}{8}$
(C) $\frac{1}{16}$ (D) $-\frac{1}{16}$
- 13.** Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3}a \cos x + 2b \sin x = c$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then the value of $\frac{b}{a}$ is [2024]
- (A) 0.1 (B) 0.5
(C) -0.5 (D) 1
- 3.5 Trigonometric functions of angles of a triangle**
- 1.** If A, B, C are the angles of ΔABC then $\cot A \cdot \cot B + \cot B \cdot \cot C + \cot C \cdot \cot A =$ [2018]
- (A) 0 (B) 1
(C) 2 (D) -1
- 2.** In ΔABC , if $\tan A + \tan B + \tan C = 6$ and $\tan A \cdot \tan B = 2$ then $\tan C =$ [2019]
- (A) 4 (B) 1
(C) 3 (D) 2
- 3.** In ΔABC , with usual notations; if $\cos A = \sin B - \cos C$, then $\cos A \cdot \cos C =$ [2019]
- (A) $\frac{1}{4}$ (B) 0
(C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{4}$
- 4.** If A, B, C are angles of a ΔABC , then $\tan 2A + \tan 2B + \tan 2C =$ [2020]
- (A) $\tan 2A \tan 2B \tan 2C$
(B) $\tan A \tan B \tan C$
(C) $\tan 3A \tan 2B \tan 2C$
(D) $\tan 2A \tan 3B \tan 2C$
- 5.** If $A + B + C = 180^\circ$, then the value of $\tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right) + \tan\left(\frac{B}{2}\right)\tan\left(\frac{C}{2}\right) + \tan\left(\frac{C}{2}\right)\tan\left(\frac{A}{2}\right)$ is [2020]
- (A) 2 (B) 1
(C) -2 (D) -1
- 6.** In a triangle ABC if $\frac{\sin A - \sin C}{\cos C - \cos A} = \cot B$, then A, B, C are in [2020]
- (A) Harmonic progression
(B) Geometric progression
(C) Arithmetico-Geometric progression
(D) Arithmetic progression
- 7.** If $\alpha + \beta + \gamma = \pi$, then the expression $\sin^2\alpha + \sin^2\beta - \sin^2\gamma$ has the value [2024]
- (A) $2\sin\alpha \sin\beta \sin\gamma$
(B) $2\cos\alpha \sin\beta \sin\gamma$
(C) $2\sin\alpha \cos\beta \sin\gamma$
(D) $2\sin\alpha \sin\beta \cos\gamma$
- 8.** If A, B, C are the angles of a triangle with $\tan\frac{A}{2} = \frac{1}{3}, \tan\frac{B}{2} = \frac{2}{3}$ then the value of $\tan\frac{C}{2}$ is [2024]
- (A) $-\frac{7}{9}$ (B) $\frac{7}{9}$
(C) $\frac{9}{7}$ (D) $-\frac{9}{7}$

Concept Fusion

- 1.** The maximum value of $(\cos \alpha_1) \cdot (\cos \alpha_2) \dots (\cos \alpha_n)$ under the constraints $0 \leq \alpha_1, \alpha_2, \dots, \alpha_n \leq \frac{\pi}{2}$ and $(\cot \alpha_1) \cdot (\cot \alpha_2) \dots (\cot \alpha_n) = 1$ is [2024]
- (A) $\frac{1}{2^{\frac{n}{2}}}$ (B) $\frac{1}{2^n}$
(C) 2^n (D) $2^{\frac{n}{2}}$



Answers and Solutions to MCQs

3.1 Trigonometric functions of sum and difference of angles

- | | | |
|---------|---------|---------|
| 1. (B) | 2. (B) | 3. (B) |
| 4. (A) | 5. (C) | 6. (B) |
| 7. (B) | 8. (A) | 9. (C) |
| 10. (A) | 11. (D) | 12. (A) |
| 13. (D) | 14. (C) | 15. (D) |
| 16. (B) | 17. (B) | 18. (B) |
| 19. (C) | | |

[Note: Detailed solutions for Q.1 to Q.19 (wherever applicable) can be accessed via QR code at the end of the chapter.]

