Written as per the syllabus prescribed by the National Council of Educational Research and Training (NCERT).

CBSE
CLASS X
MATHEMATICS

Salient Features

- Extensive coverage of the syllabus in an effortless and easy to grasp format.
- Neat and labelled diagrams.
- ‘Things to Remember’ highlights important facts.
- Variety of additional problems for practice.
- Questions from previous years board papers have been solved.
- Memory Maps at the end of each chapter to facilitate quick revision.
- Sample Test Paper at the end of each chapter designed for student’s Self Assessment.
- Two Model Question Papers according to the latest paper pattern.

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01. Real Numbers

Euclid's Division Lemma

Given positive integers a and b, there exist unique integers q and r satisfying

$a = bq + r \; ; 0 \le r < b$

where 'a' is dividend, 'b' is divisor, 'q' is quotient and r is the remainder.

Note: q and r can also be zero.

Examples:

Consider the following pair of integers:

i. 29, 8

Here, a = 29 and b = 8

By using Euclid's Division lemma,

$a = bq + r \; ; 0 \le r < b$

i.e., $29 = 8 \times 3 + 5 \; ; 0 \le 5 < 8$

$$\ldots\left[\begin{array}{r} 3 \leftarrow \text{quotient (q)} \\ \text{divisor (b)} \longrightarrow 8\overline{)29} \leftarrow \text{dividend (a)} \\ -24 \\ \hline 5 \leftarrow \text{remainder (r)} \end{array}\right]$$

ii. 77, 7

Here, a = 77 and b = 7

By using Euclid's Division lemma,

$a = bq + r \; ; 0 \le r < b$

i.e., $77 = 7 \times 11 + 0 \; ; 0 \le 0 < 7$

$$\ldots\left[\begin{array}{r} 11 \leftarrow \text{quotient (q)} \\ \text{divisor (b)} \longrightarrow 7\overline{)77} \leftarrow \text{dividend (a)} \\ -77 \\ \hline 0 \leftarrow \text{remainder (r)} \end{array}\right]$$

iii. 9, 12

Here, a = 9 and b = 12

By using Euclid's Division lemma,

$a = bq + r \; ; 0 \le r < b$

i.e., $9 = 12 \times 0 + 9 \; ; 0 \le 9 < 12$

$$\ldots\left[\begin{array}{r} 0 \leftarrow \text{quotient (q)} \\ \text{divisor (b)} \longrightarrow 12\overline{)9} \leftarrow \text{dividend (a)} \\ -0 \\ \hline 9 \leftarrow \text{remainder (r)} \end{array}\right]$$

Euclid's Division Algorithm:

Euclid's division algorithm is a technique to compute the Highest Common Factor (HCF) of two given positive integers.

Euclid's Division Algorithm to find HCF of two positive integers 'a' and 'b' (a > b):

Step I: By Euclid's division lemma, find whole numbers 'q' and 'r' where $a = bq + r \; ; 0 \le r < b$

Step II: If r = 0, the HCF is b. If $r \neq 0$, apply the division lemma to b and r.

Step III: Continue the process till the remainder is zero. When the remainder is zero the divisor at that stage is the required HCF.

For the above algorithm HCF(a, b) = HCF(b, r)

Example:

Use Euclid's division algorithm to find the HCF of 1467 and 453.

Solution:

Step I: Apply Euclid's division lemma to 1467 and 453,

$1467 = 453 \times 3 + 108$

$$\ldots\left[\begin{array}{r} 3 \leftarrow q \\ b \rightarrow 453\overline{)1467} \leftarrow a \\ -1359 \\ \hline 108 \leftarrow r \end{array}\right]$$

Step II: Since $r \neq 0$

∴ apply Euclid's division lemma to 453 and 108,

$453 = 108 \times 4 + 21$

$$\ldots\left[\begin{array}{r} 4 \leftarrow q \\ b \rightarrow 108\overline{)453} \leftarrow a \\ -432 \\ \hline 21 \leftarrow r \end{array}\right]$$

Step III: Again, $r \neq 0$

∴ apply Euclid's division lemma to 108 and 21,

$108 = 21 \times 5 + 3$

$$\ldots\left[\begin{array}{r} 5 \\ 21\overline{)108} \\ -105 \\ \hline 3 \end{array}\right]$$

Step IV: Apply Euclid's division lemma to 21 and 3,

$21 = 3 \times 7 + 0$

$$\ldots\left[\begin{array}{r} 7 \\ 3\overline{)21} \\ -21 \\ \hline 0 \end{array}\right]$$

Since, r = 0

∴ HCF(1467, 453) = 3

∴ 3 = HCF(21, 3) = HCF(108, 21)
= HCF(453, 108) = HCF(1467, 453)

Things to Remember
❖ Euclid's division algorithm can be extended for all integers except zero i.e., $b \neq 0$.