20. (D)

$$\begin{aligned} \text{Given, } \cos(\alpha + \beta) &= \frac{4}{5} \\ \Rightarrow \sin(\alpha + \beta) &= \frac{3}{5} \\ \Rightarrow \tan(\alpha + \beta) &= \frac{3}{4} \quad \dots(\text{i}) \end{aligned}$$

$$\text{Also, } \sin(\alpha - \beta) = \frac{5}{13}$$

$$\begin{aligned} \Rightarrow \cos(\alpha - \beta) &= \frac{12}{13} \\ \Rightarrow \tan(\alpha - \beta) &= \frac{5}{12} \quad \dots(\text{ii}) \end{aligned}$$

$$\begin{aligned} \text{Now, } \tan 2\alpha &= \tan [(\alpha + \beta) + (\alpha - \beta)] \\ &= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)} \\ &= \frac{\left(\frac{3}{4}\right) + \left(\frac{5}{12}\right)}{1 - \left(\frac{3}{4}\right)\left(\frac{5}{12}\right)} \\ &\dots[\text{From (i) and (ii)}] \\ \Rightarrow \tan 2\alpha &= \frac{36+20}{48-15} = \frac{56}{33} \end{aligned}$$

21. (A)

$$\begin{aligned} \cot \alpha &= \frac{1}{2} \\ \Rightarrow \tan \alpha &= 2 \text{ and} \\ \sec \beta &= -\frac{5}{3} \\ \Rightarrow \cos \beta &= \frac{-3}{5} \\ \Rightarrow \sin \beta &= \frac{4}{5} \quad \dots \left[\because \beta \in \left(\frac{\pi}{2}, \pi\right) \right] \\ \Rightarrow \tan \beta &= \frac{-4}{3} \end{aligned}$$

Consider,

$$\begin{aligned} \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} \\ &= \frac{2 + \left(\frac{-4}{3}\right)}{1 - (2)\left(\frac{-4}{3}\right)} \\ &= \frac{6 - 4}{3 + 8} = \frac{2}{11} \end{aligned}$$

22. (C)

$$\begin{aligned} \cos^2 A - \sin^2 B &= \cos(A + B) \cdot \cos(A - B) \\ \therefore \cos^2 48^\circ - \sin^2 12^\circ &= \cos(60^\circ) \cdot \cos(36^\circ) \\ &= \frac{1}{2} \cdot \left(1 - 2\sin^2 \frac{36}{2}\right) \\ &= \frac{1}{2}(1 - 2\sin^2 18^\circ) \\ &= \frac{1}{2} \left[1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2\right] \\ &= \frac{\sqrt{5}+1}{8} \end{aligned}$$

23. (C)

$$\begin{aligned} \sqrt{3} \cosec 20^\circ - \sec 20^\circ &= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} \\ &= \frac{\sqrt{3} \cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ} \\ &= \frac{4\left(\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ\right)}{2 \sin 20^\circ \cos 20^\circ} \\ &= \frac{4(\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ)}{\sin 40^\circ} \\ &= 4 \frac{\sin 40^\circ}{\sin 40^\circ} = 4 \end{aligned}$$

24. (C)

$$\begin{aligned} \text{Given, } \tan A - \tan B &= x \\ \cot B - \cot A &= y \\ \Rightarrow \frac{1}{\tan B} - \frac{1}{\tan A} &= y \\ \Rightarrow \frac{\tan A - \tan B}{\tan A \cdot \tan B} &= y \\ \Rightarrow \tan A \cdot \tan B &= \frac{x}{y} \quad \dots(\text{i}) \end{aligned}$$

$$\begin{aligned} \text{Now, } \cot(A - B) &= \frac{1}{\tan(A - B)} \\ &= \frac{1 + \tan A \cdot \tan B}{\tan A - \tan B} \end{aligned}$$



$$\begin{aligned}
 &= \frac{1 + \frac{x}{y}}{\frac{x}{y}} \\
 &= \frac{y + x}{xy} \\
 &= \frac{1}{x} + \frac{1}{y}
 \end{aligned}
 \quad \dots[\text{from (i)}]$$

25. (C)

Given, $\alpha = \beta + \gamma$

$$\therefore \gamma = \alpha - \beta$$

$$\tan \gamma = \tan(\alpha - \beta)$$

$$= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \cdot \tan \beta}$$

$$\begin{aligned}
 &= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \cdot \tan\left(\frac{\pi}{2} - \alpha\right)} \quad \dots \left[\begin{array}{l} \alpha + \beta = \frac{\pi}{2} \\ \therefore \beta = \frac{\pi}{2} - \alpha \end{array} \right] \\
 &= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \cot \alpha} \\
 &= \frac{\tan \alpha - \tan \beta}{2}
 \end{aligned}$$

$$\Rightarrow 2 \tan \gamma = \tan \alpha - \tan \beta$$

$$\Rightarrow \tan \alpha = 2 \tan \gamma + \tan \beta$$

3.2 Trigonometric functions of allied angles

- | | | |
|---------|---------|---------|
| 1. (A) | 2. (B) | 3. (B) |
| 4. (B) | 5. (C) | 6. (D) |
| 7. (B) | 8. (A) | 9. (B) |
| 10. (C) | 11. (C) | 12. (B) |

Note: Detailed solutions for Q.1 to Q.12 (wherever applicable) can be accessed via QR code at the end of the chapter.]

13. (C)

$$\frac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ \cdot \tan 110^\circ} = \tan(160^\circ - 110^\circ)$$

$$= \tan 50^\circ$$

$$= \tan(90^\circ - 40^\circ)$$

$$= \cot 40^\circ$$

$$= \frac{1}{\tan 40^\circ}$$

$$= \frac{1}{\tan 2(20^\circ)}$$

$$= \frac{1}{\left(\frac{2 \tan 20^\circ}{1 - \tan^2 20^\circ}\right)}$$

$$= \frac{1 - \tan^2 20^\circ}{2 \tan 20^\circ}$$

$$= \frac{1 - p^2}{2p} \quad \dots [\because p = \tan 20^\circ (\text{given})]$$

14. (C)

$$\begin{aligned}
 &\frac{\sin^2(-160^\circ)}{\sin^2 70^\circ} + \frac{\sin(180^\circ - \theta)}{\sin \theta} \\
 &= \frac{[-\sin 160^\circ]^2}{\sin^2 70^\circ} + \frac{\sin \theta}{\sin \theta} \\
 &= \frac{\sin^2 160^\circ}{\sin^2 70^\circ} + 1 \\
 &= \frac{[\sin(180^\circ - 20^\circ)]^2}{[\sin(90^\circ - 20^\circ)]^2} + 1 \\
 &= \frac{\sin^2 20^\circ}{\cos^2 20^\circ} + 1 \\
 &= \tan^2 20^\circ + 1 \\
 &= \sec^2 20^\circ
 \end{aligned}$$

15. (B)

$$\begin{aligned}
 &\cot(A + B) = 0 \\
 &\Rightarrow A + B = 90^\circ \quad \dots \text{(i)} \\
 &\sin(A + 2B) \\
 &= \sin(A + B) \cos B + \cos(A + B) \sin B \\
 &= \sin 90^\circ \cos B + \cos 90^\circ \sin B \\
 &= \cos B \\
 &= \cos(90^\circ - A) \quad \dots [\text{From (i)}] \\
 &= \sin A
 \end{aligned}$$

16. (D)

$$\begin{aligned}
 \tan \theta &= \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha} \\
 \frac{\sin \theta}{\cos \theta} &= \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha} \\
 \therefore \sin \alpha \sin \theta + \cos \alpha \sin \theta &= \sin \alpha \cos \theta - \cos \alpha \cos \theta \\
 \therefore \cos \alpha \cos \theta + \sin \alpha \sin \theta &= \sin \alpha \cos \theta - \cos \alpha \sin \theta \\
 \therefore \cos(\alpha - \theta) &= \sin(\alpha - \theta) \\
 \therefore \alpha - \theta &= \frac{\pi}{4} \quad \dots \left[\because 0 \leq \alpha \leq \frac{\pi}{2} \right] \\
 \therefore \theta &= \alpha - \frac{\pi}{4} \\
 \therefore 2\theta &= 2\alpha - \frac{\pi}{2} \\
 \therefore \cos 2\theta &= \cos\left(2\alpha - \frac{\pi}{2}\right) \\
 &= \cos\left[-\left(\frac{\pi}{2} - 2\alpha\right)\right] \\
 &= \cos\left(\frac{\pi}{2} - 2\alpha\right) \quad \dots [\because \cos(-\theta) = \cos \theta] \\
 \therefore \cos 2\theta &= \sin 2\alpha
 \end{aligned}$$

17. (B)

$$\begin{aligned}
 &\cos(18^\circ - A) \cos(18^\circ + A) \\
 &\quad - \cos(72^\circ - A) \cos(72^\circ + A) \\
 &= \cos(18^\circ - A) \cos[90^\circ - (72^\circ - A)] \\
 &\quad - \cos(72^\circ - A) \cos[90^\circ - (18^\circ - A)] \\
 &= \sin(72^\circ - A) \cos(18^\circ - A) \\
 &\quad - \cos(72^\circ - A) \sin(18^\circ - A) \\
 &= \sin[(72^\circ - A) - (18^\circ - A)] \\
 &= \sin 54^\circ
 \end{aligned}$$

**18. (D)**

$$\begin{aligned}
 & \frac{\cot A}{1 + \cot A} \cdot \frac{\cot B}{1 + \cot B} \\
 &= \frac{1}{(1 + \tan A)(1 + \tan B)} \\
 &= \frac{1}{\tan A + \tan B + 1 + \tan A \tan B} \\
 &= \frac{1}{1 - \tan A \tan B + 1 + \tan A \tan B} \\
 &\quad \cdots [\because \tan(A+B) = \tan 225^\circ] \\
 &\quad \cdots [\Rightarrow \tan A + \tan B = 1 - \tan A \tan B] \\
 &= \frac{1}{2}
 \end{aligned}$$