NCERT Exercise 1.1

1. Use Euclid's division algorithm to find the HCF of:

i. 135 and 225 ii. 196 and 38220

iii. 867 and 255

Solution:

i. Since 225 > 135, we apply the division lemma to 225 and 135, to get

$225 = 135 \times 1 + 90$

Since the remainder $90 \neq 0$, we apply the division lemma to 135 and 90, to get

$135 = 90 \times 1 + 45$

We consider the new divisor 90 and the new remainder 45, and apply the division lemma to get

$90 = 45 \times 2 + 0$

As the remainder is zero, we stop.

Since, the divisor at this stage is 45, the HCF of 135 and 225 is 45.

ii. Since $38220 > 196$, we apply the division lemma to 38220 and 196, to get

$38220 = 196 \times 195 + 0$

As the remainder is zero, we stop.

Since the divisor at this stage is 196, the HCF of 196 and 38220 is 196.

iii. Since $867 > 255$, we apply the division lemma to 867 and 255, to get

$867 = 255 \times 3 + 102$

Since the remainder $102 \neq 0$, we apply the division lemma to 255 and 102, to get

$255 = 102 \times 2 + 51$

We consider the new divisor 102 and the new remainder 51, and apply the division lemma to get

$102 = 51 \times 2 + 0$

As the remainder is zero, we stop.

Since the divisor at this stage is 51, the HCF of 867 and 255 is 51.

---

2. Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer. ***[CBSE 2014, 2016]***

Solution:

Let a be any positive integer and b = 6.

Then, by Euclid's algorithm, a = 6q + r for some integer $q \geq 0$ and r = 0,1,2,3,4,5 because $0 \leq r \leq 6$.

$\therefore$ a = 6q or 6q + 1 or 6q + 2 or 6q + 3 or 6q + 4 or 6q + 5

Here, a cannot be 6q or 6q + 2 or 6q + 4, as they are divisible by 2.

6q + 1

6 is divisible by 2 but 1 is not divisible by 2.

6q + 3

6 is divisible by 2 but 3 is not divisible by 2.

6q + 5

6 is divisible by 2 but 5 is not divisible by 2.

Since, 6q + 1, 6q + 3, 6q + 5 are not divisible by 2, they are odd numbers.

Therefore, any odd integer is of the form

6q + 1, or 6q + 3, or 6q + 5.

---

3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution:

HCF(616, 32) will give the maximum number of columns in which they can march.

Let us use Euclid's algorithm, to find the HCF.

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

$\therefore$ the HCF of 616 and 32 is 8.

$\therefore$ the maximum number of columns in which they can march is 8.

---

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

[Hint: Let x be any positive integer then it is of the form 3q, 3q + 1 or 3q + 2. Now square each of these and show that they can be rewritten in the form 3m or 3m + 1.]

[CBSE 2015]

Solution:

Let x be any positive integer and b = 3.

Then, by Euclid's division lemma, $x = 3q + r$ for some integer $q \geq 0$ and

r = 0,1,2 because $0 \leq r < 3$

$\therefore$ $x = 3q$ or $3q + 1$ or $3q + 2$

When $x = 3q$,

$(x)^2 = (3q)^2 = 9q^2$

$= 3(3q^2)$

= 3m, where m is a integer

When $x = 3q + 1$

$(x)^2 = (3q + 1)^2 = 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1$

= 3m + 1, where m is a integer

When $x = 3q + 2$,

$(x)^2 = (3q + 2)^2 = 9q^2 + 12q + 4$

$= 3(3q^2 + 4q + 1) + 1$

= 3m + 1, where m is a integer

$\therefore$ the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

---

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Solution:

Let a be any positive integer and b = 3.