3.3 Trigonometric functions of multiple angles

- | | | |
|----------------|----------------|----------------|
| 1. (D) | 2. (D) | 3. (B) |
| 4. (C) | 5. (D) | 6. (C) |
| 7. (C) | 8. (B) | 9. (D) |
| 10. (B) | 11. (B) | 12. (B) |

[Note: Detailed solutions for Q.1 to Q.12 (wherever applicable) can be accessed via QR code at the end of the chapter.]

13. (D)

$$\begin{aligned}
 \cos 3\theta &= 4 \cos^3 \theta - 3 \cos \theta \\
 \therefore 4 \cos^3 \theta &= \cos 3\theta + 3 \cos \theta \\
 \therefore 4 \cos^3 20^\circ &= \cos 3(20^\circ) + 3 \cos 20^\circ \\
 &= \cos 60^\circ + 3 \cos 20^\circ \\
 &= \frac{1}{2} + 3 \cos 20^\circ
 \end{aligned}$$

14. (B)

Given: $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$

$$\begin{aligned}
 \tan 2B &= \frac{2 \tan B}{1 - \tan^2 B} \\
 &= \frac{2 \left(\frac{1}{3}\right)}{1 - \left(\frac{1}{3}\right)^2} \\
 &= \frac{2}{3} \times \frac{9}{8} = \frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 \tan(A+2B) &= \frac{\tan A + \tan 2B}{1 - (\tan A)(\tan 2B)} \\
 &= \frac{\left(\frac{1}{2}\right) + \left(\frac{3}{4}\right)}{1 - \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)} \\
 &= \frac{(4+6)}{8-3} \\
 &= \frac{10}{5} \\
 &= 2
 \end{aligned}$$

15. (A)

$$\text{Given } \tan \theta = \frac{a}{b} \quad \dots(i)$$

$$\begin{aligned}
 b \cos 2\theta + a \sin 2\theta \\
 &= b \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) + a \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) \\
 &= b \left[1 - \left(\frac{a^2}{b^2} \right) \right] + a \left[2 \left(\frac{a}{b} \right) \right] \\
 &= b \left(\frac{b^2 - a^2}{b^2 + a^2} \right) + a \left(\frac{2ab}{b^2 + a^2} \right) \\
 &= b \left(\frac{b^2 - a^2}{b^2 + a^2} \right) + a \left(\frac{2ab}{a^2 + b^2} \right) \\
 &= \frac{b^3 - a^2b + 2a^2b}{a^2 + b^2} \\
 &= \frac{a^2b + b^3}{a^2 + b^2} \\
 &= \frac{b(a^2 + b^2)}{a^2 + b^2} = b
 \end{aligned}$$

16. (C)

$$\begin{aligned}
 & (\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 \\
 &= (\cos^2 \alpha + \sin^2 \alpha) + (\cos^2 \beta + \sin^2 \beta) \\
 &\quad + 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \\
 &= 1 + 1 + 2 \cos(\alpha - \beta) \\
 &= 2[1 + \cos(\alpha - \beta)] \\
 &= 2 \left[2 \cos^2 \left(\frac{\alpha - \beta}{2} \right) \right] \\
 &= 4 \cos^2 \left(\frac{\alpha - \beta}{2} \right)
 \end{aligned}$$

17. (A)

$$(1 + \sqrt{1+x}) \tan x = 1 + \sqrt{1-x}$$

$$\Rightarrow \tan x = \frac{1 + \sqrt{1-x}}{1 + \sqrt{1+x}}$$

Put $x = \sin \theta$

$$\begin{aligned}
 \therefore \tan x &= \frac{1 + \sqrt{1 - \sin \theta}}{1 + \sqrt{1 + \sin \theta}} \\
 &= \frac{1 + \sqrt{\left(\cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right)^2}}{1 + \sqrt{\left(\cos \frac{\theta}{2} + \sin \frac{\theta}{2} \right)^2}} \\
 &= \frac{1 + \cos \frac{\theta}{2} - \sin \frac{\theta}{2}}{1 + \cos \frac{\theta}{2} + \sin \frac{\theta}{2}} \\
 &= \frac{2 \cos^2 \frac{\theta}{4} - 2 \sin \frac{\theta}{4} \cos \frac{\theta}{4}}{2 \cos^2 \frac{\theta}{4} + 2 \sin \frac{\theta}{4} \cos \frac{\theta}{4}}
 \end{aligned}$$



$$\begin{aligned}
 &= \frac{2\cos^2 \theta}{4} \left(1 - \tan \frac{\theta}{4}\right) \\
 &\quad - \frac{2\cos^2 \theta}{4} \left(1 + \tan \frac{\theta}{4}\right) \\
 \therefore \tan x &= \tan \left(\frac{\pi}{4} - \frac{\theta}{4}\right) \quad \dots \left[\because \tan \frac{\pi}{4} = 1\right] \\
 \Rightarrow x &= \frac{\pi - \theta}{4} \\
 \Rightarrow \sin 4x &= \sin(\pi - \theta) = \sin \theta = x
 \end{aligned}$$

- 18. (A)**
We have, $a \cos 2\theta + b \sin 2\theta = c$

$$\begin{aligned}
 \Rightarrow a \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) + b \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) &= c \\
 \Rightarrow a - a \tan^2 \theta + 2b \tan \theta &= c + c \tan^2 \theta \\
 \Rightarrow -(a+c) \tan^2 \theta + 2b \tan \theta + (a-c) &= 0 \\
 \therefore \tan \alpha + \tan \beta &= -\frac{2b}{-(c+a)} = \frac{2b}{c+a}
 \end{aligned}$$

19. (A)
 $\cos 2B = \frac{\cos(A+C)}{\cos(A-C)}$

$$\begin{aligned}
 \frac{1 - \tan^2 B}{1 + \tan^2 B} &= \frac{\cos A \cos C - \sin A \sin C}{\cos A \cos C + \sin A \sin C} \\
 \frac{1 - \tan^2 B}{1 + \tan^2 B} &= \frac{\cos A \cos C (1 - \tan A \tan C)}{\cos A \cos C (1 + \tan A \tan C)} \\
 \frac{1 - \tan^2 B}{1 + \tan^2 B} &= \frac{(1 - \tan A \tan C)}{(1 + \tan A \tan C)} \\
 (1 - \tan^2 B) (1 + \tan A \tan C) &= (1 - \tan A \tan C) (1 + \tan^2 B) \\
 1 + \tan A \tan C - \tan^2 B - \tan^2 B \tan A \tan C &= 1 + \tan^2 B - \tan A \tan C - \tan A \tan C \tan^2 B \\
 2 \tan A \tan C &= 2 \tan^2 B \\
 \tan^2 B &= \tan A \cdot \tan C \\
 \therefore \tan A, \tan B, \tan C &\text{ are in G.P.}
 \end{aligned}$$

20. (D)
 $\tan x = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$

$$\begin{aligned}
 \therefore 1 + \tan^2 x &= \sec^2 x \\
 \therefore \sec^2 x &= \frac{25}{16} \\
 \therefore \sec x &= \frac{-5}{4} \quad \dots \left[\because \pi < x < \frac{3\pi}{2}\right] \\
 \therefore \cos x &= \frac{-4}{5} \\
 \therefore \cos \frac{x}{2} &= -\sqrt{\frac{1+\cos x}{2}} \quad \dots \left[\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}\right] \\
 &= -\sqrt{\frac{1}{10}} \\
 &= \frac{-1}{\sqrt{10}}
 \end{aligned}$$

21. (B)
 $\sin(\theta - \alpha)$, $\sin \theta$ and $\sin(\theta + \alpha)$ are in H.P.
 $\Rightarrow \frac{1}{\sin(\theta - \alpha)}, \frac{1}{\sin \theta}, \frac{1}{\sin(\theta + \alpha)}$ will be in A.P.

$$\begin{aligned}
 \therefore \frac{2}{\sin \theta} &= \frac{1}{\sin(\theta - \alpha)} + \frac{1}{\sin(\theta + \alpha)} \\
 \Rightarrow \frac{2}{\sin \theta} &= \frac{\sin(\theta + \alpha) + \sin(\theta - \alpha)}{\sin(\theta - \alpha) \sin(\theta + \alpha)} \\
 \Rightarrow \frac{2}{\sin \theta} &= \frac{2 \sin \theta \cos \alpha}{\sin^2 \theta - \sin^2 \alpha} \\
 \Rightarrow \sin^2 \theta - \sin^2 \alpha &= \sin^2 \theta \cos \alpha \\
 \Rightarrow \sin^2 \theta (1 - \cos \alpha) &= \sin^2 \alpha \\
 \Rightarrow \sin^2 \theta \left(2 \sin^2 \frac{\alpha}{2}\right) &= 4 \sin^2 \frac{\alpha}{2} \cos^2 \frac{\alpha}{2} \\
 \Rightarrow 1 - \cos^2 \theta &= 2 \cos^2 \frac{\alpha}{2} \\
 \Rightarrow \cos^2 \theta &= 1 - 2 \cos^2 \frac{\alpha}{2} \\
 \Rightarrow 2 \cos^2 \theta - 1 &= 1 - 4 \cos^2 \frac{\alpha}{2} \\
 \Rightarrow \cos 2\theta &= 1 - 4 \cos^2 \frac{\alpha}{2}
 \end{aligned}$$