Then, by Euclid's division lemma, a = 3q + r for some integer $q \geq 0$ and

r = 0,1,2 because $0 \leq r < 3$

$\therefore$ a = 3q or 3q + 1 or 3q + 2

When a = 3q,

$a^3 = (3q)^3 = 27q^3$

$= 9(3q^3)$

= 9 m, where m is a integer

When a = 3q + 1,

$a^3 = (3q + 1)^3 = 27q^3 + 27q^2 + 9q + 1$

$= 9(3q^3 + 3q^2 + q) + 1$

= 9m + 1, where m is a integer

When $a = 3q + 2$,
$a^3 = (3q + 2)^3 = 27q^3 + 54q^2 + 36q + 8$
$= 9(3q^3 + 6q^2 + 4q) + 8$
$= 9m + 8$, where m is a integer
$\therefore$ the cube of any positive integer is of the form $9m$ or $9m + 1$ or $9m + 8$.

Problems based on Exercise 1.1

1. Using Euclid's division algorithm, find the HCF of 240 and 228. ***[CBSE 2012]***

Solution:
By Euclid's division algorithm,
$240 = 228 \times 1 + 12$
$228 = 12 \times 19 + 0$
$\therefore$ HCF (240, 228) = 12

---

2. Find the HCF of 180, 252 and 324 by Euclid's Division algorithm. ***[CBSE 2016]***

Solution:
By Euclid's division algorithm,
$324 = 252 \times 1 + 72$
$252 = 72 \times 3 + 36$
$72 = 36 \times 2 + 0$
$\therefore$ HCF (324, 252) = 36
$180 = 36 \times 5 + 0$
$\therefore$ HCF (180, 36) = 36
$\therefore$ HCF (324, 252, 180) = 36

---

3. Find the HCF by Euclid's division algorithm of the numbers 92690, 7378 and 7161. ***[CBSE 2013]***

Solution:
By Euclid's division algorithm,
$92690 = 7378 \times 12 + 4154$
$7378 = 4154 \times 1 + 3224$
$4154 = 3224 \times 1 + 930$
$3224 = 930 \times 3 + 434$
$930 = 434 \times 2 + 62$
$434 = 62 \times 7 + 0$
$\therefore$ HCF (92690, 7378) = 62
$7161 = 62 \times 115 + 31$
$62 = 31 \times 2 + 0$
$\therefore$ HCF(7161, 62) = 31
$\therefore$ HCF (92690, 7378, 7161) = 31

---

4. Using Euclid's division algorithm, find whether the pair of numbers 231, 396 are coprime or not.

Solution:
By Euclid's division algorithm,
$396 = 231 \times 1 + 165$
$231 = 165 \times 1 + 66$
$165 = 66 \times 2 + 33$
$66 = 33 \times 2 + 0$
$\therefore$ HCF(231, 396) = 33
$\therefore$ the numbers are not coprime.

5. Express the HCF of numbers 72 and 124 as a linear combination of 72 and 124. ***[CBSE 2016]***

Solution:
Since, $124 > 72$
By Euclid's division algorithm,
$124 = 72 \times 1 + 52$
$72 = 52 \times 1 + 20$
$52 = 20 \times 2 + 12$
$20 = 12 \times 1 + 8$
$12 = 8 \times 1 + 4$ … (i)
$8 = 4 \times 2 + 0$
$\therefore$ HCF (124, 72) = 4
From (i),
$4 = 12 - 8 \times 1$
$= 12 - (20 - 12 \times 1)$ …. [$\because 8 = 20 - 12 \times 1$]
$= 12 - 20 + 12 \times 1$
$= 12 \times 2 - 20$
$= (52 - 20 \times 2) \times 2 - 20$ …. [$\because 12 = 52 - 20 \times 2$]
$= 52 \times 2 - 20 \times 4 - 20$
$= 52 \times 2 - 20 \times 5$
$= 52 \times 2 - (72 - 52 \times 1) \times 5$ …. [$\because 20 = 72 - 52 \times 1$]
$= 52 \times 2 - 72 \times 5 + 52 \times 5$
$= 52 \times 7 - 72 \times 5$
$= (124 - 72 \times 1) \times 7 - 72 \times 5$ …. [$\because 52 = 124 - 72 \times 1$]
$= 124 \times 7 - 72 \times 7 - 72 \times 5$
$= 124 \times 7 - 72 \times 12$
$= 72\,m + 124\,n$,
where $m = -12$ and $n = 7$

---

6. Two tankers contain 850 litres and 680 litres of petrol. Find the maximum capacity of a container which can measure the petrol of each tanker in exact number of times. ***[CBSE 2012]***

Solution:
HCF(850, 680) will give the maximum capacity of container.
$850 = 680 \times 1 + 170$
$680 = 170 \times 4 + 0$
$\therefore$ HCF (850, 680) = 170
$\therefore$ the maximum capacity of a container which can measure the petrol of each tanker in exact number of times is 170 litres.