3.4 Factorization formulae

1. (A) 2. (C) 3. (C)
 4. (D) 5. (C) 6. (A)
 7. (C)

[Note: Detailed solutions for Q.1 to Q.7 (wherever applicable) can be accessed via QR code at the end of the chapter.]

$$\begin{aligned}
 8. (C) \quad &\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ \\
 &= \frac{1}{2} (2 \cos^2 10^\circ - 2 \cos 10^\circ \cos 50^\circ + 2 \cos^2 50^\circ) \\
 &= \frac{1}{2} [1 + \cos 20^\circ - (\cos 60^\circ + \cos 40^\circ) \\
 &\quad + 1 + \cos 100^\circ] \\
 &= \frac{1}{2} [2 + (\cos 20^\circ + \cos 100^\circ) - \cos 60^\circ - \cos 40^\circ] \\
 &= \frac{1}{2} [2 + (2 \cos 60^\circ \cos 40^\circ) - \cos 60^\circ - \cos 40^\circ] \\
 &= \frac{1}{2} \left(2 + \cos 40^\circ - \frac{1}{2} - \cos 40^\circ\right) \\
 &= \frac{1}{2} \left(\frac{3}{2}\right) \\
 &= \frac{3}{4}
 \end{aligned}$$



9. (C)

$$\cos x + \cos y - \cos(x+y) = \frac{3}{2}$$

$$\therefore 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - \left(2\cos^2\left(\frac{x+y}{2}\right) - 1\right) = \frac{3}{2}$$

$$\left[\begin{array}{l} \because \cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \text{ and} \\ \dots \\ \cos\theta = 2\cos^2\left(\frac{\theta}{2}\right) - 1 \end{array} \right]$$

$$\therefore 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 2\cos^2\left(\frac{x+y}{2}\right) = \frac{3}{2} - 1$$

$$\therefore 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 2\cos^2\left(\frac{x+y}{2}\right) = \frac{1}{2}$$

$$\therefore 4\cos^2\left(\frac{x+y}{2}\right) - 4\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) + 1 = 0$$

Substituting $\cos\left(\frac{x+y}{2}\right) = t$, we get

$$4t^2 - 4t\cos\left(\frac{x-y}{2}\right) + 1 = 0$$

As t is real, we get $b^2 - 4ac \geq 0$

$$\Rightarrow \left[-4\cos\left(\frac{x-y}{2}\right)\right]^2 - 4 \times 4 \times 1 \geq 0$$

$$\Rightarrow 16\cos^2\left(\frac{x-y}{2}\right) - 16 \geq 0$$

$$\Rightarrow \cos^2\left(\frac{x-y}{2}\right) \geq 1$$

$$\Rightarrow \cos^2\left(\frac{x-y}{2}\right) = 1$$

...[$\because -1 \leq \cos\theta \leq 1$, for all values of θ]

$$\Rightarrow \frac{x-y}{2} = 0$$

$$\Rightarrow x = y$$

10. (B)

$$\cos 20^\circ + 2\sin^2 55^\circ - \sqrt{2} \sin 65^\circ$$

$$= \cos 20^\circ + 1 - \cos 2(55^\circ) - \sqrt{2} \sin 65^\circ$$

...[$2\sin^2\theta = 1 - \cos 2\theta$]

$$= \cos 20^\circ - \cos 110^\circ - \sqrt{2} \sin 65^\circ + 1$$

$$= 2\sin 65^\circ \sin 45^\circ - \sqrt{2} \sin 65^\circ + 1$$

$$= 2\sin 65^\circ \left(\frac{1}{\sqrt{2}}\right) - \sqrt{2} \sin 65^\circ + 1$$

$$= \sqrt{2} \sin 65^\circ - \sqrt{2} \sin 65^\circ + 1 = 1$$

11. (A)

$$\tan(x+100^\circ) = \tan(x+50^\circ) \tan(x) \tan(x-50^\circ)$$

$$\Rightarrow \frac{\tan(x+100^\circ)}{\tan(x-50^\circ)} = \tan(x+50^\circ) \tan(x)$$

$$\Rightarrow \frac{2\sin(x+100^\circ)\cos(x-50^\circ)}{2\cos(x+100^\circ)\sin(x-50^\circ)}$$

$$= \frac{2\sin(x+50^\circ)\sin x}{2\cos(x+50^\circ)\cos x}$$

$$\Rightarrow \frac{\sin(2x+50^\circ) + \sin 150^\circ}{\sin(2x+50^\circ) - \sin 150^\circ} \\ = \frac{\cos(50^\circ) - \cos(2x+50^\circ)}{\cos(2x+50^\circ) + \cos 50^\circ}$$