---

7. A sweetseller has 420 kaju barfis and 130 badam barfis. She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the maximum number of barfis that can be placed in each stack for this purpose?

Solution:

HCF(420, 130) will give the maximum number of barfis that can be placed in each stack.

By Euclid's division algorithm,

$420 = 130 \times 3 + 30$

$130 = 30 \times 4 + 10$

$30 = 10 \times 3 + 0$

$\therefore$ HCF(420, 130) = 10

$\therefore$ the sweetseller can make stacks of 10 for both kinds of barfi.

---

8. The length, breadth and height of a room are 8m 25 cm, 6m 75 cm and 4 m 50 cm respectively. Find the length of the longest rod that can measure the three dimensions of the room exactly. ***[CBSE 2012]***

Solution:

Since, 1m = 100 cm

$\therefore$ 8 m 25 cm = 825 cm

6 m 75 cm = 675 cm

4 m 50 cm = 450 cm

HCF(825, 675, 450) will give the length of the longest rod.

$825 = 675 \times 1 + 150$

$675 = 150 \times 4 + 75$

$150 = 75 \times 2 + 0$

$\therefore$ HCF(825, 675) = 75

$450 = 75 \times 6 + 0$

$\therefore$ HCF(450, 75) = 75

$\therefore$ HCF (825, 675, 450) = 75

$\therefore$ the length of the longest rod is 75 cm.

NCERT Exemplar

1. Write whether every positive integer can be of the form 4q + 2, where q is an integer. Justify your answer.

Solution:

No, every positive integer cannot be only of the form 4q + 2.

Justification:

Let a be any positive integer. Then by Euclid's division lemma, we have

$a = bq + r$, where $0 \le r < b$

Putting b = 4, we get

$a = 4q + r$, where $0 \le r < 4$

Hence, a positive integer can be of the form, 4q, 4q + 1, 4q + 2 and 4q + 3.

---

2. "The product of two consecutive positive integers is divisible by 2". Is this statement true or false? Give reasons.

Solution:

True.

Justification:

Let a, a + 1 be two consecutive positive integers.

By Euclid's division lemma, we have

$a = bq + r$, where $0 \le r < b$

For b = 2 , we have

$a = 2q + r$, where $0 \le r < 2$ ...(i)

Putting r = 0 in (i), we get

a = 2q, which is divisible by 2.

a + 1 = 2q + 1, which is not divisible by 2.

Putting r = 1 in (i), we get

a = 2q + 1, which is not divisible by 2.

a + 1 = 2q + 2, which is divisible by 2.

Thus for $0 \le r < 2$, one out of every two consecutive integers is divisible by 2.

$\therefore$ The product of two consecutive positive integers is divisible by 2.

---

3. "The product of three consecutive positive integers is divisible by 6". Is this statement true or false? Justify your answer.

Solution:

True.

Justification:

At least one out of every three consecutive positive integers is divisible by 2.

$\therefore$ The product of three consecutive positive integers is divisible by 2.

At least one out of every three consecutive positive integers is divisible by 3.

$\therefore$ The product of three consecutive positive integers is divisible by 3.

Since the product of three consecutive positive integers is divisible by 2 and 3.

$\therefore$ It is divisible by 6 also.

---

4. Write whether the square of any positive integer can be of the form 3m + 2, where m is a natural number. Justify your answer.

Solution:

No.

Justification:

Let a be any positive integer. Then by Euclid's division lemma, we have

$a = bq + r$, where $0 \le r < b$

For b = 3, we have

$a = 3q + r$, where $0 \le r < 3$ ...(i)

$\therefore$ The numbers are of the form 3q, 3q + 1 and 3q + 2.

$\therefore$ $(3q)^2 = 9q^2 = 3(3q^2)$

$= 3m$, where m is a integer.

$(3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1$

$= 3m + 1$,

where m is a integer.

$(3q + 2)^2 = 9q^2 + 12q + 4$,

which cannot be expressed in the form 3m + 2.

$\therefore$ Square of any positive integer cannot be expressed in the form 3m + 2.