By componendo-dividendo, we get

$$\frac{2\sin(2x+50^\circ)}{2\sin(150^\circ)} = \frac{2\cos 50^\circ}{-2\cos(2x+50^\circ)} \\ \Rightarrow 2\sin(2x+50^\circ) \cos(2x+50^\circ) \\ = -2\sin(150^\circ) \cos(50^\circ)$$

$$\Rightarrow \sin(4x+100^\circ) = -\cos 50^\circ$$

$$\Rightarrow \sin(4x+100^\circ) = \sin(270^\circ - 50^\circ)$$

$$\Rightarrow 4x+100^\circ = 220^\circ$$

$$\Rightarrow 4x = 120^\circ$$

$$\Rightarrow x = 30^\circ$$

12. (A)

$$\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)$$

$$= \left(1 + \cos\frac{\pi}{8}\right)\left(1 - \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 - \cos\frac{3\pi}{8}\right)$$

...[$\because \cos(\pi - \theta) = -\cos\theta$]

$$= \left(1 - \cos^2\frac{\pi}{8}\right)\left(1 - \cos^2\frac{3\pi}{8}\right)$$

$$= \sin^2\frac{\pi}{8} \sin^2\frac{3\pi}{8}$$

$$= \frac{1}{4} \left(2\sin\frac{\pi}{8} \cdot \sin\frac{3\pi}{8}\right)^2$$

$$= \frac{1}{4} \left(\cos\frac{\pi}{4} - \cos\frac{3\pi}{8}\right)^2$$

$$= \frac{1}{8}$$

13. (B)

α and β are roots of $\sqrt{3} a \cos x + 2b \sin x = c$

$$\therefore \sqrt{3} a \cos \alpha + 2 b \sin \alpha = c \quad \dots(i)$$

$$\sqrt{3} a \cos \beta + 2 b \sin \beta = c \quad \dots(ii)$$

Subtracting (ii) from (i), we get

$$\sqrt{3} a (\cos \alpha - \cos \beta) + 2b(\sin \alpha - \sin \beta) = 0$$

$$\Rightarrow \sqrt{3}a \left[-2\sin\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right) \right]$$

$$+ 2b \left[2\cos\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right) \right] = 0$$

$$\Rightarrow -\sqrt{3}a \left[2\sin\left(\frac{\pi}{6}\right) \sin\left(\frac{\alpha-\beta}{2}\right) \right]$$

$$+ 2b \left[2\cos\left(\frac{\pi}{6}\right) \sin\left(\frac{\alpha-\beta}{2}\right) \right] = 0$$

$$\Rightarrow -\sqrt{3}a \sin\left(\frac{\alpha-\beta}{2}\right) + 2b \left[\sqrt{3} \sin\left(\frac{\alpha-\beta}{2}\right) \right] = 0$$

$$\Rightarrow -\sqrt{3}a + 2\sqrt{3}b = 0 \Rightarrow \frac{b}{a} = \frac{1}{2} = 0.5$$



3.5 Trigonometric functions of angles of a triangle

1. (B) 2. (C) 3. (B)
 4. (A) 5. (B) 6. (D)

[Note: Detailed solutions for Q.1 to Q.6 (wherever applicable) can be accessed via QR code at the end of the chapter.]

7. (D)

$$\begin{aligned} & \sin^2 \alpha + \sin^2 \beta - \sin^2 \gamma \\ &= \sin^2 \alpha + \sin(\beta + \gamma) \sin(\beta - \gamma) \\ &= \sin^2 \alpha + \sin(\pi - \alpha) \sin(\beta - \gamma) \\ &= \sin \alpha [\sin \alpha + \sin(\beta - \gamma)] \\ &= \sin \alpha [\sin(\beta + \gamma) + \sin(\beta - \gamma)] \\ &= 2 \sin \alpha \sin \beta \cos \gamma \end{aligned}$$

8. (B)

$$\begin{aligned} A + B + C &= \pi \\ \therefore \tan\left(\frac{A+B}{2}\right) &= \tan\left(\frac{\pi}{2} - \frac{C}{2}\right) \\ \Rightarrow \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2} \tan\frac{B}{2}} &= \cot\frac{C}{2} \\ \Rightarrow \frac{\frac{1}{3} + \frac{2}{3}}{1 - \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)} &= \cot\frac{C}{2} \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{9}{7} &= \cot\frac{C}{2} \\ \Rightarrow \tan\frac{C}{2} &= \frac{7}{9} \end{aligned}$$

Concept Fusion

1. (A)

Here, $(\cot \alpha_1)(\cot \alpha_2) \dots (\cot \alpha_n) = 1$

$$\begin{aligned} \therefore \cos \alpha_1 \cdot \cos \alpha_2 \dots \cos \alpha_n &= \sin \alpha_1 \cdot \sin \alpha_2 \dots \sin \alpha_n \quad \dots(i) \\ \text{Now, } (\cos \alpha_1 \cdot \cos \alpha_2 \dots \cos \alpha_n)^2 &= (\cos \alpha_1 \cdot \cos \alpha_2 \dots \cos \alpha_n) \\ &\quad (\cos \alpha_1 \cdot \cos \alpha_2 \dots \cos \alpha_n) \\ &= (\cos \alpha_1 \cdot \cos \alpha_2 \dots \cos \alpha_n) \\ &\quad (\sin \alpha_1 \cdot \sin \alpha_2 \dots \sin \alpha_n) \quad \dots[\text{From (i)}] \\ &= \frac{1}{2^n} \sin 2\alpha_1 \cdot \sin 2\alpha_2 \dots \sin 2\alpha_n \end{aligned}$$

$\dots[\because \sin 2A = 2 \sin A \cos A]$

But each of $\sin 2\alpha_i \leq 1$

$$\therefore (\cos \alpha_1 \cdot \cos \alpha_2 \dots \cos \alpha_n)^2 \leq \frac{1}{2^n}$$

But each of $\cos \alpha_i$ is positive

$$\therefore \cos \alpha_1 \cdot \cos \alpha_2 \dots \cos \alpha_n \leq \sqrt{\frac{1}{2^n}} = \frac{1}{2^{\frac{n}{2}}}$$

Scan the adjacent QR code in *Quill - The Padhai App* to view the solutions for questions from 2004 to 2021.





AVAILABLE BOOKS FOR COMPETITIVE EXAMINATIONS

● For NEET-UG & JEE (Main) Exam

ABSOLUTE SERIES

- Physics Vol - I & II
- Chemistry Vol - I & II
- Mathematics Vol - I & II
- Biology Vol - I & II

CHALLENGER SERIES

- Physics Vol - I & II
- Chemistry Vol - I & II
- Mathematics Vol - I & II
- Biology Vol - I & II

PSP SERIES (37 YEARS) (PREVIOUS SOLVED PAPERS)

- Physics
- Chemistry
- Biology

PSP SERIES (12 YEARS) (PREVIOUS SOLVED PAPERS)

- Physics
- Chemistry
- Biology

NEET-UG TEST SERIES

- Physics
- Chemistry
- Biology

ADDITIONAL BOOKS

- NEET-UG 10 Mock Tests With Answer Key & Hints
- Previous 12 Years NEET Solved Papers With Solutions
- JEE MAIN Numerical Value Type Questions (NVT)

● For MHT-CET Exam

STD. XI & XII TRIUMPH SERIES

- Physics
- Chemistry
- Mathematics
- Biology

SOLUTIONS TO MCQs

- Physics Solutions to MCQs
- Chemistry Solutions to MCQs
- Mathematics Solutions to MCQs
- Biology Solutions to MCQs

MHT-CET TEST SERIES

- Physics With Answer Key & Solutions
- Chemistry With Answer Key & Solutions
- Mathematics With Answer Key & Solutions
- Biology With Answer Key & Solutions

PSP SERIES (26 YEARS) (PREVIOUS SOLVED PAPERS)

- Physics
- Chemistry
- Mathematics
- Biology

ADDITIONAL BOOKS

- MHT-CET PCB Solved Papers 2024
- MHT-CET PCM Solved Papers 2024
- MHT-CET 10 Model Question Papers (Physics, Chemistry, Biology)
- MHT-CET 10 Model Question Papers (Physics, Chemistry, Mathematics)
- MHT-CET 22 Model Question Papers (Physics, Chemistry, Biology)
- MHT-CET 22 Model Question Papers (Physics, Chemistry, Mathematics)
- MHT-CET 22 Model Question Papers (Physics, Chemistry, Mathematics, Biology)

PSP SERIES (10 YEARS) (PREVIOUS SOLVED PAPERS)

- Physics
- Chemistry
- Mathematics
- Biology

Visit Our Website

Published by:

Target Publications® Pvt. Ltd.
Transforming lives through learning



Explore our range of
MHT-CET Books